ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 9, Issue 1, January 2020

The Influence of Eccentricity Ratio on the Kinematic Characteristics of Elliptical Gear Hobbing

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Abstract-According to the velocity linkage relationship of non-circular gear hobbing processing, the acceleration equations are deduced. The kinematic characteristics of elliptical gears are studied by the method of equal polar angle. The MATLAB software is used to obtain the function curves with respect to time of the velocity and acceleration of the hob and the work piece in the process of elliptical gear, and the influence on the kinematic characteristics is analyzed by comparing eccentricity ratio. Results show that the smaller the eccentricity ratio is, the smaller the fluctuation of the velocity and acceleration is, the more beneficial the improvement of the gear machining accuracy.

Key words: **elliptical gear, eccentricity ratio, kinetic characteristics, MATLAB.**

I. INTRODUCTION

Gear is one of the most important parts in mechanical transmission, especially non-circular gear, has been widely favored because of its smooth transmission, high bearing capacity, compact structure and other characteristics. But complex shape, design and machining difficulties of non-circular gear lead to the slow progress [1-2]. In China, some scholars have done some related research, for example, the mathematical models of the non-circular gear hobbing processing were deduced by Hu Chibing and Tan Weiming [3-5], and electronic gear box and pulse synthesis circuit technology were applied in machine tool, which can hob non-circular gear. However, there is little research on the analysis of the kinematic characteristics of gear hobbing. In this paper, according to the relevant theoretical knowledge of non-circular gear hobbing, the kinematic characteristics of elliptical gear hobbing are analyzed by changing the eccentricity ratio, which provides theoretical basis for the selection of eccentricity for the elliptical gear in the design phase.

II. THE NON-CIRCULAR GEAR HOBBING KINEMATIC RELATIONSHIP

According to the hobbing principle and the forming principle of non-circular gear tooth profile, NC hobbing of non-circular gear can be abstracted as the process that hob pitch line(non-slip tool) rolls along the non-circular gear's pitch curve without sliding. As shown in Figure 1, the installation center of the work piece and the center of rotation are O1, and gear pitch curve of the work piece is based on O1. Besides, θ is polar angle, and r is polar radius. If cutting point is P, μ is the angle between the tangent and the polar radius, and the mould of r is denoted by r.

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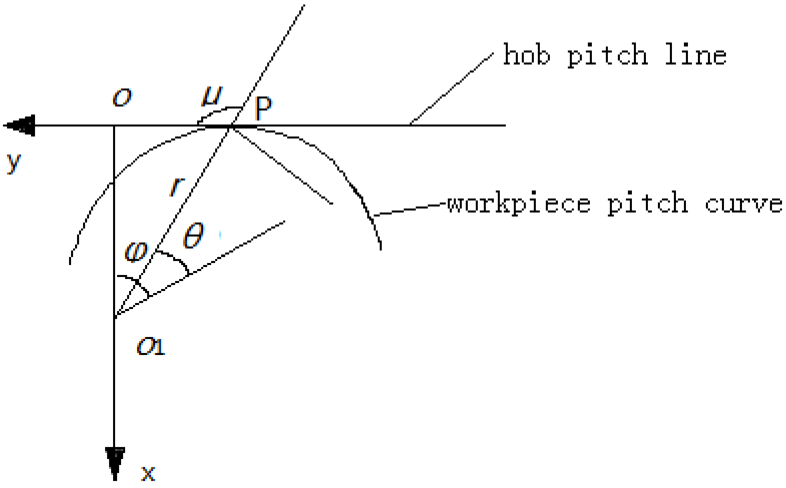

Fig. 1 Schematic drawing of non-circular gear hobbing

The movement model of the non-circular gear hobbing represented by the displacement form is known by [1].And the equations are

$$\Psi = \frac{2}{\mathrm{Km}}\left(\int_0^{\theta}\sqrt{r^2+(\mathrm{d}r/\mathrm{d}\theta)^2}\,\mathrm{d}\theta - \frac{r(\mathrm{d}r/\mathrm{d}\theta)}{\sqrt{r^2+(\mathrm{d}r/\mathrm{d}\theta}}\right.$$
$$\varphi = \theta + \arctan\left(-\frac{\mathrm{d}r/\mathrm{d}\theta}{r}\right)$$
$$x = \frac{r^2}{\sqrt{r^2+(\mathrm{d}r/\mathrm{d}\theta)^2}} \quad (1)$$

Where Ψ,K,m, φ and x are hob corner, numbers of hob head, work piece modulus, Work piece rotation, the center distance between hob pitch line and work piece pitch curve, respectively.

The key to the non-circular gear hobbing is that the hob pitch line and the work piece pitch curve must be kept in a pure rolling with no relative sliding. When the Straight teeth non-circular gear is hobbed, rotary motion of hob, relative motion along the X axis, feed motion along the Y axis and the rotation motion of the work piece must be linked to control, however, the feed motion along the Z axis is independent. According to the displacement form of the kinematic model, the velocity form of the linkage relationships are described and deduced. Reference [3] found that the formula (1) should be taken the derivative of the time to get some relationships. Hob angular velocity equation is

$$\omega_b = \frac{\mathrm{d}\Psi}{\mathrm{d}t} = \frac{2}{\mathrm{Km}} \cdot \frac{r^4+2r^2(\mathrm{d}r/\mathrm{d}\theta)^2 - r^3(\mathrm{d}^2r/\mathrm{d}\theta^2}{[r^2+(\mathrm{d}r/\mathrm{d}\theta)^2]^{\frac{3}{2}}} \quad (2)$$

Work piece angular velocity equation is

$$\omega_c = \frac{\mathrm{d}\varphi}{\mathrm{d}t} = \frac{\mathrm{d}\varphi}{\mathrm{d}\theta} \cdot \frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{\mathrm{d}\theta}{\mathrm{d}t} \cdot \frac{r^2+2(\mathrm{d}r/\mathrm{d}\theta)^2 - r(\mathrm{d}^2}{r^2+(\mathrm{d}r/\mathrm{d}\theta)^2} \quad (3)$$

By formula (3) ,the equation is

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$$\frac{d\theta}{dt}=\frac{r^2+(dr/d\theta)^2}{r^2+2(dr/d\theta)^2-r(d^2r/d\theta^2)} \tag{4}$$

The velocity equation of the hob and the work piece in the center distance which is along the X axis is

$$v_x=\frac{dx}{dt}=\frac{r(dr/d\theta)[r^2+2(dr/d\theta)^2-r(d^2r/d\theta^2)]}{[r^2+(dr/d\theta)^2]^{\frac{3}{2}}} \tag{5}$$

The formula (3) and formula (4) are simplified to get the relationship between ω_c and v_x,and the equation is

$$v_x=\frac{r}{\sqrt{r^2+(dr/d\theta)^2}}\cdot\frac{dr}{d\theta} \tag{6}$$

Taking the displacement compensation scheme of the channeling knife as an example, hob pitch line velocity in the tangential direction of the cutting point and the work piece velocity in the direction of Y axis must be consistent by [4].And Channeling velocity equation is

$$v_y=\frac{Km}{2}\cdot\omega_b=\frac{r^2}{\sqrt{r^2+(dr/d\theta)^2}} \tag{7}$$

By formula (5) ,the angular velocity equation is

$$\omega_b=\frac{2}{km}\cdot\frac{r^2}{\sqrt{r^2+(dr/d\theta)^2}} \tag{8}$$

Besides, work piece velocity equation is

$$v_c= \tag{9}$$

Assume that the angular velocity of the work piece is determined, and then the acceleration kinematic models can be deduced by the velocity linkage relationships. The acceleration of the hob and the work piece in the direction of X axis is deduced and simplified by formula (6),and the equation is

$$a_x=\frac{dv_x}{dt}=\left(\frac{r}{\sqrt{r^2+(dr/d\theta)^2}}\right)'\cdot\frac{dr}{d\theta}\cdot\omega_c+\frac{r}{\sqrt{r^2+(dr/d\theta)^2}}\cdot\left(\frac{dr}{d\theta}\right)'\cdot\omega_c+\frac{r}{\sqrt{r^2+(dr/d\theta)^2}}\cdot\frac{dr}{d\theta}\cdot\omega_c'=$$
$$\frac{(dr/d\theta)^4+r^3(d^2r/d\theta^2)}{\sqrt{r^2+(dr/d\theta)^2}}\cdot\frac{{\omega_c}^2}{r^2+2(dr/d\theta)^2-r(d^2r/d\theta^2)} \tag{10}$$

The Channeling acceleration in the direction of Y axis is deduced and simplified by formula (7), and the equation is

$$a_y=\frac{dv_y}{dt}=\left(\frac{r^2}{\sqrt{r^2+(dr/d\theta)^2}}\right)'\cdot\omega_c+\frac{r^2}{\sqrt{r^2+(dr/d\theta)^2}}\cdot\omega_c'=r\cdot\frac{dr}{d\theta} \tag{11}$$

The work piece acceleration is deduced and simplified by formula (9), and the equation is

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$$a_c=\frac{dv_c}{dt}=r'\cdot\omega_c+r\cdot\omega_c'=\frac{dr}{d\theta}\cdot\frac{d\theta}{dt}\cdot\omega_c+r\cdot\omega_c'=\frac{r^2+(dr/d\theta)^2}{r^2+2(dr/d\theta)^2-r(d^2r/d\theta^2)}\cdot\frac{dr}{d\theta}\cdot \quad (12)$$

III. KINEMATIC CHARACTERISTIC ANALYSIS OF ELLIPTICAL GEAR HOBBING

In the following elliptical straight teeth gear processing, for example, its pitch curve equation is

$$r=\frac{a(1-}{1-ecc} \quad (13)$$

Where a,e,n and θ are the semimajor,the eccentricity ratio, the order number and the polar angle, respectively. The formula (13) needs to be taken the first derivative of θ and the equation is

$$r'=\frac{dr}{d\theta}=\frac{aen(e^2-1)}{[1-ecos(} \quad (14)$$

The formula (13) needs to be taken the second derivative of θ and the equation is

$$r''=\frac{d^2r}{d\theta^2}=\frac{2ae^2n^2\sin^2(n\theta)(e^2-1)}{[ecos(n\theta)-1]^3}+\frac{aen^2\cos(n\theta)(}{[ecos(n\theta)-} \quad (15)$$

The formula (13) and formula (14) are taken into equation (6), and then the equation is

$$v_x=\frac{a^2en\sin(n\theta)(e^2-1)^2}{\sqrt{\frac{a^2(e^2-1)^2}{[ecos(n\theta)-1]^2}+\frac{a^2e^2n^2\sin^2(n\theta)(e^2-1)^2}{[e\cos(n\theta)-1]^4}\cdot[e\cos(n\theta)-1]^2}} \quad (16)$$

The formula (13) and formula (14) are taken into equation (7), then the equation is

$$v_y=\frac{a^2(e^2-1)^2}{\sqrt{\frac{a^2(e^2-1)^2}{[e\cos(n\theta)-1]^2}+\frac{a^2e^2n^2\sin^2(n\theta)(e^2-1)^2}{[e\cos(n\theta)-1]^4}\cdot[e\cos(n\theta)-1]^2}} \quad (17)$$

The formula (13) is taken into equation (9) ,then the equation is

$$v_c=\frac{a(1-e^2)}{1-ecos(n} \quad (18)$$

The formula (13), formula (14) and formula (15) are taken into the corresponding formula (10), formula (11) and formula (12) respectively, then equations is

$$a_x=\frac{A^3\cdot C+B^2}{\sqrt{A^2+B}\cdot(A^2-A\cdot C+2B)} \quad (19)$$

$$a_y=\frac{en\sin(n\theta)}{e\cos(n\theta)-1}\cdot A^2 \quad (20)$$

$$a_c=\frac{en\sin(n\theta)}{e\cos(n\theta)-1}\cdot\frac{A\cdot(A^2+B)}{A^2-A\cdot C+2B} \quad (21)$$

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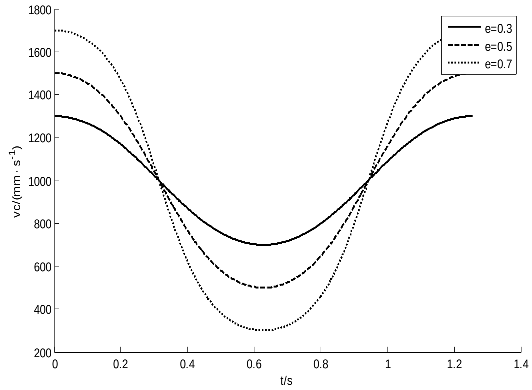
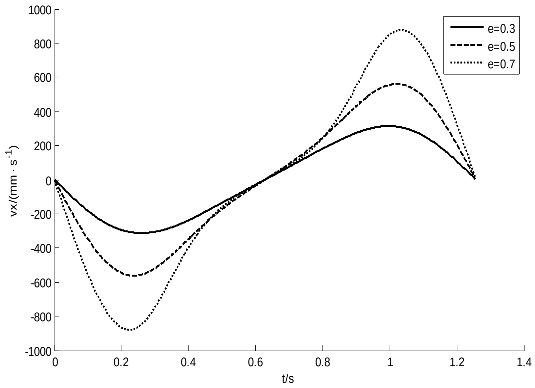
Where, $A=\frac{a(e^2-1)}{e\cos(n\theta)-1}$, $B=\frac{a^2e^2n^2\sin^2(n\theta)(e^2-1)^2}{(e\cos(n\theta)-1)^4}$, $C=\frac{2ae^2n^2\sin^2(n\theta)(e^2-1)}{(e\cos(n\theta)-1)^3}+\frac{aen^2\cos(n\theta)(e^2-1)}{(e\cos(n\theta)-1)^2}$.

The first order elliptic straight teeth gear is the simplest type of non-circular gear, so it is taken as an example to analyze the effect of the change of eccentric ratio on its kinematic characteristics. Other basic parameters are that a is 200mm ,and ω_c is 5 rad/s.

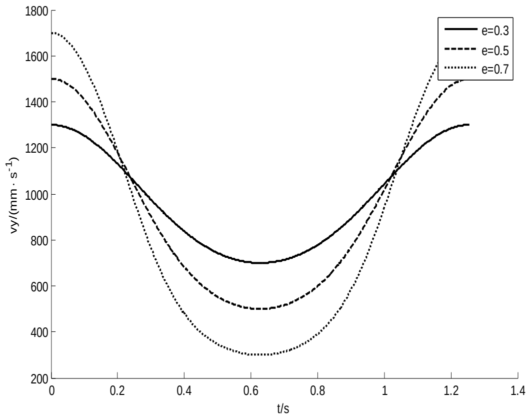
After analyzing the relationship between the hob and the workpiece, and determining parameters, MATLAB software is used to program and draw function curves. When the e are 0.3, 0.5, 0.7, the velocity and acceleration curves are shown in Figure 2 and Figure 3.



(a) Work piece velocity curves



(b) Hob velocity curves in the direction of X axis



(c) Hob velocity curves in the direction of Y axis

Fig. 2 the velocity curves when e are 0.3, 0.5, 0.7

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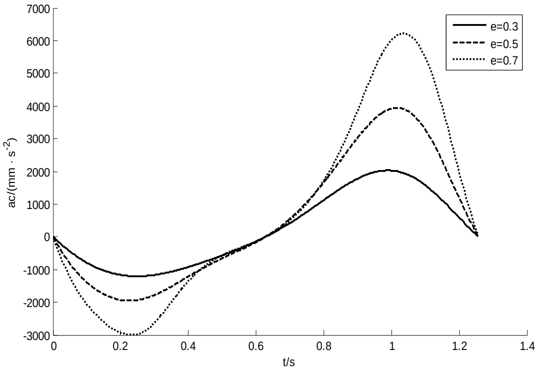
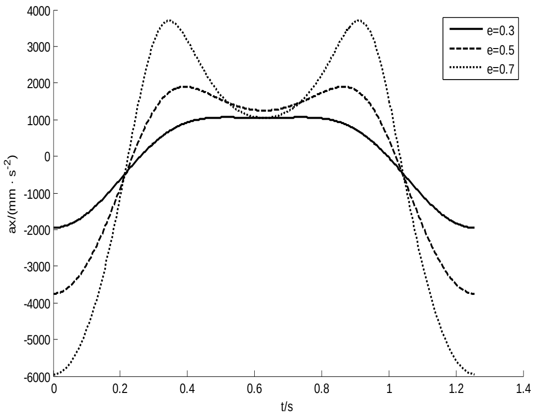
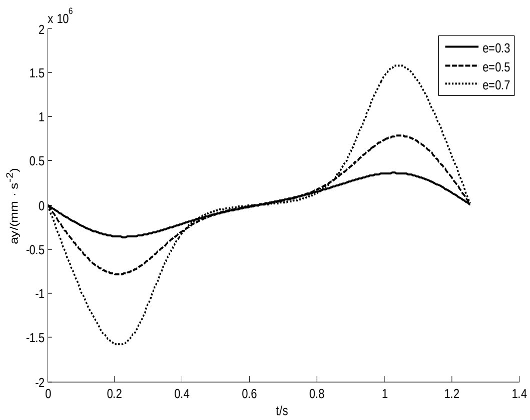
Figure 2 shows the velocity curves with respect to time. During the time of elliptical gear rotating around the shaft for a circle. It can be seen from the chart, the larger the eccentricity ratio is, the bigger the velocity fluctuation is, and it is not stable when processing, then the gear precision is not guaranteed. From Figure 2 (a) and Figure 2 (c), in the middle of the gear processing, the workpiece velocity and hob velocity in the direction of Y axis reach the lowest. In the range of the polar angle which is from 0 to 2π, when processing polar angle starts from zero, should be π at this time. At the same time, back to the initial point in the gear processing, the starting and end velocity is equal. From Figure 2 (b), velocities of all curves are equal to zero when is π, and starting velocity and ending velocity are zero. So it can be explained that there is a certain rule to be followed in the elliptical gear hobbing.



(a) work piece acceleration curves



(b) hob acceleration curves in the direction of X axis



(c) hob acceleration curves in the direction of Y axis

Fig. 3 the acceleration curves when e are 0.3,0.5,0.7

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International Journal of Engineering Science and Innovative Technology (IJESIT)
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As it can be seen from Figure 3, with the increase of the eccentricity ratio, the fluctuation of the acceleration curve gets bigger, which leads to the increase of the cutting force.Therefore,in the whole process, the influence of the vibration of the machine tool is increased. On the one hand, it reduces the life of the machine tool; on the other hand, the accuracy of the gear is greatly cut down.

IV. CONCLUSION

In summary, the smaller the eccentricity ratio is, the smaller the velocity and the acceleration of the hob spindle are. Besides, the more stable motion of the hobbing machine is, the more conducive to improve the gear machining accuracy is. Therefore, through the study of the kinematic characteristics of elliptical gear, the effect on the eccentricity ratio can provide reference for the selection of eccentric ratio at the design stage.

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