ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 8, Issue 5, September 2019

Numerical study of fluids concentration levels in industrial mixing process

Janet Chebet Langat[1], Johanah Kibet Sigey [2], Jeconia Okelo Abonyo[3], Emmah Marigi [4] & Bethsheba Menge [5]

1, 2 &3: Jomo Kenyatta University of Agriculture and Technology (JKUAT), Nairobi, Kenya
4: Dedan Kimathi University of Science and Technology (DeKUT), Nyeri, Kenya.
5: Department of Mathematics and Physics, Technical University of Mombasa, Kenya

ABSTRACT: The numerical investigation of concentration levels of fluids in industrial mixing process has been studied. The study assumes a two-dimensional laminar flow of an incompressible fluid mixture. The equations governing the flow of fluids in the mixing chamber are the equation of conservation of mass, equation of conservation of linear momentum, and advection-diffusion equation. The governing equations are non-dimensionalized to obtain the flow parameters. The resulting model equation is solved numerically using the finite difference method. The study found that the rotational and extensional local flows inside the mixing chamber dominate the mixing index and hence producing steady stirring. The study also found that the inertial force has a crucial impact on the mixing since it causes powerful vortex flow in the mixing chamber and diminishes the damping effect on diffusion, which leads to a higher mixing performance. The results obtained from this study are useful in manufacturing industries to provide new insights into mixing phenomena in both fluids and dyes as well as provide a solution to transport problems. The results will also shed light on how physical systems mix for different parameters for example, mixing in fluid flows and size segregation in granular flows.

Key Words: **Chaotic advection and diffusion, Granular flows, fluid diffusivity coefficient, fluids mixing process, fluid characteristic velocity.**

I. INTRODUCTION

Micro fluidic devices such as micro pump, micro mixer, micro valve, and even micro reactor have been widely utilized in micro-electromechanical systems and employed as powerful instruments to perform biological experiments. These microfluidic devices have been developed for a broad range of applications such as chemical engineering and genomic analysis. Effective mixing of various fluids is required in many mixing chambers. Mixing of fluids is a fundamental process in many natural and industrial flow fields. Batch mixers, similar to food mixers, are a primary method for the processing of many materials. However, there are inherent drawbacks to a batch mixing process (e.g. inconsistent mixture quality, residual voids and fissures, limited pot life, etc.). The study of these devices is complicated by several factors which include rheological properties of the fluids, complex geometries, and device scaling issues. Recent interest in developing more efficient, controllable mixing processes has focused attention on developing a better understanding of the physics of these devices.

II. LITERATURE REVIEW

Many researchers have investigated batch mixing configurations numerically while fluid properties and power consumption have also been a focus. Computational models have been utilized to study other mixer configurations. For instance, Youcefi et al. (1997) experimentally and computationally investigated the effect of fluid elasticity on the flow induced by a rotating flat plate impeller. In their work torque measurements were also made and used to determine the power number as a function of the fluid type and Reynolds number. It was found that the power number data collapsed onto a single curve for all Newtonian and non-Newtonian pseudo plastic fluids when the effective Reynolds number was used for the pseudo plastic fluids. Zhou et al. (2000) investigated the power consumption in a double planetary mixer with Newtonian and non-Newtonian fluids. It was found that the power curve for non-Newtonian fluids was found to collapse to the curve for Newtonian fluids when the Metzner-Otto Reynolds number was used for scaling. Anderson et al. (2000, 2001, and 2002) have successfully used the mapping method of Spencer and Wiley to study realistic mixing flows by advecting the boundaries of grid cells and then projecting the deformed grid onto the original one to obtain the mapping

ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 8, Issue 5, September 2019
matrix that solves the purely advective problem. Conor (2014) investigated numerically the advection-diffusion equation is to gain insights into mixing phenomena in both fluid and granular flows. The ratio of the finite-time Lyapunov exponents at the starting location for the two reacting species determines the final average concentration of each species, showing the importance of the details of the chaotic velocity field on advection-reaction-diffusion systems. Clifford et al. (2004) studied the effects of Reynolds number on the mixing in a simplified pin planetary mixer. Mixing patterns were derived using flow visualization and compared to compute fields. The mixing patterns were found to have a Reynolds number dependence. Ian and Changying (2018) performed experimental tests and computational fluid dynamics simulations to examine the flow characteristics of four impeller designs (anchor, saw-tooth, counter-flow and Rushton turbine), in achieving solution homogeneity. The impellers were used to mix potassium sulphate granules, from which values of electrical conductivity of the solution were measured and used to estimate the distribution pattern of dissolved solid concentrations within the vessel.

III. METHODOLOGY

Equation of continuity

This is derived from the principle of conservation of mass which states that under normal conditions matter can neither be created nor destroyed. The general form of continuity equation, in vector notation, is given by

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \left(\rho \vec{V} \right) = 0 \tag{1}$$

Equation (1) is essential in determining whether the flow is possible or not. If the continuity equation is satisfied, then the fluid flow is said to be possible. By performing the vector dot product, equation (1) becomes

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} + \frac{\partial (\rho w)}{\partial z} = 0 \tag{2}$$

Since the fluid is assumed to be incompressible (i.e., density is a constant) and the flow is two-dimensional, equation (2) reduces to the simpler form

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3}$$

Equation of conservation of linear momentum

This is derived from Newton's second law of motion which states that the rate of change of momentum of a body is equal to the resultant external force producing the change and takes place in the direction of the force. The general equation of motion is given by:

$$\rho \left[\frac{\partial \vec{V}}{\partial t} + \vec{V} \cdot \left(\vec{\nabla} \vec{V} \right) \right] = -\vec{\nabla} p + \mu \nabla^2 \vec{V} + \vec{F}_B \tag{4}$$

Advection-diffusion equation

In nature, transport occurs in fluids through the combination of advection and diffusion. The advection-diffusion equation is given by:

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) \tag{5}$$

ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 8, Issue 5, September 2019

Dimensionalized advection-diffusion equation

The dimensionless form of the advection-diffusion equation (5) is given by (Landau and Lifshitz, 1959);

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=\frac{1}{Pe}\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right) \quad (6)$$

The Peclet number (Pe) is a dimensionless parameter defined as the ratio heat transport by convection to heat transport by conduction. Thus, Pe is the product of Reynolds number (Re) and Prandtl number (Pr). It is expressed mathematically by

$$Pe=\frac{UL}{D} \quad (7)$$

Substituting equation (3.9) into (3.8) yields

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=\frac{D}{UL}\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right) \quad (8)$$

Discretization of governing equation

Equation (8) is discretized to study the effects of diffusivity (D) characteristic length (L) and characteristic velocity (U) on fluid concentration levels. The time derivative, $\frac{\partial C}{\partial t}$, is replaced by forward difference approximation while the space derivatives $\frac{\partial C}{\partial x},\frac{\partial C}{\partial y},\frac{\partial^2 C}{\partial x^2}$ and $\frac{\partial^2 C}{\partial y^2}$ are replaced by central difference approximation, as shown below.

$$\begin{aligned}
\frac{\partial C}{\partial t}&=\frac{C_{i,j}^{n+1}-C_{i,j}^{n}}{\Delta t}\\
\frac{\partial C}{\partial x}&=\frac{C_{i+1,j}^{n}-C_{i-1,j}^{n}}{2\Delta x}\\
\frac{\partial C}{\partial y}&=\frac{C_{i,j+1}^{n}-C_{i,j-1}^{n}}{2\Delta y}\\
\frac{\partial^2 C}{\partial x^2}&=\frac{C_{i+1,j}^{n}-2C_{i,j}^{n}+C_{i-1,j}^{n}}{(\Delta x)^2}\\
\frac{\partial^2 C}{\partial y^2}&=\frac{C_{i,j+1}^{n}-2C_{i,j}^{n}+C_{i,j-1}^{n}}{(\Delta y)^2}
\end{aligned} \quad (9)$$

Substituting these approximations (11) into equation (10), and letting $u=v=1$, we obtain

$$\frac{C_{i,j}^{n+1}-C^{n}{}_{i,j}}{\Delta t}+\frac{C_{i+1,j}^{n}-C^{n}{}_{i-1,j}}{2\Delta x}+\frac{C_{i,j+1}^{n}-C^{n}{}_{i,j-1}}{2\Delta y}=\frac{D}{UL}\left[\frac{C_{i+1,j}^{n}-2C_{i,j}^{n}+C_{i-1,j}^{n}}{(\Delta x)^2}+\frac{C_{i,j+1}^{n}-2C_{i,j}^{n}+C_{i,j-1}^{n}}{(\Delta y)^2}\right] \quad (10)$$

ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 8, Issue 5, September 2019

Taking $\phi = \frac{\Delta t}{(\Delta x)} = \frac{\Delta t}{(\Delta y)}$, $\mu = \frac{\Delta t}{(\Delta x)^2} = \frac{\Delta t}{(\Delta y)^2}$, $\xi = \frac{D}{UL}$, $\Delta x = \Delta y$ and multiplying equation (10) by $2\Delta t$ and re-arranging, we get

$$(\phi - \mu\xi)C^n_{i+1,j} - (\phi + \mu\xi)C^n_{i-1,j} + (2\mu\xi - 2)C^n_{i,j} = (\mu\xi - \phi)C^n_{i,j+1} + (\mu\xi + \phi)C^n_{i,j-1} - 2C^{n+1}_{i,j} \quad (11)$$

Taking $\Delta x = \Delta y = 0.1$, and $\Delta t = 0.01$, $\Rightarrow \phi = 0.1$ and $\mu = 2$ yields

$$(0.1 - 2\xi)C^n_{i+1,j} - (0.1 + 2\xi)C^n_{i-1,j} + (4\xi - 2)C^n_{i,j} = (2\xi - 0.1)C^n_{i,j+1} + (2\xi + \phi)C^n_{i,j-1} - 2C^{n+1}_{i,j} \quad (12)$$

The system of equations (12) are subjected to the following initial and boundary conditions

$$C(x,y,0) = 0, \quad t = 0 \quad (13)$$

$$C(0,y,t) = 0, \quad C(x,y,1) = e^t, \ t > 0, \ x \neq y \quad (14)$$

Applying the conditions (12) and (13) to the system (14), and

Substituting $\xi = \frac{D}{UL}$ yields

$$\begin{bmatrix} \left(\frac{4D}{UL}-2\right) & \left(0.1-\frac{2D}{UL}\right) & 0 & 0 & 0 & 0 \\ -\left(0.1+\frac{2D}{UL}\right) & \left(\frac{4D}{UL}-2\right) & \left(0.1-\frac{2D}{UL}\right) & 0 & 0 & 0 \\ 0 & -\left(0.1+\frac{2D}{UL}\right) & \left(\frac{4D}{UL}-2\right) & \left(0.1-\frac{2D}{UL}\right) & 0 & 0 \\ 0 & 0 & -\left(0.1+\frac{2D}{UL}\right) & \left(\frac{4D}{UL}-2\right) & \left(0.1-\frac{2D}{UL}\right) & 0 \\ 0 & 0 & 0 & -\left(0.1+\frac{2D}{UL}\right) & \left(\frac{4D}{UL}-2\right) & \left(0.1-\frac{2D}{UL}\right) \\ 0 & 0 & 0 & 0 & -\left(0.1+\frac{2D}{UL}\right) & \left(\frac{4D}{UL}-2\right) \end{bmatrix} \begin{bmatrix} C^0_{1,1} \\ C^0_{2,2} \\ C^0_{3,3} \\ C^0_{4,4} \\ C^0_{5,5} \\ C^0_{6,6} \end{bmatrix} = \begin{bmatrix} -5.4365 \\ -14.778 \\ -40.1707 \\ -109.196 \\ -296.826 \\ -806.857 \end{bmatrix} \quad (16)$$

Solving the above matrix equation (16), we get the solutions for profile values.

IV. RESULTS AND DISCUSSION

A. Effects of diffusion coefficient on fluid concentration level

The simulation results for the effects of varying diffusion coefficient (D) of the tracer (or fluid) on fluid concentration level are shown in Figure 2 below.

ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 8, Issue 5, September 2019

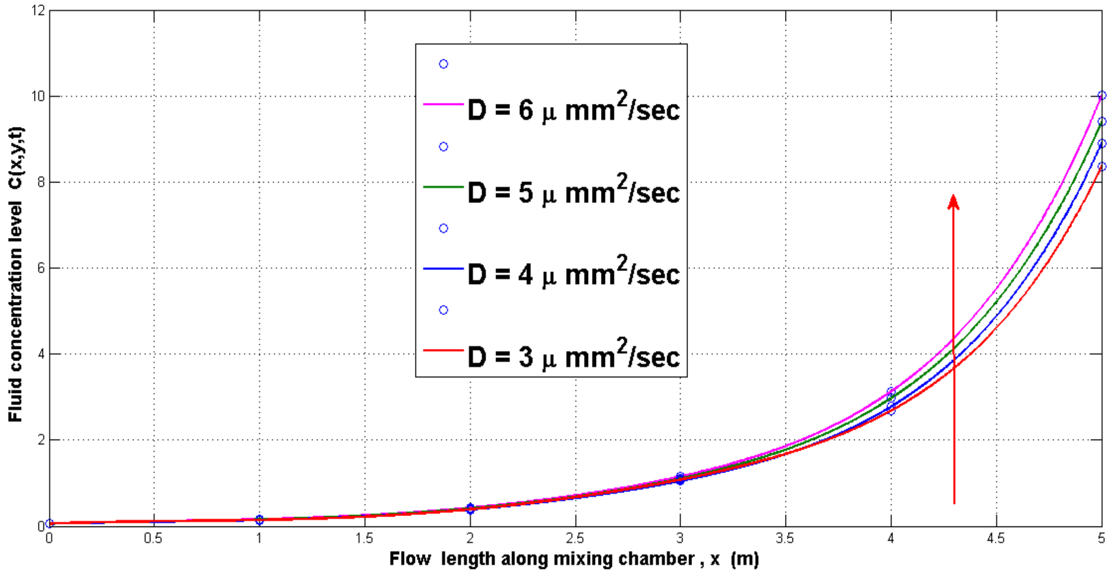

Fig 2: Graph of fluid concentration level against flow length along mixing chamber at varying diffusion coefficient

From Figure 2, it is observed that an increase in diffusion coefficient (D) causes an increase in the magnitude of the fluid concentration profiles. This is because diffusion affects the parameters of system pressure, component properties and balance time, which in turn affect the efficiency of displacement. The term molecular diffusion includes both the mass transfer diffusion and self-diffusion, which are quantitatively described by the diffusion coefficient. Mass transfer diffusion mainly occurs in the non-equilibrium condition of the chemical potential gradient.

B. Effects of Characteristic Length on Fluid Concentration level

The simulation results for the effects of varying characteristic length (L) of the mixing chamber on fluid concentration level are shown in Figure 3 below.

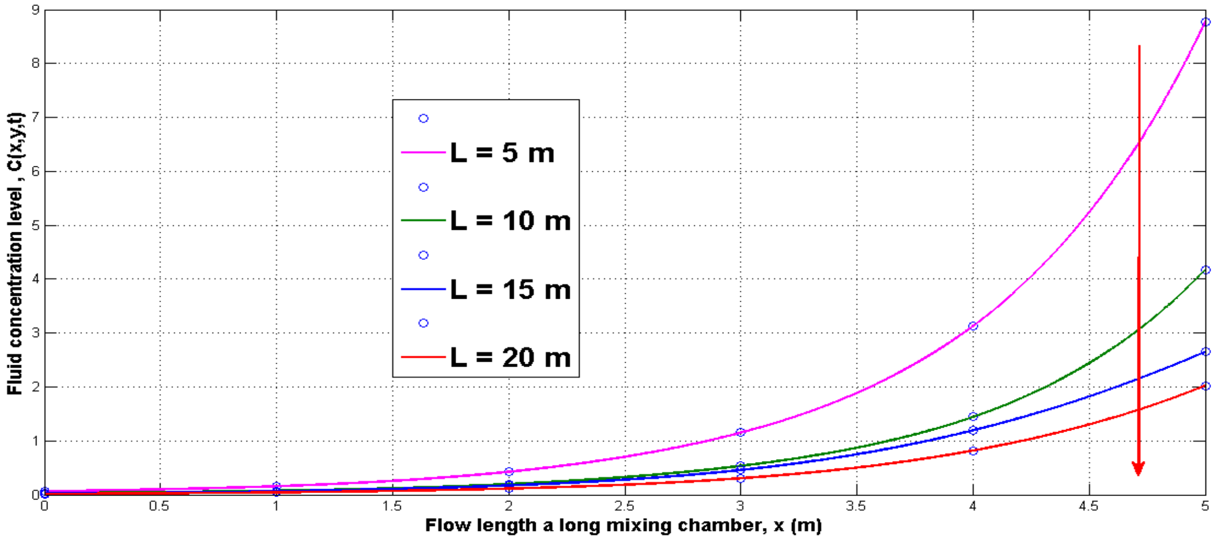

Fig 3: Graph of fluid concentration level against flow length along mixing chamber at varying chamber characteristic length

ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 8, Issue 5, September 2019
Figure 3, it is observed that an increase in the characteristic length (L) of the mixing chamber results in a decrease in the magnitude of the fluid concentration profiles. This is because the mixing at any location depends on the upstream (vortex) flow, which weakens with an increase in chamber length. Moreover, increasing the chamber length means that the contact areas between the fluids and the chamber are enlarged as well. Since the wall of the inlet channel is tangential to the wall of the mixing chamber and the inertial force accelerates the flow, the flowing fluids meet with strong inertia.

Effects of Characteristic Velocity on Fluid Concentration level

The simulation results for the effects of varying characteristic velocity (U) of the mixing chamber on fluid concentration level are shown in Figure 4 below.

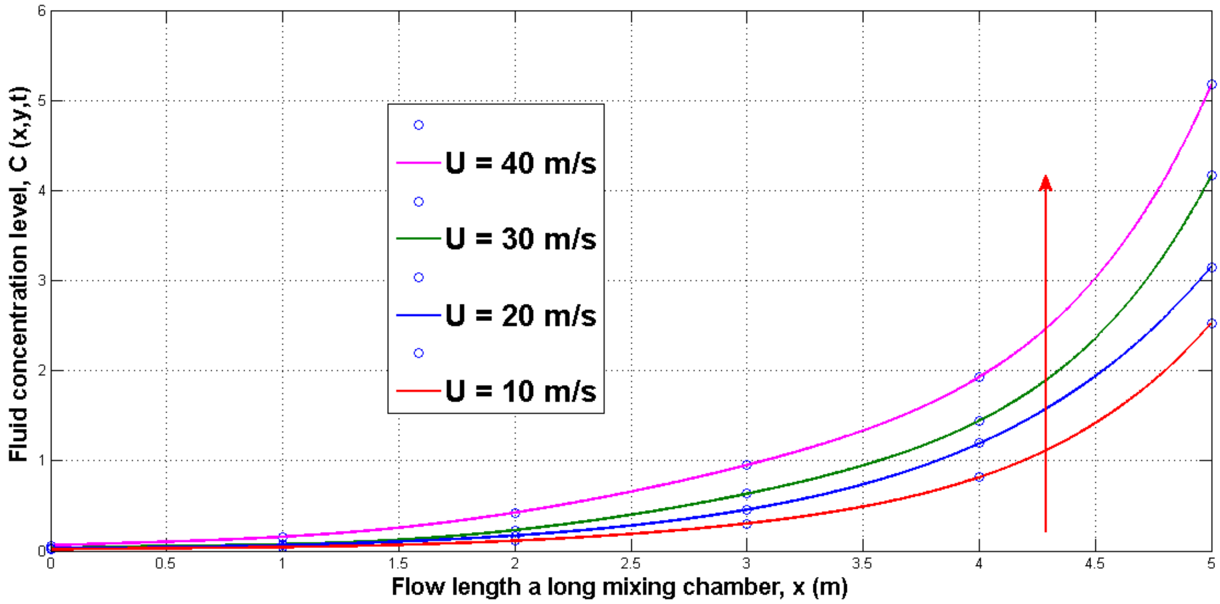

Fig 4: Graph of Fluid Concentration level against flow length along mixing chamber at varying characteristic fluid velocity

Figure 4, it is observed that an increase in fluid characteristic velocity (U) results in an increase in the magnitude of the fluid concentration profiles. This is because the fluid velocity is directly proportional to the shear rate and the large shear rate accelerates the uniform mixing by molecular diffusion. Increasing the characteristic velocity (U) will increase the temperature of the fluid, resulting in a decrease in the viscosity and shear stress of the fluid.

Validation

The results obtained in this study are in agreement with other related studies such as in the study by Ian and Changing (2018).

V. CONCLUSION

From the numerical results which have been presented in terms of concentration profiles, it can be concluded that:

1. The rotational and extensional local flows inside the mixing chamber dominate the mixing index, thereby producing steady stirring.
2. The combination of the radial dispersion and the vortex flow improve the mixing inside the chamber.
3. The inertial force has a crucial impact on the mixing since it causes powerful vortex flow in the mixing chamber and diminishes the damping effect on diffusion, which leads to a higher mixing performance.

ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 8, Issue 5, September 2019

REFERENCES

[1] Anderson P. D. Gorodetskyi O., M. Giona (2012). "Exploiting numerical diffusion to study transport and chaotic mixing for extremely large P´eclet values," Europhys.Lett. 97.

[2] Anderson P.D and H. Meijer,(2000). "Chaotic mixing analyses by distribution matrices," Appl. Rheol. **10**.

[3] Conor P. S, Fan Y., Paul B. Umbanhowar, Julio M. Ottino, and Richard M. Lueptow, (2014)."Modelling size segregation of granular materials: The roles of segregation, advection and diffusion," J. Fluid Mech. **741** pp 252-279.

[4] Conor P. Schlick (2014). Applications of advection-diffusion based methodologies to fluid and granular flows. A PhD Dissertation Field of Engineering Sciences and Applied Mathematics Norh Western university.

[5] Clifford, M. J., Cox, S. M., and Finn, M. D., (2004). "Reynolds number effects in a simple planetary mixer," Chemical Engineering Science, **59(16)**, pp. 3371-3379.

[6] Ian T., and Changying J., (2018).A Study of the Mixing Performance of Different Impeller Designs in Stirred Vessels Using Computational Fluid Dynamics. Designs, **2(10).**

[7] Youcefi, A., AnneArchard, D., Boisson, H. C., and Sengelin, M., (1997). "On the influence of liquid elasticity on mixing in a vessel agitated by a two-bladed impeller, "Journal of Fluids Engineering-Transactions of the Asme, **119(3)**, pp. 616-622.

[8] Zhou, G., Tanguy, P. A., and Dubois, C., (2000). "Power consumption in a double planetary mixer with non-Newtonian and viscoelastic materials," Chemical Engineering Research & Design, **78(A3)**, pp. 445-453.

AUTHOR BIOGRAPHY

JANET CHEBET LANGAT; Mrs. Langat holds a Bachelor of Education Science degree, and specialized in Double Mathematics from Catholic University of Eastern Africa, Kenya. She is undertaking her final research project for the requirement of Msc. in Applied Mathematics of the Jomo Kenyatta University of Science and Technology (JKUAT), Kenya. She is a teacher at Kapsimotwo Secondary School, Kenya. Phone number +254726064250.Email address janetsigei2018@gmail.com

JOHANA K. SIGEY: Prof. Sigey holds a Bachelor of Science degree in mathematics and computer science First Class honours from Jomo Kenyatta University of Agriculture and Technology, Kenya, Master of Science degree in Applied Mathematics from Kenyatta University and a PhD in applied mathematics from Jomo Kenyatta University of Agriculture and Technology, Kenya. Affiliation: Jomo Kenyatta University of Agriculture and Technology, (JKUAT), Kenya. He is currently the Director, JKuat, and Kisii CBD. He has been the substantive chairman - Department of Pure and Applied mathematics, JKUAT (2007-2012). He has published 40 papers on heat transfer, MHD and Traffic models in respected journals. Teaching experience: 2000 to date- postgraduate programme: (JKUAT); Supervised student in Doctor of philosophy: thesis (6 completed, 10 ongoing); Supervised student in Masters of Science in Applied Mathematics: (45 completed, 10 ongoing) .Phone number +254722795482

JECONIAH OKELO ABONYO: Prof. Okelo holds a PhD in Applied Mathematics from Jomo Kenyatta University of Agriculture and Technology as well as a Master of Science degree in Applied Mathematics and first class honors in Bachelor of Education, Science; specialized in Mathematics with option in Physics, both from Kenyatta University. He has dependable background in Applied Mathematics in particular fluid dynamics, analyzing the interaction between velocity field, electric field and magnetic field. Affiliation: Jomo Kenyatta University of Agriculture and Technology (JKUAT). 2011 to date: Deputy Director, SODeL Examination, Admission & Records (JKUAT). He has published 44 papers on heat transfer in respected international Journals. He is a Professor in the Department of Pure and Applied Mathematics and Supervisor at JKUAT. Supervision of postgraduate students; PHD: thesis (3 completed and 3 ongoing); Msc in Applied Mathematics: (13 completed and 8 ongoing)

ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 8, Issue 5, September 2019

EMMAH MARIGI. Dr. Marigi holds PhD in Applied Mathematics (JKUAT-2013), Msc in Applied Mathematics (JKUAT-2007) and B.Ed (Science) from KU-1987. Her Research Interest is Modeling Mathematics and area of expertise is Applied Mathematics. Her responsibilities include: Senior Lecturer (DeKUT-2013 to date), Dean of Science (DeKUT-2012 to date), and Head of Mathematics (2012), Examination Coordinator (2010-2012). She has done 5 Publications in the Applied Mathematics. She has done postgraduate Supervision of PhD thesis (1 ongoing) and Msc (1 completed and 5 on-going).

BETHSHEBA MENGE. Dr. Menge holds a PhD in Applied Mathematics from Jomo Kenyatta University of Agriculture and Technology (2014), as well as a Master of Science degree in Applied Mathematics (2002) and Bachelor of Education, Science (1989), both from Kenyatta University. She is currently working as a senior lecturer of Mathematics at Technical University of Mombasa, department of Mathematics and Physics. She has published over ten papers in international journals. Her research area is in turbulent flows.