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A numerical study on transverse vibration of euler-bernoulli beam

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Abstract*: Numerical solutions for static and dynamic stability parameters of an axially loaded uniform beam resting on a two simply supported foundations were considered using Finite Difference Method where Central Difference Scheme (CDS) was developed. The vertical displacement of a simply supported beam under a uniformly varying load, varying linear density of the bridge and application of viscoelastic dampers were considered. The main differential equation was given by the Euler-Bernoulli beam equation which is the fourth-order differential equation. Euler-Bernoulli beam equation was discretized, with the MATLAB software used in solving the equation. The results were discussed graphically and a conclusion drawn. It was found that on increasing density of the bridge material it results to decrease in beam vibrations; an increase in external force led to an increase in beam vibrations. When viscoelastic dampers were used, transverse beam vibrations were reduced. Viscoelastic dampers used minimize resonance brought about the external forces applied including moving vehicular masses, seismic and wind induced vibrations which are magnified by the excitation of the frequencies resulting from nature.

***Key words*: Finite Difference Method, Central Difference Scheme (CDS), Density, External force, Damping.**

I. INTRODUCTION

A. Background Information

A beam is a structural constituent that has an ability to weather load fundamentally by resisting against bending. Beams are characterised by their length, cross-section area, and nature of material. The works of this study has been modelled to one dimensional beam. It can be horizontal, vertical or inclined at an angle. Beams generally hold vertical gravitational forces but can also be made to hold horizontal loads. The loads carried by a beam are channelled to columns, walls, or girders, which then channel the force to the adjacent structural compression members. Compression parts are structural elements subjected only to axial compressive forces.

The bending effect is the single most significant factor in a transversely vibrating beam. The Euler-Bernoulli model includes the strain energy and the kinetic energy due to the bending and lateral displacement respectively. In real life, applications such as rail tracks, bridges, pavements, underground pipelines, foundation beams and even animal vertebra columns have been modelled as beams resting on elastic foundations. To investigate the dynamics of such applications, the vibration behaviour of these models need to be accurately determined. Finite element method, transfer matrix method, Rayleigh-Ritz method, differential quadrature element method, Galerkin procedure and perturbation techniques are some numerical methods used to investigate the vibration behaviour of different types of linear or non-linear beams resting on linear or non-linear foundations.

Jacob-Bernoulli first discovered that the bending of an elastic beam at any point directly varies as the bending moment at that point. Daniel Bernoulli formulated the differential equation of motion of a vibrating beam. Later, Jacob Bernoulli's theory was adopted by Euler in his investigation of the shape of elastic beams under various loading conditions. The theory of the Euler-Bernoulli beam is the most commonly used since it is not complex and provides reasonable mathematical and engineering estimations for many problems. It provides a means of determining the load carrying and deflection properties of beams. It covers the case for small warps of a beam that. The Euler-Bernoulli beam equation is used to determine deflection of beam elements while Plate or shell theory determines plate or shell elements. Resonance is the rise in the amplitude of an oscillation of a system under the influence of a periodic force whose frequency is close to that of the system's natural frequency (Resnick and Halliday, 1977). At resonant frequencies, small periodic driving forces have the ability to lead to large amplitude oscillations since the system stores vibration energy. When damping is small, the resonant frequency is approximately equal to the natural frequency of the system, which is a frequency of unforced vibrations. Resonance phenomena occur with all types of vibrations or waves. At times, oscillations of a mechanical system may match the system's natural frequency of vibration that leads to swaying motions and even catastrophic collapse of improperly constructed structures. During the design of structures, engineers must

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ensure the mechanical resonance frequencies of the constituent parts do not match driving vibrational frequencies of motors or other oscillating parts (Ogata, 2005).

Averting resonance disasters is a major aim in every building, bridge or any structural construction project. As a countermeasure, shock absorbers can be installed to absorb resonant frequencies and thus dissipate the absorbed energy. When a single moving mass crosses a beam there are particular ratios of the beam natural period and the time spent by the mass crossing the structure that, in absence of damping, result in the cancellation phenomenon. Hence, both resonance and cancellation situations are related to the free vibration response of beams but resonance needs a repetitive loading pattern to develop while cancellation takes place within the passage of every single mass (Billah *et al*, 1991).

B. Statement of the problem

Bridges or structures as a whole are generally prone to visible oscillations or vibrations that may lead to collapse of the structure. Beams experience compressive, shear and tensile stresses as a result of the loads applied to them. The possibilities to foretell such situations could be of special interest to researchers in an endeavour to provide solutions to high speed road and railway bridge failures. A number of researchers have studied on varying the cross-section area of the beam, using various numerical and analytical methods in solving the Euler-Bernoulli beam equation, vibrational analysis due to natural frequencies and partially distributed moving masses. Therefore, the properties studied in this research include; the effect of varying density of the bridge material, application of VEDs and uniformly varying external load on the transverse displacement of a beam on excited bridges.

C. Objectives of the study

The specific objectives of the study are;

i) To determine the effects of varying beam density on transverse vibration of the Euler-Bernoulli Beam.
ii) To determine the effects of uniformly varying external load applied on transverse vibration of the Euler-Bernoulli Beam.
iii) To examine the effectiveness of the dampers acting on the beam on transverse vibration of the Euler-Bernoulli Beam.

D. Significance of study

The effects of damping parameter, density of beams and external force on beam and column vibrations that are studied in this research, helps us to understand the level to which they influence foundation settlements as well their contribution to collapse of architectural structures. This in turn will lead to mitigate the adverse effects result through beam vibrations of architectural structures for the good of humanity. The next section will look into the literature review of what earlier researchers did in regard to Euler-Bernoulli beam.

II. LITERATURE REVIEW

Over the years, many researchers have numerically analysed the beam vibration on excited structures such as bridges, buildings, trains which results from natural frequencies and those resulting from high speed movements on those structures. Many analytical and numerical methods have been employed. Achawakorn *et al* (2012) studied the vibration analysis of exponential cross-section beam using Galerkin's method and FEM utilised to compare the results. The analytical method and the FEM give the conformable results, with slightly different value of the natural frequencies of the beam. Taha M. and Abohadima S. (2009) studied the dynamic analysis of non-uniform beams on elastic foundations by transforming the fourth order differential equation of beam vibration under appropriate boundary conditions to the Bessel equation by factorization. Modes of shapes and damped natural frequencies of the beam are obtained for broad range of beam-foundation system characteristics. The effect of foundation stiffness is more pronounced on frequencies of lower modes. Taha M. and Abohadima S. (2015) analysed non-uniform beams on elastic foundations making use of the recursive differentiation method. Foundation stiffness influence was detectable on both the critical load and natural frequency in the case of thinner beams. Effect of the end conditions reduces as the thinness parameter of the beam increases. Neither the critical load nor the natural frequency corresponding to the first mode is always the smallest one in the case of beams on the elastic foundations.

Mahmoud *et al* (2013) applied the Differential Transformation Method for free vibration analysis of rectangular beams with uniform and non-uniform cross-sections. Mahmoud (2015) using the same method, for free

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vibration analysis of Euler-Bernoulli beam with circular cross-section. Krishna *et al* (2016) studied numerical vibration analysis of rectangular beams for different end conditions such as for simply supported and cantilever type. It was concluded that the natural frequencies for aluminium are slightly higher than that of mild steel for both end conditions. The experimental, theoretical and numerical results were found to be in close agreement.

Xia *et al* (2006) did an analysis of resonance mechanism and conditions of train-bridge system where they investigated through theoretical derivatives, numerical simulations and experimental data analyses. The vehicle resonance is induced by the periodical action of regular array of bridge spans and their deflections. The resonance of the train-bridge system is affected by the span, total length, lateral and vertical stiffness of the bridge, the compositions of the train, and the axle arrangements and natural frequencies of the vehicles.

Museros P. and Martinez R. (2009) studied the cancellation phenomenon for simply supported beams and plates subjected to moving loads. For certain speeds of a single moving load, the free vibrations which usually remain after the passage of the load simply become invisible, leaving the structure completely at rest. Besides, when this kind of structures is subjected to trains of loads, resonance situations emerge very often due to the regularity of the distance between consecutive loads. For certain relations of the bridge length with respect to the characteristic load distance, the cancellation and resonance speeds coincide, and the latter is totally suppressed. This explains why some resonance peaks expected in the response on high-speed bridges do not appear in practice. Meher S. (2012) studied dynamic response of a beam structure to a moving mass using Green's function. It was established that as velocity of the load increases, position of the maximum response on the beam occurs far from the midpoint. At very high velocity the maximum deflection of the beam occurs near the end of the beam. The dynamic response of the beam is more stirred from the load velocity than mass ratio of the system. Greco A. and Santini A. (2002) studied dynamic response of a flexural non-classically damped continuous beam under moving loads. Complex mode superposition method is presented for the dynamic analysis of uniform simply supported beam with two rotational viscous dampers attached at its ends subjected to a constant moving loading. It was shown that the effectiveness of the dampers in reducing the response amplitude strongly depends on the loading speed. Kamaitis *et al* (2014) examined the effects of cable flexural rigidity on the free vibrations of suspension bridges. It was found that the bending stiffness of the main cable contributes to a considerable effect on natural frequencies for this type of suspension system. A simplified expression of predicting natural bending frequencies of the suspension bridge taking into account the bending stiffness of the cable has been developed for the application as the first step in the design process. Akinpelu (2012) who did a paper on response of viscously damped Euler-Bernoulli beam to uniform partially distributed moving loads, analysed PDEs for both moving force and mass problem. He realised that when the mass of the load increases, there was a corresponding increase in the amplitude of the displacement.

Wang *et al* (2010) studied resonance characteristics of two-span continuous beam under moving high speed trains. Each span was modelled as a Bernoulli-Euler beam and the moving trains as a series of 2 DOF mass-spring-damper system at the axle locations. It was found that the two-span continuous beam had two critical velocities causing the resonance response, which was different from simple supported beam and the bigger damping ratios lead to the smaller deflection. Wang *et al* (2013) did study on vibration suppression of train-induced multiple resonant responses of two-span continuous bridges using VE dampers. The installed VE dampers concurrently suppressed the multiple resonant peaks of train-bridge system and further improved the riding comfort of moving vehicles.

Mehrabi M. (2017) modelled a viscoelastic damper and its application in the structural control where they clarified the fundamental properties of VED system by analysing building structures under cyclic loading. They found out that the damper showed good performance in terms of the column compression force resulting from the brace action. Issa S. (2012) did a paper on vibration absorbers for simply supported beams subjected to constant moving loads where linear vibration absorbers were explored. It was shown that the optimal absorber was an undamped absorber which should be attached at a fixed location irrespective of the modal damping or mass ratios.

Omari *et al* (2016) studied a model of the Euler-Bernoulli beam equation that governs excited bridge vibrations was solved using finite difference method. It was found that transverse vibrations of the bridge beams increase with increasing bridge length and decreasing the cross-sectional area of the bridge. The increase in forcing term on the bridge beam led to a rise on the transverse vibration of the bridge beams.

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Following the researchers' works as illustrated in the literature review, it's evident that varying the density of the bridge material, application of the VEDs and uniformly varying external load on transverse displacement of excited beams has not been adequately done, hence the study of this project.

III. METHODOLOGY

A. Geometry of the model

The following diagram shows a beam resting on two elastic supports at either ends with two viscoelastic dampers connected in between. There is uniformly varying loads up to a maximum at one end.

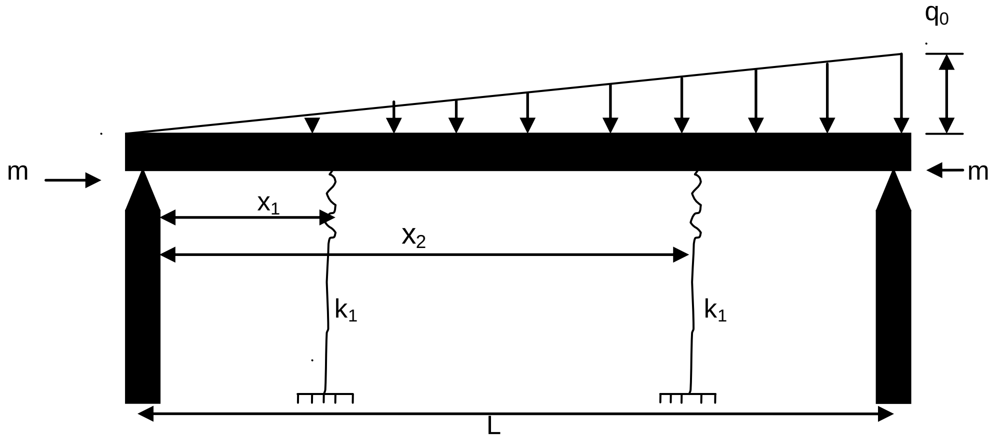

Fig 1: Simple Model of Euler-Bernoulli Beam

Where L represents the length of the beam in study, k_1 and k_1 as viscoelastic dampers, q_0 as the maximum external load, x_1 and x_2 represent lengths from one of the end supports, m as end supports.

B. Assumptions

1. Plane cross-sections remain plane and perpendicular to the longitudinal axis after bending.
2. Cross-sections of the beam do not deform in a significant manner under the application of transverse or axial loads and can be assumed as rigid.

C. Euler-Bernoulli beam equation

The Euler-Bernoulli beam equation demonstrates the relationship between the beam deflection and the applied load for excited beam as

$$\frac{\partial^2}{\partial x^2}\left[EI(x)\frac{\partial^2 u(x,t)}{\partial x^2}\right]=q(x) \quad (1)$$

Where the curve *u(x,t)* denotes the deflection of the beam at some position *x, q(x)* is the distributed load, *E* is elastic modulus and *I* is the moment of inertia. According to Euler-Bernoulli theory, for the excited vibrations of the Euler-Bernoulli beam with elastic supported foundations is modelled by the equation (Leissa *et al* 2011)

$$EI(x)\frac{\partial^4 u(x,t)}{\partial x^4}+\rho A(x)\frac{\partial^2 u(x,t)}{\partial t^2}=f(x,t) \quad (2)$$

Where A(x) is the cross-section area at position x, *I(x)* is moment of inertia of *A(x), E* denotes Young's modulus, ρ is linear mass density of the beam material, *u(x,t)* is the transverse deflection at position x and time t. When the VE dampers are used, the Euler-Bernoulli equation becomes (Wang *et al* 2010)

$$EI(x)\frac{\partial^4 u(x,t)}{\partial x^4}+\rho A(x)\frac{\partial^2 u(x,t)}{\partial t^2}+k\frac{\partial u}{\partial t}=f(x,t) \quad (3)$$

Where *k* is the damping of the beam, *f(x,t)* is external load from the moving vehicles and external damping force induced by the dampers acting on the beam.

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D. Method of solution

When analysing the Euler-Bernoulli equation, Finite Difference method (FDM) with central difference scheme is used to obtain solutions and proper boundary conditions are applied numerically to attain the expected results. FDM was chosen as the method for solving the Euler-Bernoulli equation since it converges, its stable and consistent. Equation (3) is used in discretization where partial derivatives will be replaced with their finite difference approximations.

E. Discretization of the governing equation

In this section, equation governing Euler-Bernoulli beam transverse vibrations is discretized. Using Central Difference numerical scheme, u_t is substituted with forward difference while u_{tt} and u_{xxxx} are substituted with central difference in (3).Then equation (3) becomes

$$EI\left[\frac{U_{i+2,j}-4U_{i+1,j}+6U_{i,j}-4U_{i-1,j}+U_{i-2,j}}{(\Delta x)^4}\right]+\rho A\left[\frac{U_{i,j+1}-2U_{i,j}+U_{i,j-1}}{(\Delta t)^2}\right]+k\left[\frac{U_{i,j+1}-U_{i,j}}{\Delta t}\right]=F(x,t)$$

(4)

Taking $EI = 80N/mm^2$, $\Delta t = 0.01, \Delta x = 0.2$, i=1,2,3,…,6, j = 1and then initial and boundary conditions u(0,t) = 1 , u(x,0) = 0 , u(0,t) = 0 and u(x,t) = 0 respectively substituted into (4) we obtain the matrix-vector equation as shown below.

$$\begin{bmatrix} (3000-200\rho A-k) & -2000 & 0 & 0 & 0 & 0 \\ -2000 & \ddots & -2000 & 0 & 0 & 0 \\ 0 & -2000 & \ddots & -2000 & 0 & 0 \\ 0 & 0 & -2000 & \ddots & -2000 & 0 \\ 0 & 0 & 0 & -2000 & \ddots & -2000 \\ 0 & 0 & 0 & 0 & -2000 & (3000-200\rho A-k) \end{bmatrix}\begin{bmatrix} U_{11} \\ U_{2,1} \\ U_{3,1} \\ U_{4,1} \\ U_{5,1} \\ U_{6,1} \end{bmatrix}=\begin{bmatrix} 0.01F+2000 \\ 0.01F-500 \\ 0.01F \\ 0.01F \\ 0.01F \\ 0.01F \end{bmatrix} \quad (5)$$

We solve equation (5) using MATLAB and obtain the results of varying force (F), density (ρ) and damping parameter (k).

IV. RESULTS AND DISCUSSION

A. Introduction

The simulated results exhibits relations between beam transverse vibrations and various parameters as obtained by computing numerically are given in Figs 2, 3, 4a and 4b. The simulation results given focus on the effects of the beam density, external force and damping parameter on beam transverse vibrations.

B. Effects of density on beam transverse vibrations

We solve equation 5 using MATLAB and get the results of the effects of beam on bridge transverse vibrations as shown in table1below.

Table 1. Beam vibrations for varying beam density

Density of the beam	**Beam length**					
	0	**1**	**2**	**3**	**4**	**5**
$\rho = 7000kg/m^3$	-23.705	-13.462	14.7527	14.7527	-13.462	-23.705
$\rho = 7500kg/m^3$	-14.447	-9.527	10.3990	10.3990	-9.527	-14.447
$\rho = 8000kg/m^3$	-11.004	-7.9108	7.8429	7.8429	-7.9108	-11.004

The results in table 1 above is represented graphically as seen in figure 2 below

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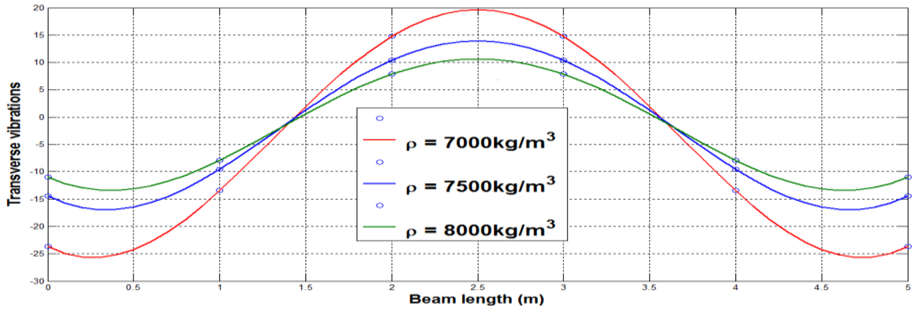

Fig 2: Graph of beam transverse vibrations against beam length at varying density

The effect of beam transverse vibration against beam length at varying density on beam vibration at different values of varying density. From the graph, the one with least density has the highest amplitude while the one with higher density has lower amplitude. The graph illustrates that beam vibrations increase with decrease in density values. The graphs intersect at a particular point, where the beam vibrations are zero.

C. Effects of external force on the beam vibrations

The central difference scheme in Equation (5) is used to find the effects of varying external force on the beam vibration at three different values of varying external forces i.e F = 100 kN, F = 150 kN and F = 200 kN. We take $\Delta t = 0.01$ A = 1m^2 and $\Delta x = 0.2$ with constant values $EI = 80 N / mm^2$, $\rho = 7000 kg / m^3$ and substitute into (5) we get the results as shown in table 2.

Table 2. Beam vibrations for varying external force

External force	**Beam length**					
	0	**1**	**2**	**3**	**4**	**5**
F = 100 kN	21.204860	7.0912260	-21.23856	-21.23856	7.0912260	21.204860
F = 150 kN	28.778080	9.6246210	-28.72052	-28.72052	9.6246210	28.778080
F = 200 kN	36.511800	12.157420	-36.28249	-36.28249	12.157420	36.511800

When the values in the table above are considered, we plot the values of transverse vibrations against beam length at varying external force F(x, t). The results in the table 2 above is represented graphically as seen in figure 3 below

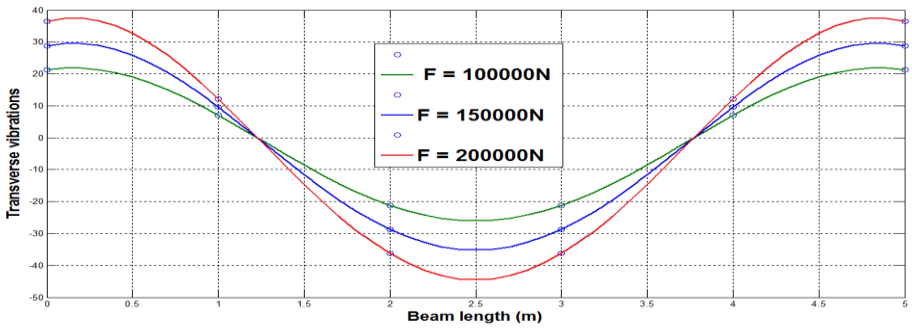

Fig 3: Graph of beam transverse vibrations against beam length at varying external force

The graph illustrates the effect of transverse vibrations against bridge length at varying external force. A larger force of 200kN has the greatest amplitude as compared to the one with a smaller value of force. This shows that beam vibrations increase as we move from a lesser beam length and lesser external force to a bigger beam length and bigger external force. The effect of transverse vibrations is highest when the external force is also highest (200 kN) and the beam length is also highest (5m), while the effect of transverse vibrations is lowest when the external force is also lowest (100 kN). The graphs intersect at two particular points where there is no vibration of particles.

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D. Effects of damping parameter on the beam vibrations

The central difference scheme in Equation (5) is used to find the effects of varying damping parameter on the beam vibration at three different values of damping parameter i.e. k = 0Ns/m, k = 10Ns/m, k = 20Ns/m and k = 30 Ns/m. We take $\Delta t = 0.01$ A = 1m^2 and $\Delta x = 0.2$ with constant values of $EI = 80N/mm^2$, $\rho = 7000kg/m^3$ and substitute into (5) we get the results as shown in table 3.

Table 3. Beam vibrations for varying damping parameter

Damping parameter	Beam length					
	0	**1**	**2**	**3**	**4**	**5**
k = 0 Ns/m	44.25231	17.041580	-44.9580	-44.9580	17.041580	44.25231
k = 10 Ns/m	42.042330	14.705850	-40.79970	-40.79970	14.705850	42.042330
k = 20 Ns/m	37.827190	12.824460	-37.45800	-37.45800	12.824460	37.827190
k = 30 Ns/m	32.517101	10.164600	-29.16302	-29.16302	10.164600	32.517101

The results in the table 3 above is represented graphically as seen in figure 4 below

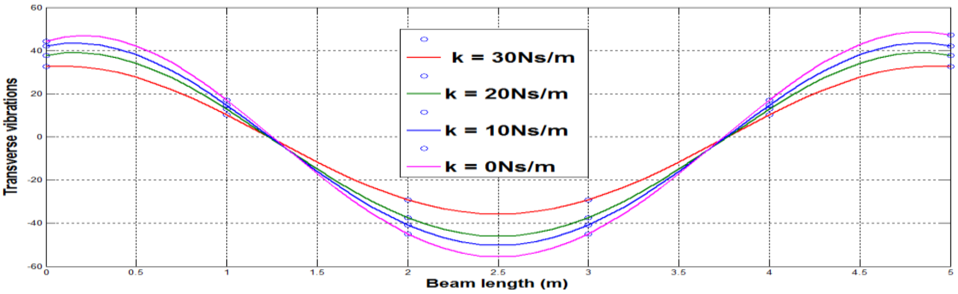

Fig 4(a) Graph of beam transverse vibrations against beam length at varying damping parameter

In the results show sinusoidal graphs which give deflections that are symmetrical. Maximum beam vibration occurs at mid-span. The effect of transverse vibrations against beam length at varying damping parameter, decrease as we move from a short beam length and small damping parameter to a bigger beam length and bigger damping parameter. The intersection of the graphs signify a point where there are no or minimum beam vibrations. Using table 3 to plot a graph of transverse vibrations against damping parameter, k(N.s/m) is as shown below.

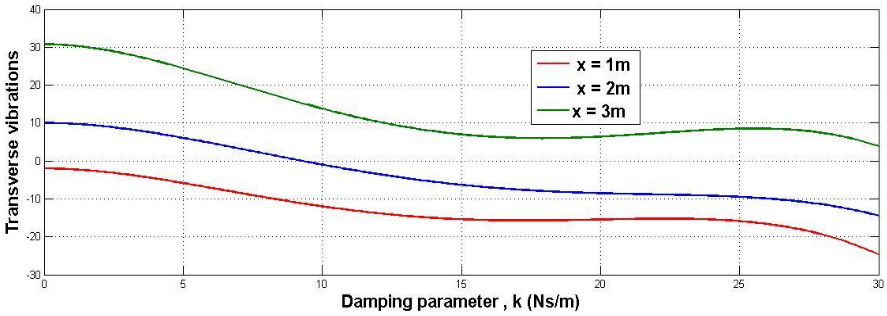

Fig 4(b): Graph of beam transverse vibrations against damping parameter at varying beam length

The figure 4(b) shows a graph of transverse beam vibrations against damping parameter with varying beam length. When damping parameter is none or small, the beam vibrations are great which decreases with increase in damping parameter. When the beam length increases from 1m to 3m, transverse beam vibrations, with no damping parameter, are high. But as the damping parameter increases, the transverse beam vibrations decrease with the change in beam length uniformly increases.

E. Discussion

When the density of the beams increases, the transverse beam vibrations decreases. This results from the fact that a material with a low density, its particles are far apart. Its particles are held by slightly weaker forces of cohesion unlike that material with higher density. A material with higher density has stronger forces of cohesion holding its particles together hence its particles are closely packed. Hence, when a moving mass that leads to beam vibrations is applied, the beam with higher density vibrates least as the particles are held in a close knit by

stronger forces of cohesion. For less dense material, however, the particles are set into vibrations easily due to the fact that its particles weakly held.
When external force applied increases, transverse beam vibration also increases. As the beam length increases, the force exerted upon it also increases with maximum deflection at mid-span. This is brought about by a number of factors, such as, the vehicular mass, vehicular velocities, natural frequency and seismic vibrations. The beam vibrations are brought about the weight exerted upon the beam stressing the forces holding the particles together. Since the force applied is not constant, the particles experiences random changes shifting them from one point to the other. This leads to a set of vibrations being generated. When a larger force is applied on a beam, the particles of the beam are set into vigorous vibrations due to high kinetic transferred to them and random changes on the particles. This leads to the entire beam vibrating in response to individual particles.

VEDs incorporated during structural constructions like bridge building, are utilised as energy dissipaters. When they are used and increased in a construction, the transverse beam vibrations are also decreased significantly though not fully since the particles comprising the beam are not in their lowest temperature attainable. VEDs reduce or rather convert resonant vibrations into heat energy through shear deformation. The graphs intersect for all the three Figs 2, 3 and 4a at particular points. At these points, the beam vibrations are at the minimum or zero.

F. Validation

The earlier researchers who did a study employing Euler-Bernoulli beam equation as their governing equation obtained similar results as depicted by graphs include;-
Akinpelu F. O. (2012), "The response of viscously damped Euler-Bernoulli beam to uniform partially distributed moving loads." It was found that as the mass of the load increased, the amplitude of displacement also increased.
Omari *et al* (2016), "Mathematical analysis of elastically supported beams with application to excited bridge vibrations". It was found that as the forcing term increased, the beam vibration increased.
Wang *et al* (2013),"Vibration suppression of train-induced multiple resonant responses of two-span continuous bridges using VE dampers". The installed VE dampers minimized the multiple resonant peaks.
From the results obtained in this research, it was noted that an increase in external force applied leads to an increase in transverse beam vibrations, denoted by rise of amplitude of displacement. Viscoelastic dampers utilised in structural construction tend to reduce beam vibrations significantly, besides, when using materials of higher density lead to a corresponding reduction in beam vibrations.

V. CONCLUSION AND RECOMMENDATIONS

A. Conclusion

In this study, the transverse beam vibrations were investigated numerically under specified different boundary conditions. Numerical solution of the fourth order Euler-Bernoulli beam equation was carried out. Finite Difference Method was used in calculating the dynamic response of the uniform Euler-Bernoulli simply supported beam with Central Difference Scheme used. The solutions obtained for Euler-Bernoulli beam equation take into consideration the effects of beam density, external force and damping parameter. From the results, it was found out that;

i) The transverse beam vibrations of the bridge increase with the increasing bridge length.
ii) An increase in the external force on the bridge causes an increase in transverse beam vibration.
iii) Inclusion of the viscoelastic dampers in the bridge construction results in a decrease of transverse beam vibrations.
iv) An increase in beam density leads to decrease in beam vibrations.

B. Recommendation

The study dealt with the effects of density, external force and damping parameter on beam transverse vibrations of the bridge. From this study there are a number of areas that arose as result that requires further research which may be experimental or theoretical. These areas of study include:

i) Solving the model equation using Timoshenko beam theory and then compares the results.
ii) Investigate the effects of varying the damping parameter when using non-linear Euler-Bernoulli beam on transverse beam vibration.

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NOMECLATURE

ρ	Density (kg/m^3)
u(x,t)	Deflection of the beam(m)
L	Span or length of a beam (m)
U	Strain energy
E	Modulus of elasticity (N/m^2)
I	Moment of inertia (m^4)
EI	Flexural stiffness of a beam (N/m^2)
A	Area of the cross-section (m^2)
f(x,t)	External load (N)
K	Damping parameter (N.s/m)

ABBREVIATIONS

FDM	Finite Difference Method
VED	Visco-elastic damper
JKUAT	Jomo Kenyatta University of Agriculture and Technology
CDS	Central Difference Scheme
FEM	Finite Element Method Partial
PDE	Differential Equation
2DOF	Two-Degree-of-Freedom
FDA	Finite Difference Approximation

M	Mass per unit length (kg/m)
q_0	External load applied at a point on the beam (N)
t	Time (s)
X	Axial co-ordinate

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