ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 8, Issue 2, March 2019

Static-based damage detection using measured responses

Hee-Chang Eun[a,], Dong-Ho Cho[b], Young-Ji Choi[c]

[a] Department of Architectural Engineering, Kangwon National University, Korea

[b] Department of Architectural Engineering, Kangwon National University, Korea

[c] Department of Architectural Engineering, Kangwon National University, Korea

Abstract: The detection of damaged member in structure can be performed by comparing the change in the physical property before and after the occurrence of the damage. Estimating the residual force in the equilibrium equation or the stiffness variation due to structural damage, this work presents the static-based damage detection approach. The initial structure is disassembled by the stiffness matrices of all members, the change in the stiffness using measured static responses is estimated, and the damaged member is traced by evaluating the elemental stiffness change with respect to the initial stiffness. The method is straightforwardly derived minimizing a performance index in the satisfaction of measured response data. The validity of the proposed method is illustrated in a numerical experiment of truss structure.

Keywords: **damage detection, static approach, residual force.**

I. INTRODUCTION

Damaged member or region is detected based on the variation in static or dynamic responses due to the local change of physical properties at damage region. The static responses are deeply related with the stiffness of all physical parameters. Most of the damage detection methods require knowing fundamental information for a structure at intact state as a baseline data set. Finite element method should be one of the most commonly used methods to analyze complex structures.

The localized deterioration of stiffness in a structure will produce a local change in responses at the region. Static damage identification methods are usually simpler than the dynamic ones. The equipment in static testing is comparatively cheaper. The static methods have attracted much attention because the accurate deformation or strain of the structure can be obtained rapidly and economically. Sheena et al.(1982) presented an analytical method to assess the stiffness matrix by minimizing the difference between the actual and the analytical stiffness matrix subjected to the measured displacement constraints. Minimizing the difference between the applied and the internal forces, Sanayei and Scampoli(1991) presented a finite element method for static parameter identification of structures by the systematic identification of plate-bending stiffness parameters for a one-third scale, reinforced-concrete pier-deck model. Sanayei and Onipede (1991) provided an analytical method to identify the properties of structural elements from static test data such as a set of applied static forces and another set of measured displacements. Stohr et al. (2006) observed that specific static measurements such as the influence lines of the inclination are useful for damage detection. Lee and Eun(2008) presented an analytical method for damage detection by utilizing displacement curvature and all static deflection data to be expanded from the measured deflection data. Serker and Wu (2010) provided the damage detection method using the

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ISO 9001:2008 Certified
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change in the strain ratio between two measurement locations. Eun et al.(2013) compared the static-based damage detection methods depending on strain and deflection sensors.

The damage detection methods using residual force vector have been introduced. Using the residual force vector, Yang and Liu (2007) provided three damage identification techniques by the algebraic solution of the residual force equation, MREU technique, and the natural frequency sensitivity method. Yang (2010) presented the damage identification method using flexibility disassembly. Li et al. (2016) suggested the damage detection method using the residual force vector and response sensitivity analysis.

The detection of damaged member can be performed by comparing the change in the physical property of the corresponding members before and after the occurrence of the damage. Estimating the residual force in the equilibrium equation or the stiffness variation due to structural damage, this work presents the static-based approach for damage detection. The initial structure is disassembled by the stiffness matrices of all members, the change in the stiffness using measured static responses is estimated, and the damaged member is traced by evaluating the elemental stiffness change with respect to the initial stiffness. The method is straightforwardly derived minimizing a performance index in the satisfaction of measured response data. The validity of the proposed method is illustrated in a numerical experiment of truss structure.

II. FORMULATION FOR DAMAGE DETECTION

Nondestructive test is performed to evaluate the structural performance. The damage detection at specific locations in a single member or at specific members in structure is included in the test. The local damage in a single member with a constant section locates at the discontinuous section to display the stiffness deterioration. The local damage can be detected by investigating the responses due to external excitation because it exists at location to exhibit the abrupt change of the response.

The structure is composed of the members of various different types and sizes, and it is joined at nodes. The detection of damaged member can be performed by comparing the physical property of all members before and after the occurrence of the damage unlike the local damage detection. This work disassembles the elemental stiffness matrices of all members, estimates the change in the elemental stiffness in the satisfaction of measured static responses, and detects the damage member comparing the stiffness change.

Using finite element analysis, the equilibrium equation of an entire structure with n degrees of freedom (dofs) can be written by

$$\hat{\mathbf{K}}_e \mathbf{u}_e = \mathbf{f}_e \qquad (1)$$

Where $\hat{\mathbf{K}}_e$ is the $n \times n$ positive-definite stiffness matrix, and $\mathbf{u}_e$ and $\mathbf{f}_e$ are the $n \times 1$ displacement and force vectors, respectively. Supposing that the structure is composed of *m* finite elements, the elemental stiffness matrix is established and the global stiffness matrix is assembled by *m* elements. The global stiffness matrix can be written by

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$$\mathbf{K}_e = \sum_{i=1}^{m} \mathbf{K}_e^{(i)} \qquad (2)$$

Where $\mathbf{K}_e$ is an $N \times N$ disassembled stiffness matrix, N is the total dofs of the disassembled stiffness matrix, $\mathbf{K}_e^{(i)}$ is the *i*-th elemental stiffness matrix with *l* dofs, and *m* is the number of elements. The $N \times N$ stiffness matrix can be obtained by the elemental decomposition technique as

$$\hat{\mathbf{K}}_e = \mathbf{C}\mathbf{K}_e\mathbf{C}^T \qquad (3)$$

Where $\mathbf{C}$ is the $n \times N$ stiffness connectivity matrix between dofs.

The columns of matrix $\mathbf{C}$ embody the stiffness contribution to the global stiffness matrix in terms of the elemental stiffness parameters.

Assuming that r members in the structure are damaged, the equilibrium equation and the global stiffness matrix can be modified as

$$\hat{\mathbf{K}}_d \mathbf{u}_d = \mathbf{f}_e \qquad (4)$$

$$\mathbf{K}_d = \sum_{i=1}^{m} \mathbf{K}_d^{(i)} \qquad (5)$$

Where the subscript ‘ d ’ indicates the damaged state, the stiffness matrix of damaged structure $\hat{\mathbf{K}}_d = \hat{\mathbf{K}}_e - \Delta\mathbf{K}$, and the displacement vector at damaged state $\mathbf{u}_d = \mathbf{u}_e + \Delta\mathbf{u}$. And $\Delta\mathbf{K}$ and $\Delta\mathbf{u}$ indicate the change in the stiffness matrix and the displacement vector, respectively. The corresponding $N \times N$ stiffness matrix can be obtained by the elemental decomposition technique as

$$\hat{\mathbf{K}}_d = \mathbf{C}\mathbf{K}_d\mathbf{C}^T \qquad (6)$$

Where $\mathbf{K}_d$ is the $N \times N$ diagonal matrix to be composed of the *m* elemental stiffness parameters expressed by Eq. (5).

Substituting $\hat{\mathbf{K}}_d = \hat{\mathbf{K}}_e - \Delta\mathbf{K}$ and $\mathbf{u}_d = \mathbf{u}_e + \Delta\mathbf{u}$ into Eq. (4) and arranging the result, it can be written by

$$\hat{\mathbf{K}}_e \Delta\mathbf{u} = \Delta\mathbf{K}\mathbf{u}_d \qquad (7)$$

Inserting Eq. (3) and $\Delta\mathbf{K} = \mathbf{C}\left(\Delta\hat{\mathbf{K}}\right)\mathbf{C}^T$ into Eq. (7), it yields

$$\mathbf{C}\mathbf{K}_e\mathbf{C}^T \Delta\mathbf{u} = \mathbf{C}\left(\Delta\hat{\mathbf{K}}\right)\mathbf{C}^T\mathbf{u}_d \qquad (8)$$

Where $\Delta\hat{\mathbf{K}}$ is the change in the elemental stiffness. In order to obtain the equation related with the variation in the elemental stiffness matrix from Eq. (8), the performance index F defined as the following is minimized.

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$$F = \mathbf{K}_e^{-1/2}\left(\mathbf{T}^T \Delta\hat{\mathbf{K}}\mathbf{T}\right)\mathbf{K}_e^{-1/2} \quad (9)$$

Where $\mathbf{T}$ is the transformation matrix, the superscript 'T' indicates the matrix transpose.
In order to utilize the condition to minimize the performance index of Eq. (9), Eq. (8) is modified as

$$\mathbf{C}\mathbf{K}_e^{1/2}\mathbf{K}_e^{-1/2}\mathbf{T}^T\left(\Delta\hat{\mathbf{K}}\right)\mathbf{T}\mathbf{K}_e^{-1/2}\mathbf{K}_e^{1/2}\mathbf{C}^T\mathbf{u}_d = \hat{\mathbf{K}}_e(\Delta\mathbf{u}) \quad (10)$$

Solving Eq. (10) with respect to $\mathbf{C}\mathbf{K}_e^{1/2}\mathbf{K}_e^{-1/2}\mathbf{T}^T\left(\Delta\hat{\mathbf{K}}\right)\mathbf{T}\mathbf{K}_e^{-1/2}\mathbf{K}_e^{1/2}$ using the solution of the generalized inverse, it is derived by

$$\mathbf{C}\mathbf{K}_e^{1/2}\mathbf{K}_e^{-1/2}\mathbf{T}^T\left(\Delta\hat{\mathbf{K}}\right)\mathbf{T}\mathbf{K}_e^{-1/2}\mathbf{K}_e^{1/2} = \hat{\mathbf{K}}_e(\Delta\mathbf{u})\mathbf{R}^+ + \mathbf{Y}\left(\mathbf{I} - \mathbf{R}\mathbf{R}^+\right) \quad (11)$$

Where $\mathbf{R} = \mathbf{C}^T\mathbf{u}_d$ and '+' denotes the generalized inverse matrix, and $\mathbf{Y}$ is an arbitrary matrix.

Introducing the condition to minimize Eq. (9) into Eq. (11), using the property of the generalized inverse of $\mathbf{R}^+\mathbf{R}\mathbf{R}^+ = \mathbf{R}^+$, and solving Eq. (11) with respect to the arbitrary matrix, it follows

$$\mathbf{Y} = \mathbf{K}(\Delta\mathbf{u})\mathbf{R}^+\left(\mathbf{I} - \mathbf{R}\mathbf{R}^+\right) + \mathbf{W}\mathbf{R}\mathbf{R}^+ \quad (12)$$

Where $\mathbf{W}$ is another arbitrary matrix?
Substituting Eq. (12) into Eq. (11) and arranging, the result leads to

$$\mathbf{C}\mathbf{K}_e^{1/2}\mathbf{K}_e^{-1/2}\mathbf{T}^T\left(\Delta\hat{\mathbf{K}}\right)\mathbf{T}\mathbf{K}_e^{-1/2}\mathbf{K}_e^{1/2} = \hat{\mathbf{K}}_e(\Delta\mathbf{u})\mathbf{R}^+ \quad (13)$$

Letting $\mathbf{Q} = \mathbf{C}\mathbf{K}_e^{1/2}$ and solving Eq. (13) with respect to $\mathbf{K}_e^{-1/2}\mathbf{T}^T\left(\Delta\hat{\mathbf{K}}\right)\mathbf{T}\mathbf{K}_e^{-1/2}\mathbf{K}_e^{1/2}$, it yields

$$\mathbf{K}_e^{-1/2}\mathbf{T}^T\left(\Delta\hat{\mathbf{K}}\right)\mathbf{T}\mathbf{K}_e^{-1/2}\mathbf{K}_e^{1/2} = \mathbf{Q}^+\mathbf{K}_e(\Delta\mathbf{u})\mathbf{R}^+ + \left(\mathbf{I} - \mathbf{Q}^+\mathbf{Q}\right)\mathbf{G} \quad (14)$$

Where $\mathbf{G}$ is an arbitrary matrix. Applying the minimization condition of Eq. (9) to Eq. (14) and solving the result with respect to the arbitrary matrix $\mathbf{G}$, it is expressed by

$$\mathbf{G} = -\left(\mathbf{I} - \mathbf{Q}^+\mathbf{Q}\right)\mathbf{Q}^+\mathbf{K}_e(\Delta\mathbf{u})\mathbf{R}^+ + \mathbf{Q}^+\mathbf{Q}\mathbf{D} \quad (15)$$

Where $\mathbf{D}$ is an arbitrary matrix. Inserting Eq. (15) into Eq. (14) and premultiplying both sides of the result by $\mathbf{K}_e^{1/2}$

$$\mathbf{T}^T\left(\Delta\hat{\mathbf{K}}\right)\mathbf{T} = \mathbf{K}_e^{1/2}\mathbf{Q}^+\mathbf{K}_e(\Delta\mathbf{u})\mathbf{R}^+ = \mathbf{K}_e^{1/2}\left(\mathbf{C}\mathbf{K}_e^{1/2}\right)^+\mathbf{K}_e(\Delta\mathbf{u})\left(\mathbf{C}^T\mathbf{u}_d\right)^+ \quad (16)$$

Equation (16) indicates the change of the elemental stiffness transformed to the global coordinate system. Thus, Eq. (16) can be utilized as the damage index to evaluate the health state of structural members. The elemental damage can be investigated by

ISSN: 2319-5967
ISO 9001:2008 Certified
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$$(D.I.)_{(i)} = \frac{\mathbf{T}_{(i)}^T \left(\Delta\hat{\mathbf{K}}_{(i)}\right)\mathbf{T}_{(i)}}{\mathbf{T}_{(i)}^T \left(\mathbf{K}_{e(i)}\right)\mathbf{T}_{(i)}}, \qquad i = 1,2,\ldots m \qquad (17)$$

Where the subscript i indicates the element number, and $\mathbf{T}_{(i)}$, $\left(\Delta\hat{\mathbf{K}}_{(i)}\right)$, and $\mathbf{K}_{e(i)}$ are the transformation matrix, the change in the elemental stiffness, and the elemental stiffness corresponding to the *i*th element at intact state, respectively. The damage locates at the element to represent the abrupt increase of the damage index of Eq. (17). The application of Eq. (17) is illustrated in the following example.

III. NUMERICAL EXPERIMENT

The proposed method is verified through a numerical experiment to detect damage in a truss structure. Two damage scenarios of single and multiple damages are considered. The utilization of the proposed method is examined from the numerical results. The measured data contain external noise to contaminate the accurate data. This work also investigates the sensitivity of external noise on the proposed method.

The damage detection of a plane truss structure model with damaged element was considered. In Fig. 1, the nodal points and members are numbered. The truss is composed of 6 nodes, 9 members, and 9 dofs. All members have the same elastic modulus of 200GPa, cross-sectional area of $2.5\times10^{-3}\text{m}^2$, and density of 7860kg/m^3. The simply supported truss has a single span. Its length is 12m, its height is 3m and each bay is 4m long. The elemental stiffness matrices to disassemble the global truss structure at intact state can analytically be expressed by Eq. (2). And the corresponding responses can be calculated by finite element method under the action of the external force 10N in the downward of node 3. The numerical results of the elemental stiffness matrices and the responses should be saved for subsequent analysis.

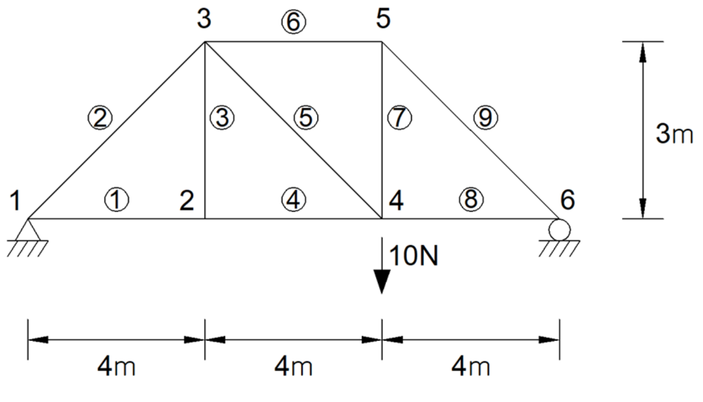

Fig. 1: A three-bay truss structure

The detection of a single damage of 20% section loss at element ⑥ and of multiple damages of 20% section loss at elements ② and ④ was considered in this numerical experiment. The responses of the truss structure at damaged state are measured by measurement sensors. In this application, the measured data were replaced by

ISSN: 2319-5967
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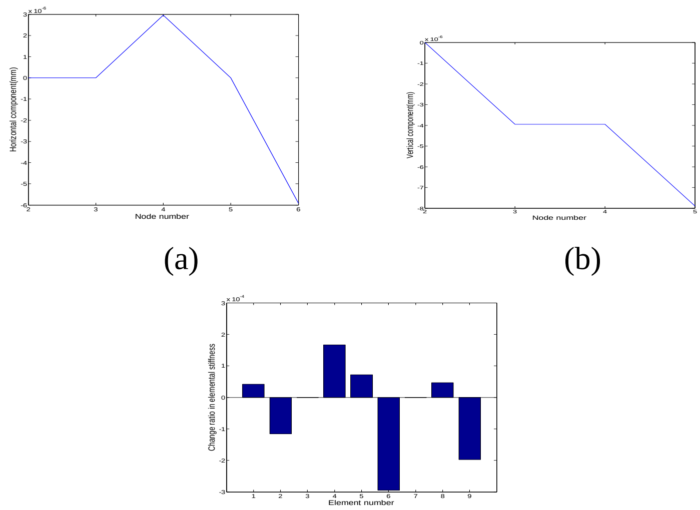
the numerically simulated data. The numerical results on the single damage detection scenario using the noise-free response data are displayed in Fig. 2. Figures 2(a) and (b) represent the difference in the horizontal and vertical displacements before and after the occurrence of the damage, respectively. Inserting the elemental stiffness at intact state and the damaged responses into Eq. (16), the change in elemental stiffness at damaged state can be calculated. The ratio of the stiffness change with respect to the stiffness corresponding to the first row and the first column of the elemental stiffness matrix at intact state $D.I._{(i)} = \Delta k_{(i)}(1,1)/k_{(i)}(1,1)$ was established as the damage index. Figure 2(c) displays the ratio according to the elements. The ratios corresponding to the elements ③ and ⑦ could rarely be predicted because the corresponding horizontal stiffness at intact state are zeros. Thus, the ratios were displayed as zeros in this plot. It is shown in the plot that the stiffness variation is abruptly changed at damaged members and the damage member can explicitly be detected.

The errors included in measured data lead to some deviation from the actual results. Simulated data were established by adding a series of random numbers to represent the errors to actual response data. The measured displacements $\mathbf{u}_d^m$ can be calculated of the analytical displacements $\mathbf{u}_d^0$ as:

$$\mathbf{u}_d^m = \mathbf{u}_d^0 (1 + \alpha\sigma) \tag{18}$$

where α denotes the relative magnitude of the error, and σ is a random number variant in the range $[-1, \quad 1]$. Using $\alpha = 0.01$, this work investigated the sensitivity of the noise included in the measured data. Figures 3(a) and (b) exhibits the measured responses. Despite the existence of the noise, it is found that the damaged elements can be explicitly detected as shown in Fig. 3(c).



(a) (b)

(c)

Fig. 2: Detection of single damage using noise-free displacements: (a) horizontal displacement components, (b) vertical displacement components, (c) stiffness change ratio

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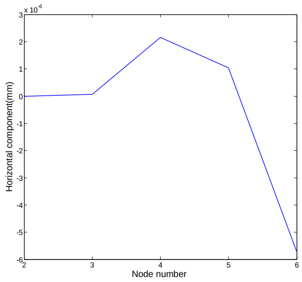
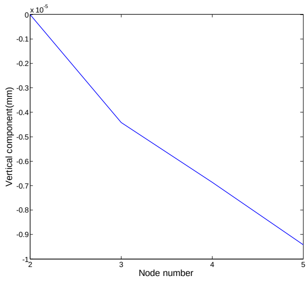
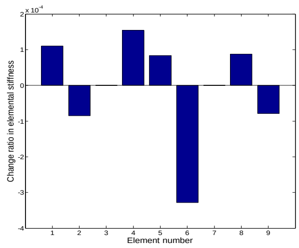
Next numerical experiment considered the damage detection of multiple damages. The damages of 20% section loss locate at elements ② and ④. Figures 4 and 5 represent the displacements and stiffness ratio corresponding to noise-free displacements and displacements contaminated by 1% noise, respectively. It is observed that the damaged elements can be easily detected from the plots.

As shown in the above two applications, it is estimated that the damage detection of vertical members cannot easily be performed because the horizontal stiffness component is zeros. Thus, this work considered the damage detection of the vertical member ③ of 20% section loss. Figure 6 represents the change in elemental stiffness. The stiffness variations due to the damage are rarely found. If the stiffness change is very slight, the damage at vertical members can be expected or the damage can be detected using the measured displacements by the action of horizontal force. In the numerical experiments, it is explained that the proposed damage detection methods can be properly utilized in detecting damage.



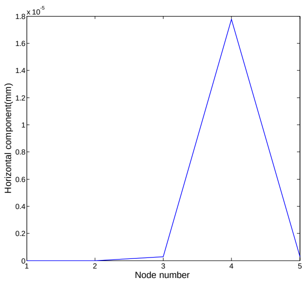


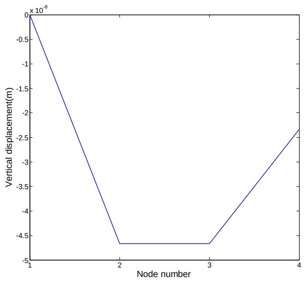
(a) **(b)**



(c)

Fig. 3: Detection of single damage using 1% noise displacements: (a) horizontal displacement components, (b) vertical displacement components, (c) stiffness change ratio





(a) **(b)**

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ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
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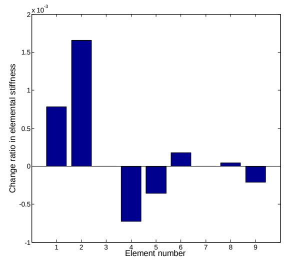
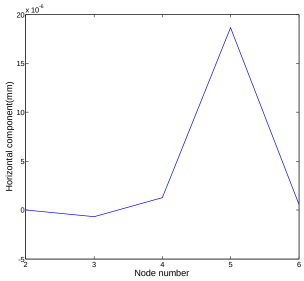
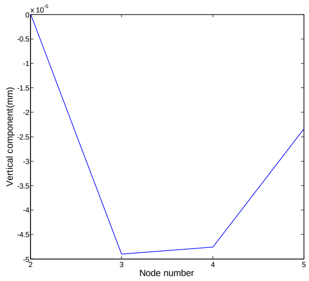


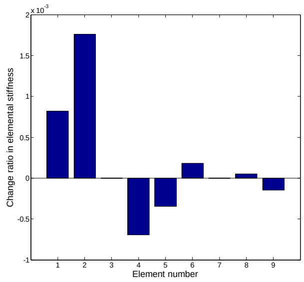
(c)

Fig. 4: Detection of multiple damages using noise-free displacements: (a) horizontal displacement components, (b) vertical displacement components, (c) stiffness change ratio





(a) (b)



(c)

Fig. 5: Detection of multiple damages using 1% noise displacements: (a) deviations of horizontal displacement components, (b) deviations of vertical displacement components, (c) stiffness change ratio

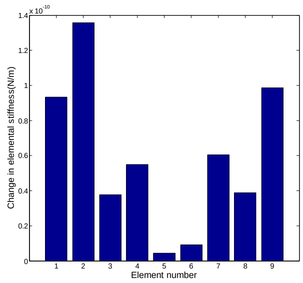


Fig. 6: Change in elemental stiffness matrix using noise-free displacements

ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
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IV. SUMMARY

Structural damage causes the change in responses due to stiffness deterioration. This work presented the damage detection based on the variation in elemental stiffness. The global stiffness matrix of a structure is disassembled by elemental stiffness matrices. Using measured static responses, estimating the residual force in the equilibrium equation or the stiffness variation due to structural damage, and comparing the elemental stiffness change, this work presents the static-based damage detection approach. The method is straightforwardly derived minimizing a performance index in the satisfaction of measured response data. The validity of the proposed method was illustrated in a numerical experiment to consider the detection of single damage and multiple damages in truss structure.

ACKNOWLEDGMENT

This research is supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education (NRF-2016R1D1A1A09918011).

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