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Effects of Grashof number and Magnetic Parameter on MHD Stokes free Convective fluid flow Past an Infinite vertical porous plate in presence of a variable transverse Magnetic field

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Abstract: This research has determined the effects of Grashof number and Magnetic field parameter on MHD free convective flow of an incompressible fluid past an infinite vertical porous plate with joule heating in presence of a variable transverse magnetic field .The derivation of the implicit Scheme schemes has been presented. The momentum equation governing the fluid flows is solved using finite difference method. The resulting set of algebraic equations is solved using Matlab in computer programme. The results are presented in tabular and graphical form to show the effects of Grashof number and Magnetic field parameter on MHD fluid velocity profiles. The results showed that a rise in Grashof number and Magnetic field parameter on MHD lead to increase in fluid velocity profiles.

Index terms: **Momentum Equation, Grashof number, Magnetic field parameter, Finite Difference method and implicit scheme.**

I. INTRODUCTION

A. Literature Review

Onyango *et al* (2017) studied unsteady magneto hydrodynamic viscous incompressible electrically conducting fluid flow between two parallel porous plates of infinite length in x and z directions subjected to a constant pressure gradient in the presence of a uniform transverse magnetic field applied parallel to the y axis with the plate moving with a time dependent velocity. They considered cases, one where the plates are moving in the same direction and another in the opposite direction while fluid suction/injection takes place through the walls of the channels with a constant velocity for suction and injection. It was found that velocity of the fluid increases with the increase in the Reynolds number for both suction on the upper plate and injection on the lower plate and injection on the upper plate and suction on the lower plate. The increase in the magnetic number leads to an increase in the velocity of the fluid in the case where fluid suction is done on the upper plate and injection on the lower plate and the case where fluid injection is done on the upper plate and suction on the lower plate. The velocity profiles increase with increase in the pressure gradient in both cases where fluid suction is done on the upper plate and injection on the lower plate and where fluid injection is done on the upper plate and suction on the lower plate. Danial *et al* (2018) considered a laminar viscous fluid flow being electrically conducting flowing to an accelerated sheet with the fluid flow through a porous medium over a porous surface. The flow was assumed to be under the effect of applied magnetic field, radiative heat source and slip conditions. The mathematical model involved the conversion of governing partial differential equation in to ordinary differential form via similarity transform Physical nature of the problem explored by computing results for temperature and velocity field for wide ranges of influential parameters namely slip parameter A, porosity parameter K, Magnetic field parameter M, Prandtl number Pr, parameter of radiative heat Rn and mixed convection parameter. The result shows that the radiation raises the temperature with porosity increasing the fluid flow speed. The increases in magnetic field strength causes increase in Lorentz force that opposes the flow and thus higher values of, M shows reduction in flow speed and an increase in Pr causes reduction in temperature field. Job *et al* (2018) investigated the turbulent fluid flow problem of a conducting fluid past an infinite porous vertical plate in a rotating system while accounting for the hall currents, mass transfer and Joule's heating. A deviation from the laminar flow, non-porous medium, Hall currents and Joule's heating was done with construction of a mathematical formulation in which turbulence is approximated using Prandtl mixing hypothesis. The final set of partial differential equations obtained is resolved into difference equations using the forward time central space finite difference method. The rotational parameter and Hall parameter are found to

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inhibit the primary velocities while enhancing the secondary velocities. Injection enhances all the flow variables while suction inhibits all flow variables. Job *et al* (2014) solved the MHD turbulent flow past a porous vertical plate using FTCS finite difference method. Turbulence is treated using Prandtl's mixed lengths theorem. They factored into the model the mass transfer, Hall currents and Joule's heating. All these factors were found to have a profound effect on the primary and secondary velocity profiles, temperature profiles and concentration profiles. The primary velocity increases with Gr ,the secondary velocity increases with Gr but decreases with Pr, Temperature increases with E_c and M^2 , but decreases with Pr while Concentration increases with suction velocity but decreases with Sc . Mwangi *et al* (2016) determined the effects of temperature dependent viscosity on MHD natural convection flow past an isothermal sphere. The uniformly heated sphere is immersed in a viscous and incompressible fluid where viscosity of the fluid is taken as a non-linear function of temperature. The Partial Differential Equations governing the flow are transformed into non dimensional form and solved using the Direct Numerical Scheme and implemented in MATLAB. It was observed that increasing the Magnetic parameter M leads to decrease in velocity, temperature, skin friction and the rate of heat transfer. It was also noted that increase in the Grashof number Gr leads to increase in velocity and temperature whereas increase in the values of Nusset number leads to increase in temperature but there is a decrease in velocity. Thomas *et al* (2016) investigated the effect of mass transfer on unsteady Hydromagnetic convective flow, of an incompressible electrically conducting fluid, past an infinite vertical rotating porous plate in presence of constant injection and heat source. The non-linear partial differential equations governing the flow were solved numerically using the infinite differences method. It was found that an increase in Hartmann's number, Prandtl number and Schmidt number retards both the primary and secondary velocity of the fluid at all points. The effect of increasing Grashof number for heat transfer, Grashof number for mass transfer, permeability parameter, Heat source parameter, and Eckert number accelerated both the primary and secondary velocity profiles at all points. Nyariki *et al* (2017) studied unsteady MHD Couette ow of an incompressible electrically conducting fluid between two parallel infinite plates in the presence of an inclined variable magnetic field fixed relative to the moving porous plate with suction. The fluid flow is due to the uniformly accelerated movement of the lower plate of the channel with constant pressure gradient. The nonlinear partial differential equations governing the flow were solved by finite difference method and implemented in MATLAB. It was found that an increase in suction leads to a decrease in the induced magnetic field, the velocity and temperature profiles while suction has a retarding influence to fluid velocity. As the angle of inclination and the magnetic parameter increase, velocity and the induced magnetic field increase. Raj *et al* (2018) investigated homogeneous-heterogeneous chemical reaction and heat absorption effects on a two-dimensional steady hydromagnetic Newtonian nanoliquid ow along a continuously stretching sheet. The flow field is subjected to a uniform magnetic field acting in a direction perpendicular to the direction of nanoliquid flow. A mathematical model of the physical problem was presented involving nonlinear partial differential equations with appropriate boundary conditions. The equations are then transformed into nonlinear ordinary differential equations using a suitable similarity transformation with approximate solutions of the transformed equations obtained using the spectral quasi-linearization method. It was found that the applied magnetic field has a retarding influence on the nanofluid velocity and species concentration, while it does not have any significant effect on the nanofluid temperature while the homogeneous and heterogeneous reactions tend to decrease the species concentration.

In view of the foregoing pertinent literature presented above, it can be inferred that the problem of Magnetohydrodynamic Stokes free convective fluid flow past an infinite vertical porous plate in the presence of a variable transverse magnetic field has received little attention particularly when governing equations are in two dimensions. The specific governing equations used are in one dimension at variables of time (t) and space (y) only but there is need to solve them when they are in two dimension at variables of time (t) and space (x,y). Also the effects of magnetic parameter and Grashof number in presence of variable transverse magnetic field and joule heating parameter on the fluid velocity and temperature have not yet been investigated. In view of these existing gaps, there is need to bridge the gaps.

B. The Model Equations
The non-dimensionalized general governing equations for MHD Stokes free convective flow of an incompressible fluid past an infinite vertical porous plate with joule heating in presence of a variable transverse magnetic field are the momentum and energy equations; (Amenya *et al*, 2013, Sigey *et al*, 2013);

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$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+Gr\theta+M^2u \qquad (1)$$

Where Gr and M^2 are the Grashof number, magnetic parameter respectively.

II. NUMERICAL SCHEME FOR MOMENTUM EQUATION

The Discretization of momentum equation is done to obtain the implicit scheme. First, we derive the finite differential form of implicit method for the given model momentum equation (1) and then present an algorithm for the method. Discretization of momentum equation (1) is only considered in this study. The partial derivatives in momentum equation (1) are replaced by their finite approximations. This discretization gives the scheme

$$\frac{U_{i,j}^{n+1}-U_{i,j}^{n}}{(\Delta t)}+u_{i,j}^{n}\frac{U_{i+1,j}^{n}-U_{i-1,j}^{n}}{2(\Delta x)}+v_{i,j}^{n}\frac{U_{i,j+1}^{n}-U_{i,j-1}^{n}}{2(\Delta y)}=\frac{U_{i+1,j}^{n}-2U_{i,j}^{n}+U_{i-1,j}^{n}}{(\Delta x)^2}+\frac{U_{i,j+1}^{n}-2U_{i,j}^{n}+U_{i,j-1}^{n}}{(\Delta y)^2}+Gr\theta_{i,j}^{n}+M^2u_{i,j}^{n}$$

(2)

If we take $\theta_{i,j}^{n}=u_{i,j}^{n}=v_{i,j}^{n}=1$, $(\Delta x)=(\Delta y)$, and let $r=\Delta t/(\Delta x)^2=\Delta t/(\Delta y)^2$, and $\phi=\Delta t/(\Delta x)=\Delta t/(\Delta y)$ and multiply (2) by $2\Delta t$ throughout , we get implicit scheme

$$(4r+2)U_{i,j}^{n+1}-2rU_{i+1,j}^{n+1}-2rU_{i-1,j}^{n+1}=(2-4r+2M^2\Delta t)U_{i,j}^{n}+(2r-\phi)U_{i,j+1}^{n}+(2r+\phi)U_{i,j-1}^{n}-\phi U_{i+1,j}^{n}+\phi U_{i-1,j}^{n}+2Gr\Delta t\theta_{i,j}^{n}$$

(3)

We use the initial and boundary conditions

$$u(x,y,0)=\theta(x,y,0)=1 \qquad (4)$$

$$\left.\begin{aligned}&u(0,y,t)=0,\ u(10,y,t)=0, 0\le x\le 10\\&u(x,0,t)=0,\ u(x,2,t)=0, 0\le y\le 2\end{aligned}\right\} \qquad (5)$$

respectively substituted in (3), we obtain the matrix-vector equation. For the known values on the right hand side of the algebraic equations (3) we obtain the finite difference equations obtained at any space node, say, i at the time level n+1 has only three unknown coefficients involving space nodes at i , i-1 and i+1 at n+1 on the left hand side of the algebraic equations (3) . In matrix notation, the 10 algebraic equations in (3) can be expressed as AU = B where U is the unknown vector of order N-1 at any time level n+1, B is the known vector of order N-1 which has the value of U at the n time level and A is the coefficient square matrix of order N-1 by N-1 which is a triadiagonal structure. Then the 10 systems of equations in (3) can be written with 10 unknowns in matrix-vector form as;

$$\begin{bmatrix}3.25&-0.625&0&0&0&0&0&0&0&0\\-0.625&3.25&-0.625&0&0&0&0&0&0&0\\0&-0.625&3.25&-0.625&0&0&0&0&0&0\\0&0&-0.625&3.25&-0.625&0&0&0&0&0\\0&0&0&-0.625&3.25&-0.625&0&0&0&0\\0&0&0&0&-0.625&3.25&-0.625&0&0&0\\0&0&0&0&0&-0.625&3.25&-0.625&0&0\\0&0&0&0&0&0&-0.625&3.25&-0.625&0\\0&0&0&0&0&0&0&-0.625&3.25&-0.625\\0&0&0&0&0&0&0&0&-0.625&3.25\end{bmatrix}\begin{bmatrix}U_{1,1}^{1}\\U_{1,1}^{1}\\U_{1,1}^{1}\\U_{1,1}^{1}\\U_{1,1}^{1}\\U_{1,1}^{1}\\U_{1,1}^{1}\\U_{1,1}^{1}\\U_{1,1}^{1}\\U_{1,1}^{1}\end{bmatrix}=\begin{bmatrix}M^2+Gr+1.25\\M^2+Gr+1.375\\M^2+Gr+1.375\\M^2+Gr+1.375\\M^2+Gr+1.375\\M^2+Gr+1.375\\M^2+Gr+1.375\\M^2+Gr+1.375\\M^2+Gr+1.375\\M^2+Gr+1.375\end{bmatrix} \qquad (6)$$

We hold one variable constant and vary the other with four different values of each one of them. With use of these four different values of Grashof number and magnetic parameter in (5).We get set of four different values of velocity profile for Gr and M^2 is substituted into Equations (3).

III. NUMERICAL RESULTS

A. *Effects of Grashof number on the velocity profiles*

We set M^2 =3 and vary Gr with four different values i.e Gr = 1, 3, 6 ,9 in (6). With use of these four different values of Grashof number in (6), we get set of four different values of velocity profile as shown in table 1.

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Table 1: Effects of Grashof number on velocity profiles

Plate height	Gr = 1	Gr = 3	Gr = 6	Gr = 9
X=1	1.1880	1.92390	3.62390	6.623900
X=2	1.6538	3.30580	6.20580	9.205800
X=3	2.1602	4.79560	7.79560	10.79560
X=4	2.4426	5.71125	8.71125	11.71125
X=5	2.5764	6.13580	9.13580	12.1358
X=6	2.5972	6.15410	9.15410	12.1541
X=7	2.5106	5.76990	8.76990	11.7699
X=8	2.2935	4.90640	7.90640	10.9064
X=9	1.8879	3.49080	6.39080	9.39080
X=10	1.18585	1.95007	3.92007	6.92007

Table 1 represents effects of Gr on velocity profiles. The Fig 1 present the typical velocity profiles in the boundary layer for various values of the Grashof number. It is observed that an increase in Gr leads to a rise in the values of velocity due to enhancement in buoyancy force. Here, the plate height between x = 8 and x = 9 values correspond to cooling of the plate. In addition, it is observed that the velocity increases sharply at x = 6 and x = 7 of the porous vertical plate and then decays to the free stream value due to dominance of viscous force. Increase in Gr contributes to an increase in velocity when all other parameters that appear in the velocity field are held constant. Physically, the thermal variant of Grashof number is a derivative of buoyancy effects. According to Archimedes principle, less dense substance will be displaced by the heavier substance. The heated fluid expands and its density reduces. The heavier fluid sinks to displace this lighter fluid and hence the increased velocity.

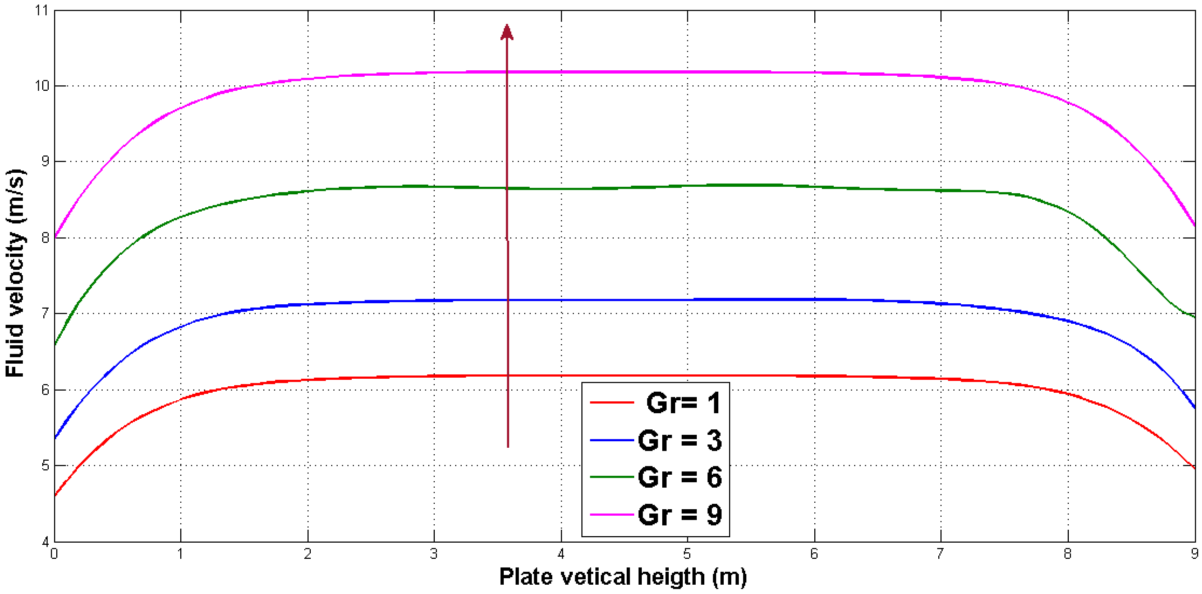

Fig 1: Fluid velocity against Plate Height at Varying Grashof Number

B. Effects of Magnetic parameter on the velocity profiles

We set Gr = 1 and vary M^2 with four different values i.e M^2 = 3, 5, 8 ,10 in (6). With use of these four different values of Magnetic parameter in (6), we get set of four different values of velocity profile as shown in table 1.

From Fig.2 and table 2 it is seen that velocity increases on increasing magnetic parameter. With an increase in M^2, the fluid velocity decreases in a region near the bottom of the plate and then increases in the region away from the bottom of the plate. For instance at x = 1 and x = 5 the velocities are 2.1900 and 3.1857 respectively for M^2 =3.0 as seen in Table 1. The magnetic parameter signifies the relative strength of the magnetic force to the viscous force, magnetic parameter increases on increasing the strength of the magnetic force. This is as a result of Lorentz force which acts in the direction of the fluid flow. This force increases the velocity of the fluid.

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Thus we conclude that the magnetic field has accelerating influence on the fluid flow. This tendency of the magnetic field may be due to Newtonian heating at the surface of the plate.

Table 2: Effects of Magnetic parameter on velocity profiles

Plate height	$M^2 = 3$	$M^2 = 5$	$M^2 = 8$	$M^2 = 10$
x = 1	2.190000	2.990000	4.190000	4.989999
x = 2	2.987988	3.947998	5.387997	6.347997
x = 3	3.147592	4.139589	5.627585	6.619583
x = 4	3.179479	4.177866	5.575447	6.673834
x = 5	3.185700	4.185316	5.68474	6.684356
x = 6	3.186161	4.185777	5.685201	6.684817
x = 7	3.182333	4.180723	5.678304	6.676692
x = 8	3.161988	4.153985	5.641981	6.633978
x = 9	3.059998	4.019997	5.459996	6.419996
x = 10	2.549999	3.349999	4.549999	5.349999

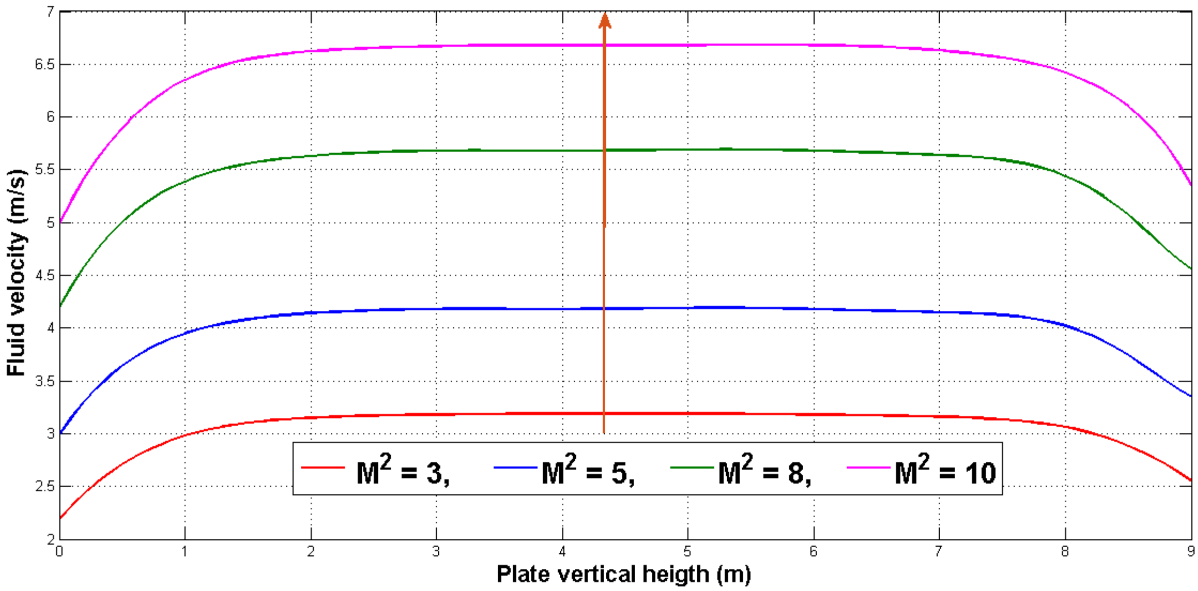

Fig 2: Fluid velocity against Plate Height at Varying Magnetic parameter

IV. CONCLUSION

We have determine the effects of Grashof number and Magnetic parameter MHD Stokes free convective flow of an incompressible fluid past an infinite vertical porous plate with joule heating in presence of a variable transverse magnetic field. The results showed that a rise in Grashof number and Magnetic field parameter on MHD lead to increase in fluid velocity profiles.

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NOMECLATURE

M^2	Magnetic Parameter
Gr	Grashof number

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Pr	Prandtl number
IFDS	Implicit Finite Difference Scheme
PDE	Partial Differential Equation
MHD	Magnetohydrodynamics
FDM	Finite Difference Method

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