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Thermo Elasto - Hydrodynamic Lubrication Analysis of a Journal Bearing

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Abstract: In this study, thermo-elasto hydrodynamic lubrication analysis of a journal bearing is done. Reynolds equation is used to obtain pressure distribution while velocity distribution is derived from the momentum equation while the energy equation is solved to obtain the temperature distribution. The partial differential equations derived are highly nonlinear and have been transformed to non-dimensional form and solved by finite differences method. Numerical results are presented graphically using MATLAB. In this study it is noted that by varying the parameters there will be a variation in pressure profiles, the load capacity and temperature profiles. The results obtained can be used in engineering in the design of more efficient journal bearings.

Index terms **- Journal bearing, Thermo Elasto hydrodynamic Lubrication, Polar Coordinates.**

I. INTRODUCTION

Plain bearings consist of a shaft which rotates freely in a supporting metal sleeve or shell. There are no rolling elements in these bearings. Their design and construction may be relatively simple, but the theory and operation of these bearings can be complex. The space between the two cylinders is very small and the flow of the fluid between them can be taken as the flow between two parallel plates. Reynolds explained the mechanism in the hydrodynamic lubrication and used the method of viscous liquid films placed between the surfaces in motion, ever since a lot of research has been done. Smith [1] studied the effect of lubricant inertia on the dynamic characteristics, in the paper it is shown that in the linearized characteristics of journal motion, the expression for disturbance force should include acceleration terms arising from the inertia of the lubricant, it is concluded that in many cases of vibration of turbine rotors in journal bearing, the effect of the acceleration terms is unimportant. Nada and Osman [3] investigated the problem of lubrication of finite hydrodynamic journal bearing lubricated by magnetic fluids with couple stresses. The results indicated that the influence of couple stresses and magnetic effects on the bearing characteristic are significantly apparent. It was concluded that fluids with couple stresses are better than Newtonian fluids. Muzakkir *et. al.* [6] in the study conducted investigations to determine the effectiveness of providing axial and circumferential grooves in minimizing wear of journal bearing operating in mixed lubrication regime. Based on the experimental results a grooving arrangement that minimized the bearing wear is recommended. Lijesh and Hirani [7] proposed bearing (FFB+ Hal Bach magnetic bearing) to deal with the extreme operating conditions like: high load and low speed FFBs. Ma and Li [8] provided a design and operation of large multiple pockets tilted pad hydrostatic journal bearing showing that the pad attitude adjusting moment increases with the pad tilt angle linearly for appropriate pocket area ration. Kalavathi [9] *et al* presented a numerical study of effect of surface roughness on finite porous journal bearing with heterogeneous slip/no-slip surface. It was shown that pressure and load carrying capacity increases with surface roughness for the bearing with slip/no-slip surface. Christiansen *et al* [10] compared two dimensional Reynolds equation and three dimensional Navier-stokes equation approaches of investigating journal bearing by performing an investigation of two inlet groove designs: the axial and circumferential groves. Yadav *et al* [11] developed a numerical method based on finite element method for direct computation of gas film damping and stiffness coefficients. The numerical results presented showed that the bearing elliptic ratio significantly affects the nonlinear trajectory of the journal. As sited above it is quite evident that even though so much research has been done on journal bearing lubrication none uses a conservative scheme in the analysis of a thermo Elasto hydrodynamic lubrication of a journal bearing.

II. MATHEMATICAL ANALYSIS

This study considers the mathematical analysis of a journal bearing that is cylindrical in shape therefore need for a polar coordinate system. The equations governing the flow of the fluid in bearing namely the Reynolds equation, momentum equations, and the equation of energy are expressed in polar form and analyzed to suit the model.The mathematical analysis was done by taking a sector of the journal bearing as shown in the diagram.

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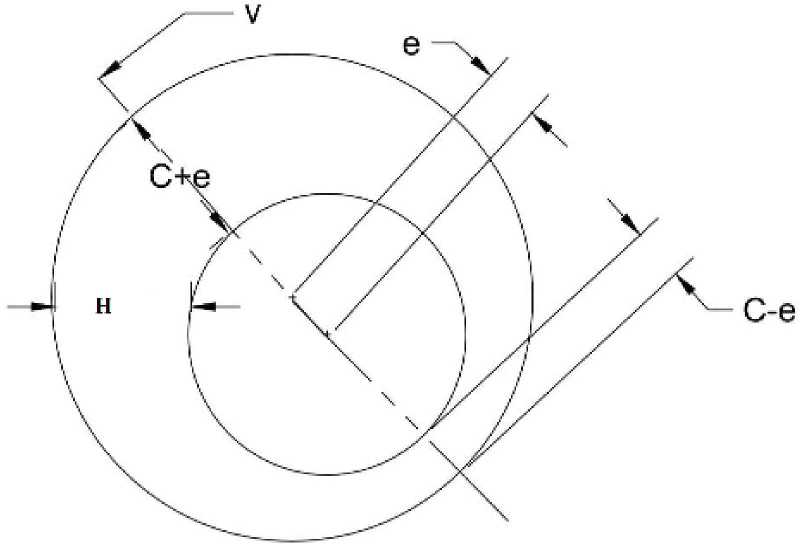

Fig 1: Bearing scheme

Film thickness

From figure 1 above, the Journal bearing scheme examined is cylindrical and finite bearing. The film thickness at any angle θ is given by Nada and Osman [3] as;

$$\overline{\mathrm{H}} = \mathrm{C} + \mathrm{e}\cos\theta \qquad (1)$$

In dimensionless form:

$$\overline{\mathrm{H}} = 1 + \mathrm{e}\cos\theta \qquad (2)$$

MODEL EQUATIONS

On making the substitutions;

$$\bar{r} = \frac{r}{r_0},\ \overline{H} = \frac{H}{H_0},\ \bar{\mu} = \frac{\mu}{\mu_0},\ \bar{u}_\theta = \frac{u_\theta}{u_0},\ \bar{p} = \frac{h_0^2 P}{r_0^2 u_0 \mu_0} = \frac{p}{p_0},\ \bar{\theta} = \theta \text{ and } \bar{T} = \frac{T}{T_0} \qquad (3)$$

and computing the necessary derivatives, the Reynolds equations and the energy equations were reduced to non - dimensional form as shown below with an assumption of unsteady flow and constant specific heat for the energy. Solutions to the Reynolds equation and energy equation give the pressure distribution and the temperature distribution in the journal bearing respectively.

$$\bar{r}\frac{\partial}{\partial \bar{r}}\left[\frac{\bar{r}\bar{H}^3}{\bar{\mu}}\frac{\partial \bar{p}}{\partial \bar{r}}\right] + \frac{\partial}{\partial \bar{\theta}}\left[\frac{\bar{H}^3}{\bar{\mu}}\frac{\partial \bar{p}}{\partial \bar{\theta}}\right] = 6S_t\left[\frac{r_0}{H_0}\right]^2 \bar{r}^2\frac{\partial \bar{H}}{\partial \bar{\theta}} \qquad (4)$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial r} + \frac{u}{r}\frac{\partial T}{\partial \theta} = \frac{1}{R_e}\frac{\partial T}{\partial r} + \frac{1}{R_e P_r}\frac{\partial^2 T}{\partial r^2} + \frac{1}{R_e P_r}\left[\frac{1}{r} + \frac{\sin^2\theta}{e^2}\right]\frac{\partial^2 T}{\partial \theta^2} + \frac{P_r C_p U_0^2 \sin^2\theta}{e^2}\left[\left[\frac{\partial u}{\partial \theta}\right]^2 + \left[\frac{\partial v}{\partial \theta}\right]^2\right] - \epsilon T\left[u\frac{\partial P}{\partial r} + \frac{v}{r}\frac{\partial P}{\partial \theta}\right] \qquad (5)$$

Rates of flow

The reduced volumetric rates of flow on the edges of the journal bearing in the increasing direction of r and θ agreed with Ashour [5] are;

$$u_r = -\frac{1}{12}\frac{h^3}{\mu}\frac{1}{r^2}\frac{\partial \bar{P}}{\partial \bar{r}} \tag{6}$$

$$u_\theta = -\frac{1}{12}\frac{h^3}{\mu}\frac{1}{r^2}\frac{\partial \bar{P}}{\partial \bar{\theta}} + \omega\frac{h}{2} \tag{7}$$

Load capacity

The load capacity is calculated once pressure distribution is determined. The load capacity is given in dimensionless form is expressed as;

$$\bar{W} = \frac{W}{K\, r_0^2} = \int_a^b \int_\alpha^\beta (P)\, d\bar{\theta}\, d\bar{r} \tag{8}$$

Simpson's $\frac{1}{3}$ rule is used to evaluate this integral in Matlab. This being the summation of 441 pressure points where the even and odd points had a coefficient of both 4 and 2 respectively, with the initial and final points having coefficient of 1.This was then multiplied by the change in value of both r and θ then divided by nine.

$$\overline{W} = \frac{\Delta r \Delta\theta}{9}\left\{ p(0,0) + p(21,21) + 4\sum_{n=2,even}^{441} p(i,j) + 2\sum_{n=3,0dd}^{441} p(i,j)\right\} \tag{9}$$

COMPUTATION PROCEDURE

Thefollowing transformations$\theta = \theta = \theta(\eta^\theta), r = r(\eta^r), \eta^r = \eta^r(r), \eta^\theta = \eta^\theta(\theta), h_\theta = \frac{1}{h_\theta}\frac{d\theta}{d\eta^\theta}, h_r = \frac{1}{h_r}\frac{dr}{d\eta^r}$and $j = h_r h_\theta$.such that $\frac{\partial}{\partial\theta} = \frac{1}{h_\theta}\frac{\partial}{\partial\eta^\theta}, r\frac{\partial}{\partial r} = \frac{1}{h_r}\frac{\partial}{\partial\eta^r}$are used and on applying the conservative finite difference scheme proposed by Morinishi [2], Equations (4) and (5) are discretized into finite difference form that forms a set of linear algebraic equations which are then solved using MATLAB. The pressure and temperature at each point is obtained. The five point stencil method is used to approximate the derivatives of the governing equation where (i,j) is the point in the grid and it's four neighbors. These yields;

$$P_{i,j} = \left(\frac{-H^3}{\mu h_r^2}\left(\frac{P_{i+1,j}+P_{i-1,j}}{(\Delta\eta^r)^2}\right) - \frac{H^3}{h_r}\left(\frac{P_{i+1,j}-P_{i-1,j}}{2\Delta\eta^r}\right)\frac{\partial}{\partial\eta^r}\left(\frac{1}{h_r\mu}\right) - \frac{1}{h_\theta^2}\frac{\partial\left(\frac{H^3}{\mu}\right)}{\partial\eta^\theta}\left(\frac{P_{i,j+1}-P_{i,j-1}}{2\Delta\eta^\theta}\right)\right) - \frac{-H^3}{\mu h_\theta^2}\left(\frac{P_{i,j+1}+P_{i,j-1}}{(\Delta\eta^\theta)^2}\right) + \frac{M_1}{h_\theta}\frac{\partial}{\partial\eta^\theta}(r^2 H) - \left(\frac{2H^2}{h^2\mu(\Delta\eta^r)^2} + \frac{2H^2}{h_\theta^2(\Delta\eta^\theta)^2}\right)$$

(10)

and

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$$T(i,j,k+1) = T(i,j,k-1) + 2\Delta t\left\{\left[\left[\frac{1}{rR_e h_r} - \frac{u(i,j,k)}{rh_r}\right]\frac{T(i+1,j,k)-T(i-1,j,k)}{2\Delta\eta^r} - \frac{u(i,j,k)}{rh_\theta}\frac{T(i,j+1,k)-T(i,j-1,k)}{2\Delta\eta^\theta} - \frac{1}{r^2R_eP_rh_r^2}\frac{T(i+1,j,k)-2T(i,j,k)+T(i-1,j,k)}{(\Delta\eta^r)^2} + \frac{1}{R_eh_\theta^2}\left[\frac{1}{r}+\frac{\sin^2\theta}{\epsilon^2}\right]\frac{T(i,j+1,k)+T(i,j-1,k)}{(\Delta\eta^\theta)^2} + \frac{P_rC_pU_0^2\sin^2\theta}{\epsilon^2}\left[\left[\frac{1}{h_\theta}\frac{u(i,j+1,k)-u(i,j-1,k)}{2\Delta\eta^\theta}\right]^2 + \left[\frac{1}{h_\theta}\frac{v(i,j+1,k)-v(i,j-1,k)}{2\Delta\eta^\theta}\right]^2\right]\right]\right\} - \epsilon T(i,j,k)\left[u(i,j,k)\frac{P(i+1,j,k)-P(i-1,j,k)}{2\Delta\eta^r} + \frac{v(i,j,k)}{r}\frac{P(i,j+1,k)-P(i,j-1,k)}{2\Delta\eta^\theta}\right] \quad (11)$$

III. RESULTS AND DISCUSSION

A glimpse to the numerical results is as shown below.

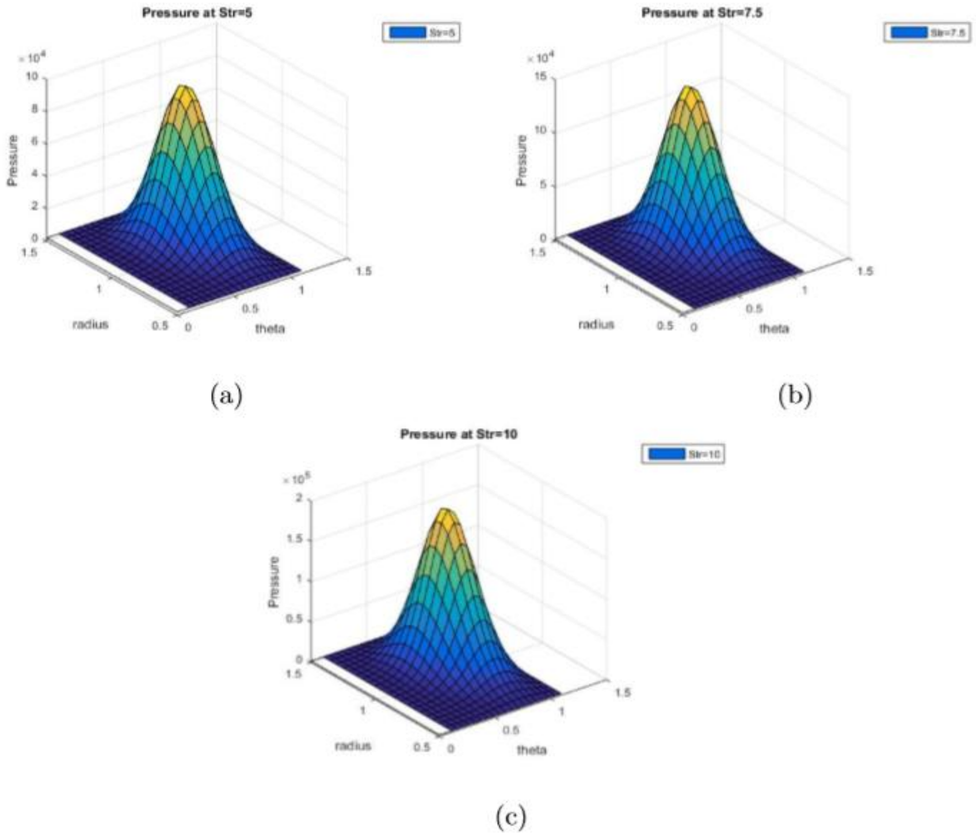

Fig 2: Pressure profiles mesh

Pressure profiles for selected measurements are compared in figures (2) and (3). Figure (2) a, b and c gives the pressure distribution over the whole sector for the three chosen values of Stribeck numbers. Figure (3) gives the pressure distribution against the radial distance for a fixed value theta and also the pressure distribution against the axial distance for a chosen value of radius. The Stribeck number was maintained at 5, 7.5 and 10 so secure three conditions within the standard bounds for elasto hydrodynamic lubrication. The eccentricity ratio was fixed at 0.4.The pressure profiles increased steadily from zero and changed rapidly in the area of smallest film thickness where it reached a maximum. In this region the film was convergent. The pressure then gradually dropped to zero. There was an agreement in the pressure distributions for the three values of Stribeck numbers. The pressures build up from the leading edge up to around 75 percent of the sector, followed by a drop pressure towards the trailing edge. This is because as the fluid enters the sector from the leading edge. There is a decrease in the velocity that causes a decrease in minimum film thickness hence pressure increases. The sharp drop in

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pressure is due to the increase in velocity as the fluid escapes from the sector resulting to an increase in film thickness thus decrease in pressure. Figure (3) it is also seen that the pressure increased steadily from zero to a maximum value then decreased steadily to zero. The discrepancies in the magnitudes of the pressure profiles is as a result of an increase in Stribeck number leads to an increase in the rotational speed of the Journal causing an increased fluid velocity resulting to a rise in pressure buildup. The distributions along the radial and theta directions showed similar trends for pressure distribution.

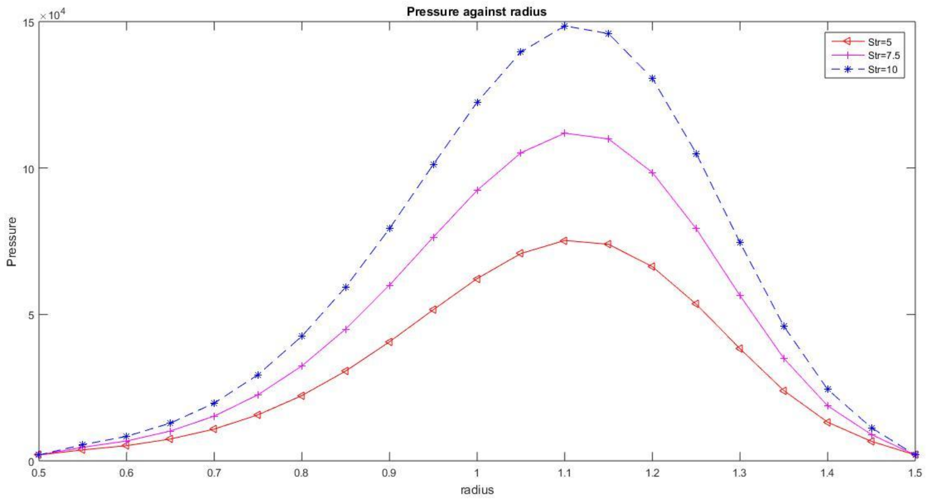

Fig 3 (a): Pressure against radius

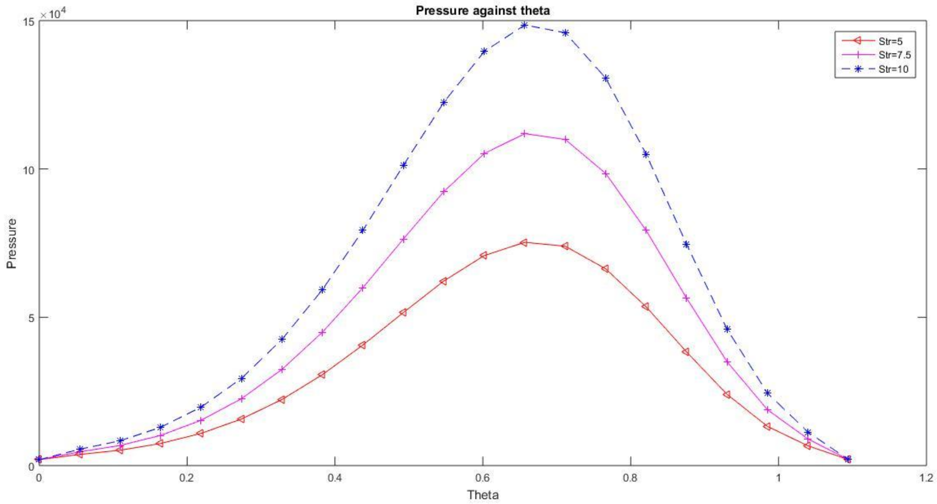

Fig 3 (b): Pressure against theta

Table 1: Bearing load

STRIBECK NUMBER	5	7.5	10
LOAD	3.3467e+04	4.8398e+04	6.3329e+04

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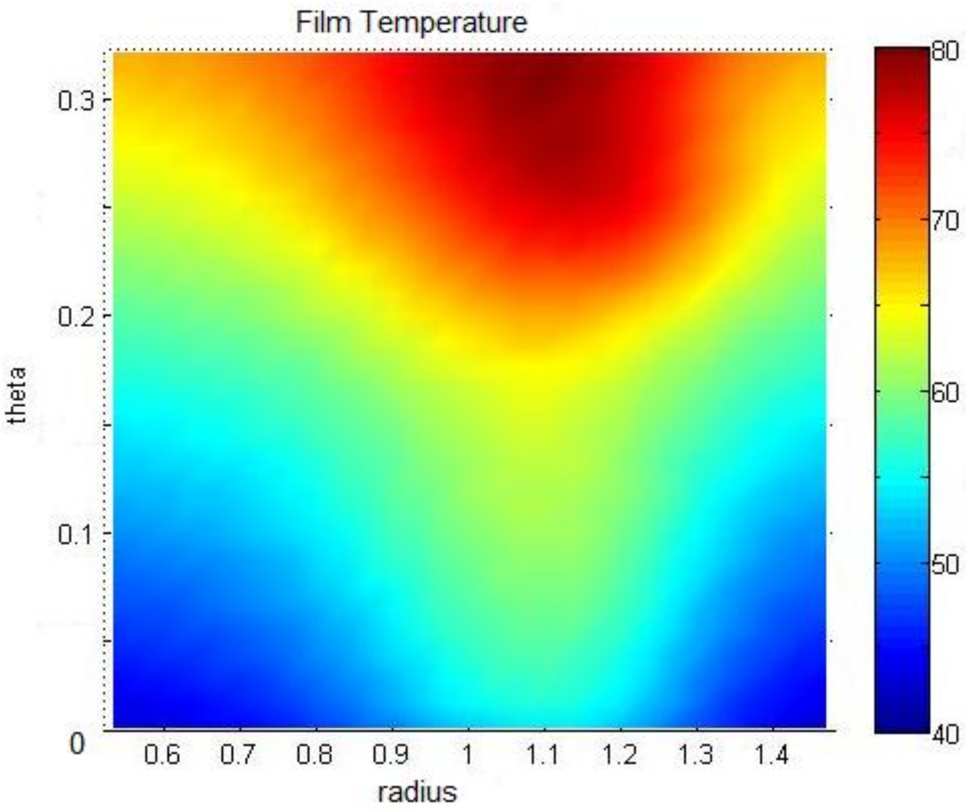

Fig 4: Fluid film temperature in the sector

Figure (4) shows the temperature distribution in the fluid film. A variation of temperature in the radial axial directions is caused by the hot oil carry over effect due to the rotation of the Journal. There is energy created in the walls of the journal due to the arising momentum generated by the slip. The temperature of the fluid adjacent to the rotating journal is higher than that of the fluid near the fixed cylinder. The bearing load is computed against Stribeck number as shown in Table 1. There is a direct proportion between the two. The pressure in the fluid film plays an essential role in determining the bearing load.

IV. CONCLUSION

Two dimensional Reynolds equation and Energy equations in r and θ directions have been solved to obtain the pressure profiles and temperature profiles by finite difference method. On obtaining the pressure, Simpson's rule is used to obtain the load. Of importance, the energy equation is solved using three loops. Two loops are used in updating the pressure and velocity while the third loop outputs the temperature. The results obtained for this model strongly agree with the results previously published by different authors. This proves the fidelity of the model especially the method used to compute the load. The results obtained in this model will go a long way in designing journal bearings.

V. RECOMMENDATIONS

In the future, the authors recommend that the grid could be made finer by increasing the number of nodes from 441 to a bigger number for more accurate result with a reduced marginal error. The negative effect of having a big number of nodes is a higher computational power will be required. Instead of using FDM (Finite difference method) which entirely depends on Taylor series expansions and assumes a higher regularity of the solution an alternative such as FEM (Finite element method) could be considered in the numerical methods for discretization of the non-linear partial differential equation formulated. It is also recommended that the research be extended in other bearings such as step bearing, thrust bearing and slide bearing.

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NOMENCLATURE

C: Bearing clearance

e: Eccentricity of the journal center

h: Lubricant film thickness

$\bar{H}$: Dimensionless film thickness

$\bar{W}$: Non dimensional load capacity $\frac{W}{Kr_0^2}$.

$\alpha - \beta$: Angular extent of the pad

P: Pressure, pa

$\bar{r}$: Non dimensional radius, $\frac{r}{r_0}$

θ: Angular co-ordinate

r: Radial co-ordinate

μ: Viscosity of oil Pa.s

i, j: Index of node in radial and circumferential directions

S_t: Stribeck number

v: Velocity along the radial directionm/s

u: Velocity along the theta directionm/s

T: Lubricant temperature

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