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Magneto hyrodynamics Fluid Flow in Convergent-Divergent Conduit

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Abstract -The stokes flow problem of a viscous, incompressible and electrically conducting fluid flow through a convergent-divergent conduit and in presence of a constant pressure gradient and uniform magnetic field applied perpendicular to the fluid flow direction considering joule heating has been studied. The governing equations for this fluid flow study includes continuity equation, momentum equation and the energy equation. The governing equations are transformed into non-linear Ordinary Differential Equations by introducing similarity transformation. The higher order non-linear Ordinary Equations are then reduced into first order Ordinary Differential Equation and then solved using bvp4c collocation and implemented in MATLAB version 7.9.0(R2018b). Observations and discussions on the effect of varying various parameters such as Hartman Number Ha, , Prandtl Number Pr, Eckert Number Ec, Joules heating parameter J, Reynolds Number Re and Unsteadiness Parameter λ on the velocity profiles and temperature profiles is done. A change in the value of the parameters mentioned above was observed to either increase, decrease or to have no effect on the Velocity and Temperature profiles respectively. The results obtained in this research is very helpful to engineers as they design the motion of liquid metals or alloys in the cooling systems of advanced nuclear reactors.

Index Terms -**Magneto-hydrodynamic, Non-linear Viscosity, Similarity Transformation, Jeffery-Hamel.**

I. INTRODUCTION

Magneto hydrodynamics (MHD) is the study of the flow of an electrically conducting fluid in presence of magnetic field. Converging-Diverging flow is the flow of viscous electrically conducting fluid between two non-parallel inclined plane walls. In this study we shall consider a fluid to be a substance that undergoes continuous deformation when acted upon by an external force however small. Alam M.S [1] studied the critical behavior of the MHD flow in convergent-divergent channels. The convergence of critical values and the change in bifurcation graph for Re and by the positive effect of Ha and the critical relationship among the parameters was discussed with the help of approximation method by Rahman and Xu (2004). He found that increasing Reynolds numbers leads to decrease in velocity field and increasing the Hartman number will lead to backflow reduction.

O.D. Makinde [2] investigated the Magneto hydrodynamic (MHD) flows in Convergent-Divergent Channels which was an extension of the classical Jeffery-Hamel flows to MHD. He interpreted that the effect of external magnetic field works as a parameter in solution of the MHD flows in Convergent-Divergent Channels. Therefore, a non-dimensional magnetic parameter Ha was involved with the flow Reynolds number and the Channel angular width. A Perturbation series of twenty-four terms in powers of parameters Re,α and Ha was obtained and they showed how the flows change and bifurcate as the flow parameters vary by using algebraic approximate method. They noted that the critical value $\alpha_c$increases uniformly for the increasing Hartman number and as αincreases then Re decreases.O.D Makinde [3] investigated and solved the non-linear 2D Navier-Stokes equations modeling the flow field using a perturbation technique applying the special type of Pade-Hermite approximation method implemented numerically on MAPLE and a bifurcation study was also performed. The increasing values of magnetic Reynolds number causes a general decrease in the fluid velocity around the central region of the channel. The flow reversal control is also observed by increasing magnetic field intensity. The bifurcation study reveals the solution branches and turning points. Voropayev [4] investigated the critical relationships among the parameters to observe the behavior of the magnetic Reynolds number and the angle. It is found that as αincreases then Re decreases and conversely Re increases when αdecreases. This implies that both channel angle and Reynolds number are inversely proportional to each other. They also found that increasing Reynolds numbers leads to decrease in velocity field and increasing the Hartman number will lead to backflow reduction. Rahimi [5] wall friction in the Newtonian and non – Newtonian fluid flow in convergent divergent channels. He found that the pressure gradient was a key parameter in the Newtonian and non – Newtonian fluid flow behaviors, and good agreement between the lubrication theory and asymptotic

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solution was found for small cone angles and small friction parameter values. He concluded that an increase in Reynolds number will lead to increase in velocity of the fluid in the converging section and in diverging channels an increase in the channel angle increases the velocity of the fluid in the wall apparently because of an increase in the size of favorable pressure gradient. Moffat [6] investigated effect of MHD on convergent divergent flow and found that analytically that velocity discontinuity was rapidly growing with the increase in the Reynolds number in the case of two dimensional flow of unmixed fluid between convergent walls with the variable viscosity. Bird [7] investigated the converging incompressible slow visco-elastic fluid flow through cones and wedges. They used perturbation method to solve the resulting solution and found that in the case of two-phase flow the outlet flow are resistance was beyond the range of other regions where the fraction value of air bubble was maximal. They also managed to control the air mass ratio at the end of the end of the cone channel to reduce the flow resistance in the region. Khan [8] studied the critical behavior of the MHD flow in convergent-divergent channels. The convergence of critical values and the change in bifurcation graph for Re and by the positive effect of Ha and the critical relationship among the parameters was discussed with the help of approximation method. He found that increasing Reynolds numbers leads to decrease in velocity field and increasing the Hartman number will lead to backflow reduction.J. Nagler [9] investigated Jeffrey-Hamel flow on non-Newtonian fluid with nonlinear viscosity and wall friction. He found that Increase in Reynolds number leads to an increase in velocity and temperature profiles, increase in Eckert number leads to an increase in velocity and temperature profiles and when the Prandtl number increases it leads to increase in velocity and there is reduction in temperature profile. Although many investigations have been made in the past two decades, only a few published papers take into account Magneto-hydrodynamic fluid flow between two inclined plates considering joule heating, both plates are rigid and the flow is under constant pressure gradient and uniform magnetic field applied perpendicular to the flow field considering joule heating has been investigated. This has varied applications in dyeing industries and polymer industries.

II. MATHEMATICAL ANALYSIS

A. Flow configuration

This study has considered the analysis of the unsteady stokes fluid flow between two inclined plates with a constant pressure gradient and uniform magnetic field applied normal to the flow field. The fluid is incompressible, good in electrical conductivity and has non-linear viscosity. The fluid flow is unsteady with motion purely radially and purely dependent on r, θ and t. The power law model which govern non-newtonian fluids has been used in this study to non-dimensionalise the momentum equation. Pressure gradient drives the motion of the fluid while viscous dissipation and joule heating provides the source of energy.

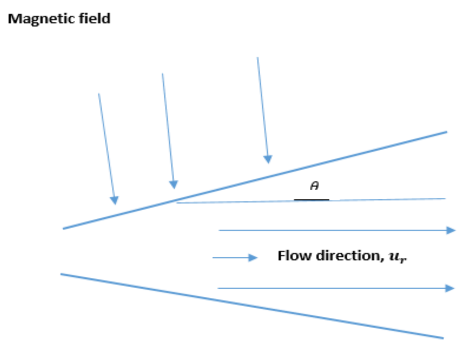

Fig 1: Flow configuration

The initial conditions are:

At the centerline:

$$u_r = u_\infty, \frac{\partial u_r}{\partial \theta} = 0, T = T_\infty, \frac{\partial T}{\partial \theta} = 0$$

At the Walls:

$$\frac{\partial u_r}{\partial \theta} = -\gamma u(\theta), T = T_w$$

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The governing equations are equation of mass, momentum and energy.

Governing equations

Equation of conservation of mass

The equation of continuity is derived from the law of conservation of mass which assumes that under normal conditions mass can neither be created nor destroyed and for a steady fluid flow; the mass in a control volume does not change.

$$\frac{\partial \rho}{\partial t} + \nabla(\rho \boldsymbol{q}) = 0 \quad (1)$$

In cylindrical coordinate system the equation (1) can be expressed as the following:

$$\frac{\partial \rho}{\partial t} + \frac{\rho}{r}\frac{\partial (r u_r)}{\partial r} + \frac{\rho}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial \theta} = 0 \quad (2)$$

Since the flow is in two-dimensions and the density of the fluid is a assumed to be constant equation (2) reduces to:

$$\frac{\rho}{r}\frac{\partial (r u_r)}{\partial r} + \frac{\rho}{r}\frac{\partial u_\theta}{\partial \theta} = 0 \quad (3)$$

We assume the flow is purely radially and dependent on r, t and θ, thus we have:

$$r u_r = f(\theta, t) \quad (4)$$

Equations of conservation of momentum

This equation is also known as the momentum equation and is derived from the Newton's second law of motion. The law requires that the sum of all the force acting on a control volume must be equal to the time rate of change of fluid momentum within the control volume.

$$\frac{\partial \boldsymbol{q}}{\partial t} + \boldsymbol{q}(\nabla.\mathbf{q}) = -\frac{1}{\rho}\nabla p + \mu\nabla^2 \boldsymbol{q} + \boldsymbol{F_r} \quad (5)$$

Momentum equation is expressed as the following in this study:

r:

$$\rho\left(\frac{\partial u_r}{\partial t} + u_r\frac{\partial u_r}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta} - \frac{u_\theta^2}{r} + u_z\frac{\partial u_r}{\partial z}\right) = -\frac{\partial p}{\partial r} + \left(\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rr}\right) + \left(\frac{1}{r}\frac{\partial}{\partial \theta}(\tau_{\theta r}) - \frac{\tau_{\theta\theta}}{r} + \frac{\partial \tau_{zr}}{\partial z}\right) + \boldsymbol{F_r} \quad (6)$$

θ:

$$\rho\left(\frac{\partial u_\theta}{\partial t} + u_r\frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta} - \frac{u_r u_\theta}{r} + u_z\frac{\partial u_\theta}{\partial z}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau_{r\theta}\right) + \left(\frac{1}{r}\frac{\partial}{\partial \theta}(\tau_{\theta\theta}) + \frac{\partial \tau_{z\theta}}{\partial z}\right) + \frac{\tau_{\theta r} - \tau_{r\theta}}{r} \quad (7)$$

The electromagnetic force $\boldsymbol{F_r}$ can be expressed as $\boldsymbol{F_r} = \boldsymbol{J} \times \boldsymbol{B}$, where $\boldsymbol{J} = \sigma(E + \boldsymbol{u_r} \times \boldsymbol{B_\theta})$ but $E = 0$ and computing for $\boldsymbol{F_r}$ we have $\boldsymbol{F_r} = -\sigma \boldsymbol{B_\theta^2} u_r \boldsymbol{r}$ and equation (6) reduces to

r:

$$\rho\left(\frac{\partial u_r}{\partial t} + u_r\frac{\partial u_r}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta} - \frac{u_\theta^2}{r} + u_z\frac{\partial u_r}{\partial z}\right) = -\frac{\partial p}{\partial r} + \left(\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rr}\right) + \left(\frac{1}{r}\frac{\partial}{\partial \theta}(\tau_{\theta r}) - \frac{\tau_{\theta\theta}}{r} + \frac{\partial \tau_{zr}}{\partial z}\right) + -\sigma \boldsymbol{B_\theta^2} u_r \boldsymbol{r} \quad (8)$$

The fluid considered in this case is non-Newtonian and power law model govern these fluids and can be expressed as equation (9)

$$\mu = \mu_0 \theta^{C(n-1)}, \; g(\theta) = \theta^c \quad (9)$$

Where c>1 indicating non-Newtonian fluid of dilatant type.

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Equation of conservation of energy

This is derived from the first law of thermodynamics, it state that the increase in internal energy dE of a system from the surrounding is equal to the amount of work done by the system to the surrounding.

$$\rho c_p \frac{DT}{Dt} = K\nabla^2 T + \mu\emptyset + \frac{J^2}{\sigma} \qquad (10)$$

Equation (10) in cylindrical coordinates is expressed as the following:

$$\rho c_p \left(\frac{\partial T}{\partial t} + u_r \frac{\partial T}{\partial r} + \frac{u_\theta}{r}\frac{\partial T}{\partial \theta} + u_z \frac{\partial T}{\partial z}\right) = k\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2}\right] + \mu\emptyset + \frac{J^2}{\sigma} \qquad (11)$$

Where

$$\mu\emptyset = 2\mu\left[\left(\frac{\partial u_r}{\partial r}\right)^2 + \left(\frac{u_r}{r}\right)^2\right] + \mu[(\frac{1}{r}\frac{\partial u_r}{\partial \theta})^2] \qquad (12)$$

Computing for ohmic heating we have:

$\boldsymbol{J} = \sigma(E + \boldsymbol{u_r} \times \boldsymbol{B_\theta})$ but $E = 0$ and $\boldsymbol{J} = \sigma(\boldsymbol{u_r} \times \boldsymbol{B_\theta})$ thus

$$\frac{J^2}{\sigma} = \frac{\sigma^2 \mu_e^2 H^2 u_r^2}{\sigma} = \sigma\mu_e^2 H^2 u_r^2 \qquad (13)$$

Using equation (13) and (12) in (11) we have:

$$\rho c_p \left(\frac{\partial T}{\partial t} + u_r \frac{\partial T}{\partial r} + \frac{u_\theta}{r}\frac{\partial T}{\partial \theta} + u_z \frac{\partial T}{\partial z}\right) = k\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2}\right] + 2\mu\left[\left(\frac{\partial u_r}{\partial r}\right)^2 + \left(\frac{u_r}{r}\right)^2\right] + \mu[(\frac{1}{r}\frac{\partial u_r}{\partial \theta})^2] + \sigma\mu_e^2 H^2 u_r^2$$

(14)

Boundary Conditions

The Boundary Conditions for the above model are:

At the centerline:

$$f(0) = 1,\ f'(0) = 0,\ \ \omega(0) = \delta^{m+1} \qquad (15)$$

At the walls:

$$f'(\alpha) = -\gamma f(\alpha),\ \ \ \omega(\alpha) = 0 \qquad (16)$$

III. NON-DIMENSIONALIZATION

Dimensional analysis is the process of expressing the units of any given physical quantity in terms of the fundamental units i.e. time, mass, length and temperature. In this study the following non-dimensional parameters shall be applied to simplify the problem:

$$u_r = -\frac{Q}{r}\frac{1}{\delta^{m+1}} f(\varphi) \qquad (17)$$

$$\frac{\omega(\varphi)}{\delta^{m+1}} = \frac{T - T_W}{T_\infty - T_W} \qquad (18)$$

Where φ is the similarity variable.

Employing equations (17) and (18) in (14), (8) and (7) we have, the final governing equations of the fluid flow model as:

Energy equation;

$$\frac{1}{P_r}\omega(\theta)'' + (m+1)\frac{r^{m+1}}{\delta^{m+1}}\lambda\omega(\theta) + \frac{Ec}{\delta^{m+1}}\theta^{c(n-1)}[4f^2 + f'^2] + \frac{R}{\delta^{m+1}} f^2 = 0 \qquad (20)$$

These are the non-dimensional parameters in equation (20);

Prandtl number, $Pr = \frac{\mu_0}{\rho\alpha}$ where $\alpha = \frac{k}{\rho C_p}$; Eckert number, $Ec = \frac{Q^2}{r^2 C_p (T_\infty - T_W)}$; Joule heating parameter, $R = \frac{\sigma\mu_e^2 H^2 Q^2}{C_p \mu_0 (T_\infty - T_W)}$ and Unsteadiness parameter is given by, $\lambda = \frac{\rho\,\delta^m}{\mu_0 r^{m-1}}\frac{d\delta}{dt}$ where m is a parameter related to wedge angle and δ is time dependent scale and r is the radius of the pipe.

Momentum equation;

$$(m+1)\frac{r^{m+1}}{\delta^{m+1}}\lambda f' + c(n-1)[c(n-1)-1]\theta^{c(n-1)-2}f' + c(n-1)\theta^{c(n-1)-1}[2f''+4f] + \theta^{c(n-1)}(f'''+4f') - 2Re\frac{1}{\delta^{m+1}}ff' - Ha^2 f' = 0 \quad (21)$$

These are the non-dimensional parameters in equation (21);

Unsteadiness parameter, $\lambda = \frac{\rho\,\delta^m}{\mu_0 r^{m-1}}\frac{d\delta}{dt}$; Reynolds number, $\mathrm{Re} = \frac{Q\rho}{\mu_0}$; and Hatman number, $\mathrm{Ha} = B_0 r\sqrt{\frac{\sigma}{\mu_0}}$

IV. NUMERICAL TECHNIQUE

Equations (20) and (21) are reduced into first order Ordinary Differential Equations together with their boundary conditions equations (15) and (16). The idea behind this is to introduce new variable, one for each variable in the original program and one for each of its derivatives up to one less than the highest derivative appearing.The formed reduced first ODEs together with the reduced Boundary Conditions are implemented in MATLAB using an inbuilt MATLAB solver called Bvp4c. Bvp4c is an inbuilt MATLAB solver based on collocation which provides a continuous solution with a 4th order level of accuracy. The method mostly solves two-point boundary value ordinary differential equations.The method uses a mesh of points to divide the interval of integration into sub-intervals. Each sub-interval will be solved based on algebraic equations according to boundary conditions provided. The solver will then estimate the error of the numerical solution on each sub interval. If the solution does not satisfy the tolerance criteria, the solver adapts the mesh and repeats the process until the criteria is satisfied. The user must at first provide the initial mesh as well as the initial approximation of the solution of the mesh points. This method was adopted to solve the following governing equations because it uses solvers that takes low computational memory, it is not expensive since it's easy to programand it is able to give optimal solutions that are accurate.

V. RESULTS AND DISCUSSION

The following are the results obtained for equations (8) and (9) and are presented graphically as shown below;

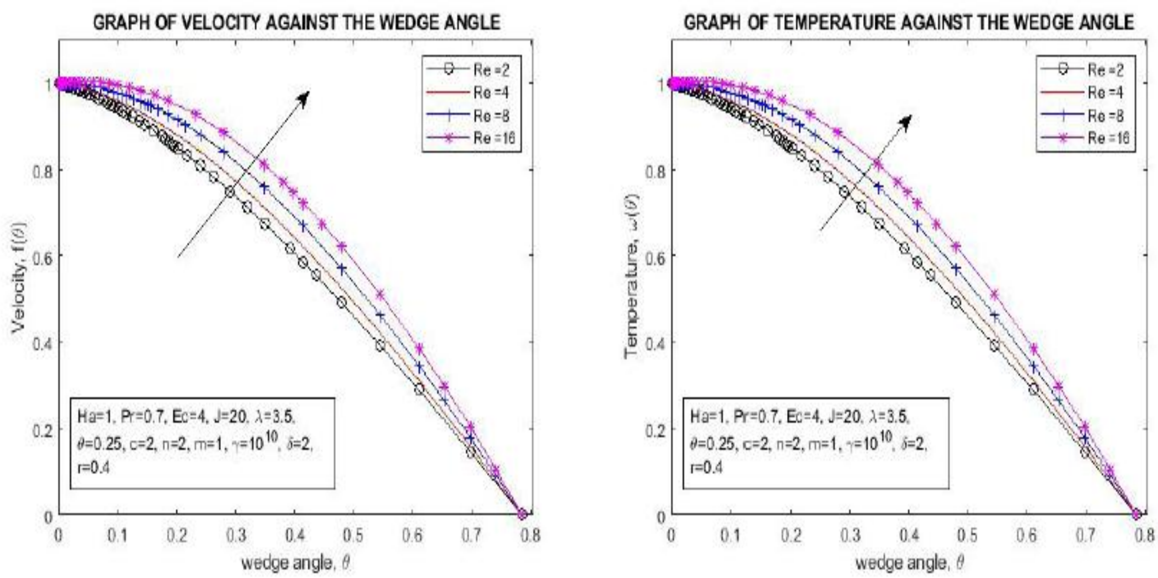

Fig 2: Velocity profiles for different values of hydrodynamic Reynolds number, Re

Reynolds represents the ratio of the inertial to viscosity forces. From the velocity profile graph it is noted an increase in Reynolds number leads to an increase in velocity of the fluid flow. Increase in Reynolds number means viscous forces reduces and inertial forces become predominant, since viscous forces opposes the motion

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of the fluid, their decrease implies that the velocity will increase since boundary layer formed does not extend more in the flow region. From temperature profiles graph it is noted that when the Reynolds number increases, the temperature of the fluid flow increases. From literature we know that when temperature increases the viscous forces reduces in liquids and thus there are inversely proportional to each other in liquid. Thus when temperature increases the viscosity reduces and thus the cause of increase in the temperature in the flow field.

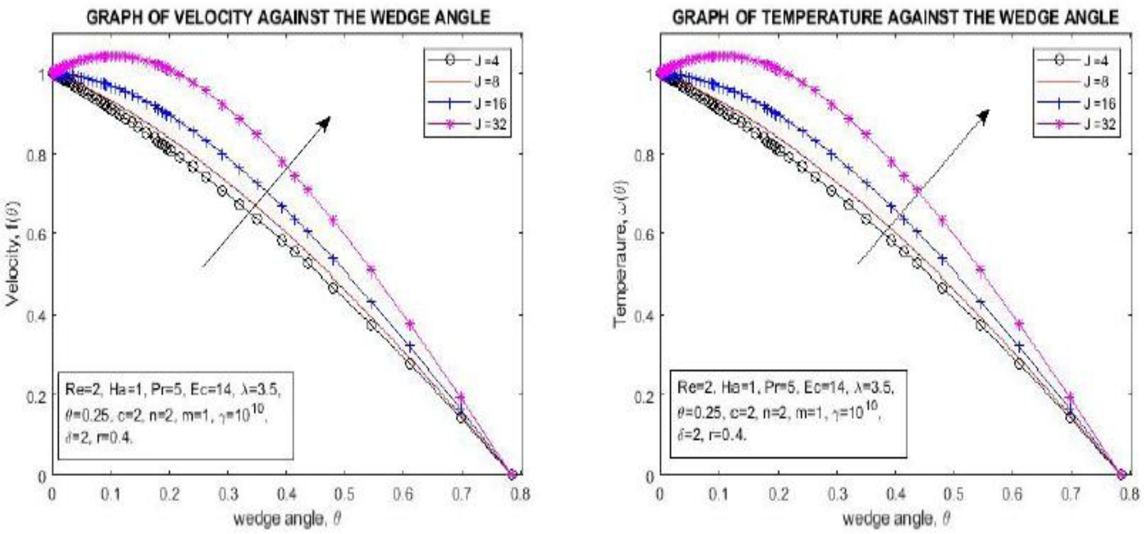

Fig 3: Velocity profiles for different values of Joule Heating, J

Joule heating is the process by which the passage of an electric current through a conductor or electrolyte releases heat due to its electrical resistance. Considering the velocity profile graphs, increase in joule heating parameter leads to an increase in velocity of the fluid flow. Increase in joule heating parameter leads to heating of the fluid particles and thus accelerating heat transfer by convection currents on the surface of the wall and thus the velocity increases. Considering the temperature profile graph, increase in joule heating parameter leads to an increase in temperature of the fluid flow. When a current passes through the electrolyte, it causes the fluid particles to vibrate more thus increasing the kinetic energy which causes heat generation and thus causes the rise of temperature of the fluid. This causes the increase of temperature of the fluid as joule heating keeps increasing.

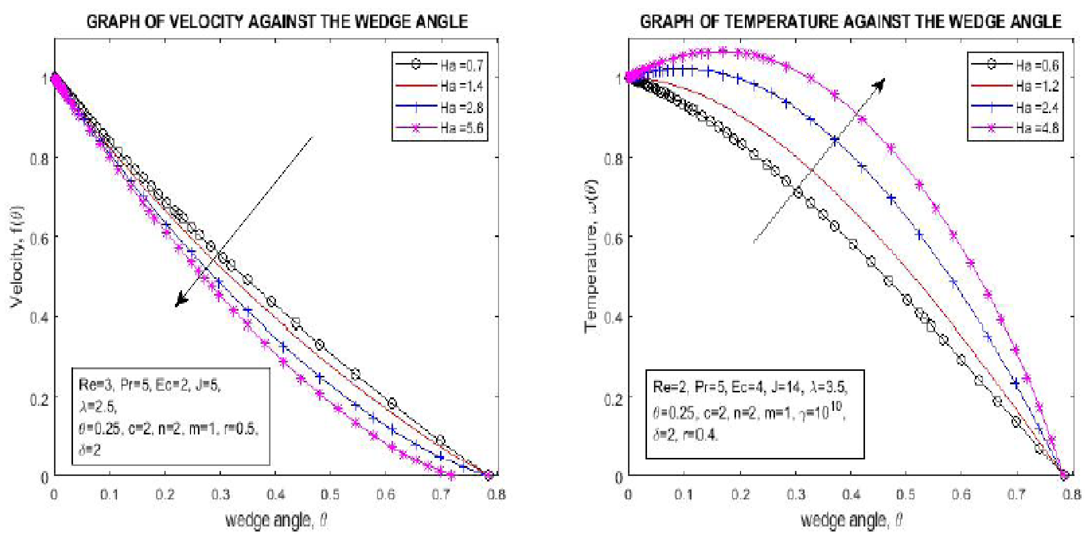

Fig 4. Velocity profiles for different values of Hartman Number, Ha

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Hartman number represents the ratio between electromagnetic force and viscous force. Considering the velocity profile graphs, increase in Hartman number leads to a decrease in velocity of the fluid flow. The presence of magnetic field which is applied normal to the direction of flow in an electrically conducting fluid induces a force called Lorentz force. This force tends to act against the fluid flow direction and hence retards the fluid flow hence leading to reduction of velocity in the flow field.

Considering the temperature profile graphs, an increase in Hartman number leads to an increase in temperature in the fluid flow region. Increase in Hartman number means that magnetic force dominates and viscous forces reduces. The increased magnetic force increases the thermal boundary layer which in turn leads to an increased fluid temperature.

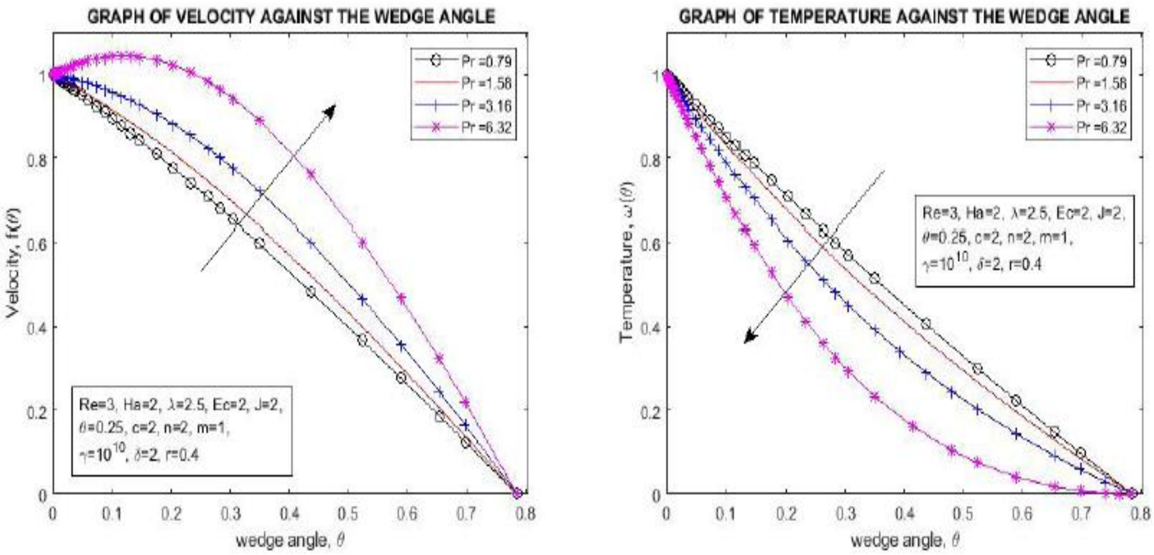

Fig 5: Temperature profiles for different values of Prandtl , Pr

The Prandtl number (Pr), it expresses ratio of viscous force to thermal force. From velocity profile graph, increase of Prandtl number leads to an increase of the fluid velocity. In this, viscous forces become less predominant and high thermal forces hence high rate of heat dissipation thus increasing the heat transfer by convectional current consequently leading to increase of fluid velocity.

In the temperature profile graph, increase in Prandtl leads to decrease in fluid temperature. From definition of Prandtl number, viscous forces are less predominant, from literature temperature and viscous forces are inversely proportional in liquids and thus temperature will decrease since the viscous forces are less predominance.

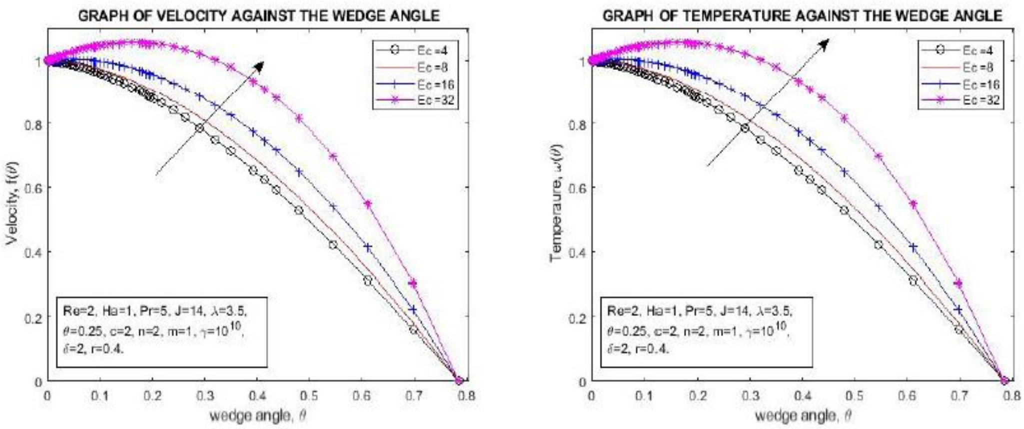

Fig 6: Temperature profiles for different values of Eckert, Ec

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Eckert number represent the conversion of kinetic energy into internal energy by the work which is done against the viscous fluid stress. In velocity profile graph, increase in Eckert number leads to an increase in fluid velocity. In this, kinetic energy is predominant and thus there is a lot of bombardment of the fluid particles and hence causing alot of vibration of the fluid particles and consequently increase of velocity of the fluid since even viscous forces are not predominant.

Considering the temperature profiles graph, an increase of Eckert number leads to an increase of temperature of the fluid. With kinetic energy being predominant and the velocity of fluid flow being high, the fluid particles are in constant collision and have high vibrations and this leads to high collisions of the particles. This increased collisions of the particles bring about dissipation of heat in the boundary layer region hence an increase in temperature profiles.

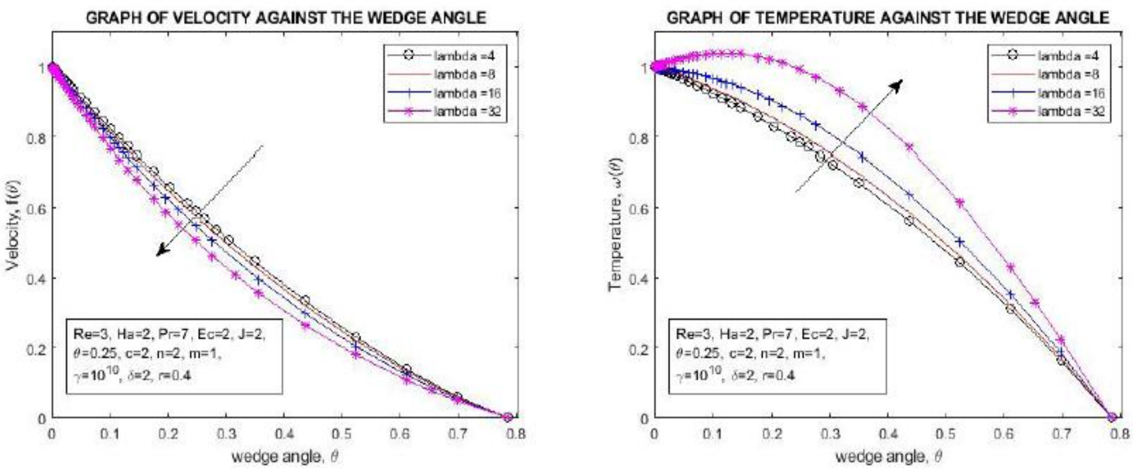

Fig 7: Temperature profiles for different values of Unsteadiness parameter, λ

Unsteadiness parameter brings about the issue of unsteady state of the flow and time-dependent scale. This caters in the issue of unsteady state of the study being considered. Considering the velocity profile graph, increase of unsteadiness parameter leads to decrease of velocity of the fluid flow. From definition of unsteadiness parameter, we see that unsteadiness parameter and time-dependent length scale have direct relation. Velocity and time-dependent length scale has an inverse relationship thus when unsteadiness parameter increases then the velocity decreases with time.

In temperature profile graph, increase in unsteadiness parameter leads to increase the temperature of the fluid flowing in the flow field. When the fluid is in motion all the kinetic energy in the moving fluid is being converted in to heat. This causes the fluid to gain more heat as time goes and thus consequently increase in temperature as the fluid flows.

VI. CONCLUSION

Magneto-hydrodynamic fluid flow in convergent-divergent conduit has been investigated presence of constant pressure gradient and magnetic field perpendicular to the flow field considering joule heating.It is found an increase in Joule heating parameter leads to an increase in the velocity and temperature profiles respectively and an increase in Hartman number leads to an increase in Temperature profiles and decrease in velocity profiles.

VII. VALIDATION

When Joule heating and Magnetohydrodynamic is not considered in the flow the results agreed with those of J. Nagler (2017).The results obtained in these research can be applied by engineers in designing duct pipes for transporting heat in underground to different rooms.

VIII. RECOMMENDATIONS

Future research can be done in MHD flow in convergent divergent conduit under varying pressure gradient and magnetic field applied at angle and MHD fluid flow in convergent-divergent conduit considering turbulent flow and compressible fluids.

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NOMENCLATURE

B	Magnetic flux density Wbm^{-2}
E	Electric field strength, Vm^{-1}
Ec	Eckert number
H	Magnetic field strength, Wbm^{-2}
J	Electric current density, Am^{-2}
M	Magnetic parameter
Pr	Prandtl number
ρ	Density of the fluid, Kg/m^3
Re	Reynold's number
J	Joules heating parameter
μ	Coefficient of Viscosity, $kgm^{-1}s^{-1}$
F_r	Electromagnetic force, N
λ	Unsteadiness parameter

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