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Some approximate properties of Cesaro means Fourier series

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Abstract: It is established $\sigma_n^a(x,f)$ ***Cesaro means some approximate features.***

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I. INTRODUCTION

Suppose, that $T=[-\pi,\pi]$ and $f:R\to R$ 2π - are periodic functions. If $f\in L(T)$ as a rule, $\sigma(f)$ represent respectively trigonometric Furrier series

$$\sigma[f](x)=\frac{a_0}{2}+\sum_{k=1}^{\infty}a_k\cos kx+b_k\sin kx, \quad (1)$$

Where

$$\left.\begin{array}{l} a_k\equiv a_k(f)=\frac{1}{\pi}\int_T f(t)\cos kt\,dt, \quad k\in N_0 \\ b_k=b_k(f)=\frac{1}{\pi}\int_T f(t)\sin kt\,dt, \quad k\in N. \end{array}\right\} . \quad (2)$$

Assume that $p\in[1,+\infty[$ - is a number. For each function $f\in L^p(T)$ the following will be considered

$$\|f\|_p=\left\{\frac{1}{2\pi}\int_T|f(t)|^p\,dt\right\}^{\frac{1}{p}}, (3)$$

And also it will consider the following: $L^\infty(T)=c(T)$, $\|f\|_c=\|f\|_\infty=\sup_{x\in T}|f(x)|$. Let us set

$$\omega^{(k)}(\sigma,f)=\sup_{|h|\le\sigma}\left\|\sum_{j=0}^{k}(-1)^{k+j}\binom{k}{j}f(t+jh)\right\|_p \;;\quad \sigma\in]0,2\pi].$$

$\omega^{(k)}(\sigma,f)$ is called module L^p of smoothness of an order to function f.

In the future we will assume that $\omega^{(1)}(\sigma,f)_p\equiv\omega(\sigma,f)_p$.

Bellow A, $A(f)$, $A(f,p)$, $A(f,a,p)$, $A(a)$, $A_1(a)$, ... indicate absolute positive or positive constants depending only on the specified parameters.

Let ω-be module of continuity. Assume that

$H_p^\omega\equiv H_p^\omega(T)=\{f:\omega(\sigma,f)_p\le A(f,p)\omega(\sigma)\}$ and $H_p^\omega=H^\omega$. If $\omega(\sigma)=\sigma^\alpha$, $\alpha\in]0,1]$, than $H_p^\omega\equiv Lip(a,p)$. $H^\omega\equiv Lipa$.

With $S_n(x,f)$ we will define, respectively partial sums of the series

$$S_n(x,f)=\frac{a_0}{2}+\sum_{k=1}^{n}a_k\cos kx+b_k\sin ks=\frac{1}{\pi}\int_T f(x+t)D_n(t)dt, \quad (4)$$

where $D_n(t)$ - is Dirichlet kernel, i.e.

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$$D_n(t)=\frac{1}{2}+\sum_{k=1}^{n}\cos kt=\frac{\sin\left(n+\frac{1}{2}\right)t}{2\sin\frac{t}{2}}, \qquad (5)$$

and

$$D_n(0)=n+\frac{1}{2}, \quad n\in N$$

Consider that $A_k^a=1,\ A_k^a=\frac{(a+1)(a+2)(a+3)\ldots(a+k)}{k!},\quad k\in N,\ a>-1.$ (6)

It is known (see e.g. A. Sigmund [7], pp. 130-131) that

$$A_k^a=\sum_{i=0}^{k}A_{k-i}^{a-1},\quad A_k^a-A_{k-1}^a=A_k^{a-1}, \qquad (7)$$

$$A(a)\le\frac{A_k^a}{k^a}\le A_1(a), \qquad (8)$$

If

$$K_n^a(t)=\frac{1}{A_n^a}\sum_{k=0}^{n}A_{n-k}^{a-1}D_k(t), \qquad (9)$$

than they are respectively called Chesaro kernel. It is known that (see e.g. A. Sigmund [2] pp. 157-164), that

$$K_n^a(t)=\varphi_n^a(t)+r_n^a(t), \qquad (10)$$

Where

$$\varphi_n^a(t)=\frac{\sin\left[\left(n+\frac{1}{2}+\frac{a}{2}\right)t-\frac{a\pi}{2}\right]}{A_n^a\left(2\sin\frac{t}{2}\right)^{1+a}} \qquad (11),$$

and

$$\left\|K_n^a\right\|_c\le A(a)n,\ (12),\quad \left|\tau_n^a(t)\right|\le\frac{A(a)}{nt^2},\ \frac{\pi}{n}\le t\le\pi,\ (13),$$

In what follows we shall use the Holder inequality for integrals. If $f_1\in L^p(T),\ f_2\in L^q(T)$ и $\frac{1}{p}+\frac{1}{q}=1,$ than

$$\|f_1f_2\|_L\le\|f_1\|_p\|f_2\|_q \quad . \qquad (14)$$

We also use the Minkowski inequality. If $f_1\in L^p(T),\ f_2\in L^p(T),\ p\in[1,+\infty[,$ than

$$\|f_1+f_2\|_p\le\|f_1\|_p+\|f_1\|_p \qquad (15)$$

Assume that function $f\in L(T)$ than $\sigma_n^a(x,f)$ $t_n^a(x,f)$ symbols denotes the Chesaro means of order $a>-1$ consequently $\sigma[f]$ i.e.

$$\sigma_n^a(x,f)=\frac{1}{A_n^a}\sum_{k=0}^{n}A_{n-1}^{a-1}S_k(x,f), \qquad (16)$$

Where $A_n^\alpha(i=1,2,\cdots)$ and $S_k(x,f)$ are given to relations (6) and (4). Using the equalities (4), (5), (9) and (16), we can write

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$$\sigma_n^{\alpha}(x,f) = \frac{1}{\pi}\int_{T} f(x+t)K_n^{\alpha}(t)dt = \frac{1}{\pi}\int_{T} [f(x+t)+f(x-t)]K_n^{\alpha}(t)dt \quad (17)$$

Zigmund proved that if the function $f \in c(T)$ and

$$\omega(\delta,f)_c = \bar{\bar{O}}(\delta^{\alpha})(\delta \to +0), \alpha \in]0;1[, \quad (18)$$

then

$$\lim_{n\to\infty} \|\sigma_n^{-\alpha}(f) - f\|_c = 0\,. \quad (19)$$

In the writings of Hardy and Littlewood mentioned provision was generalized, in particular, they showed that if $f \in c(T)$ and

$$\omega(\delta,f)_p = \bar{\bar{O}}(\delta^{\alpha}) \quad (\delta \to +0), \alpha \in]0;1[, \alpha p > 1, \quad (20)$$

then it is executed (19). They showed that when

$$f \in c(T) \cap \mathcal{L}ip(\alpha,p), \alpha \in]0;1[, \alpha p > 1,$$

Then for any $\beta \in]0;\alpha[$ numbers the following ratio is valid yeah

$$\lim_{n\to\infty} \left\|\sigma_n^{-\beta}(f) - f\right\|_c = 0 \quad . \quad (21)$$

Our goal is to evaluate the above expression

$$\|\sigma_n^{\alpha}(x,f) - f\|_c\,,$$

With the help of the modules of continuity of space $c(T)$ и $\mathcal{L}^p(T)$. These estimates will be taken some new position in relation to the behavior $\sigma_n^{\alpha}(x,f)$ means .

II. MATHEMATICAL ANALYSIS

The following in true

Theorem 1.

$$\left\|\sigma_n^{-\frac{1}{p}}(x,f) - f\right\|_c \le A(p)\left\{\omega\left(\frac{1}{n^{1-\frac{1}{p}}},f\right)_c + n^{\frac{1}{p}}\omega\left(\frac{1}{n},f\right)_p (\ln n)^{1-\frac{1}{p}}\right\} \quad (22)$$

Proof. Taking into account the relations (5), (7) and (9) , conclude that

$$\frac{2}{\pi}\int_0^{\pi} K_n^{-a}(t)dt = 1\,.$$

Therefore, according to equality (17) we find

$$\sigma_n^{-\frac{1}{p}}(x,f) - f(x) = \frac{1}{\pi}\int_0^{\pi} [f(x+t)+f(x-t)-2f(x)]K_n^{-\frac{1}{p}}(t)dt \equiv \frac{1}{\pi}\int_0^{\pi} \varphi(x,t)K_n^{-\frac{1}{p}}(t)dt \quad (23)$$

Where

$$\varphi(x,t) \equiv f(x+t)+f(x-t)-2f(x). \quad (24)$$

We will be guided by the method used in the monograph L.V. Zhizhiashvili. According to (23) equality we find

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$$\sigma_n^{-\frac{1}{p}}(x,f) - f(x) = \frac{1}{\pi}\int_{\pi}^{\frac{\pi}{n}} \varphi(x,t)K_n^{-\frac{1}{p}}(t)dt + \frac{1}{\pi}\int_{\frac{\pi}{n}}^{\pi} \varphi(x,t)K_n^{-\frac{1}{p}}(t)dt$$
$$\equiv U_{n,p}^{(1)}(x,f) + U_{n,p}^{(2)}(x,f) \qquad (25)$$

In force ratio (12) and (25) write on

$$\left\|U_{n,p}^{(1)}(x,f)\right\|_c \le A(p)n\left\|\int_0^{\frac{\pi}{n}} \varphi(x,t)dt\right\|_c \le A(p)\omega\left(\frac{1}{n},f\right)_c \qquad (26)$$

Consider the expression $U_{n,p}^{(2)}(x,f)$ from equality (25). Applying equality (10) we will have

$$U_{n,p}^{(2)}(x,f) = \frac{1}{\pi}\int_{\frac{\pi}{n}}^{\pi} \varphi(x,t)\varphi_n^{-\frac{1}{p}}(t)dt + \frac{1}{\pi}\int_{\frac{\pi}{n}}^{\pi} \varphi(x,t)r_n^{-\frac{1}{p}}(t)dt \equiv U_{n,p}^{(3)}(x,f) + U_{n,p}^{(4)}(x,f) \ (27)$$

Where $\varphi_n^{-\frac{1}{p}}(t)$ is determined by (11) equality , and for $r_n^{-\frac{1}{p}}(t)$ expression rightly assessment (13). Since

$$\sin\left[\left(n+\frac{1}{2}-\frac{1}{2p}\right)t+\frac{\pi}{2p}\right] = \sin\left[\left(n+\frac{1}{2}-\frac{1}{2p}\right)t\right]\cos\frac{\pi}{2p} + \cos\left[\left(n+\frac{1}{2}-\frac{1}{2p}\right)t\right]\sin\frac{\pi}{2p} =$$
$$= \left[\sin nt\cos\left(\frac{1}{2}-\frac{1}{2p}\right)t + \cos nt\sin\left(\frac{1}{2}-\frac{1}{2p}\right)t\right]\cos\frac{\pi}{2p} +$$
$$+ \left[\cos nt\cos\left(\frac{1}{2}-\frac{1}{2p}\right)t - \sin nt\sin\left(\frac{1}{2}-\frac{1}{2p}\right)t\right]\sin\frac{\pi}{2p} \qquad (28)$$

Therefore, for evaluation $\left\|U_{n,p}^{(3)}(x,f)\right\|_c$ enough (see(11), (28)) corresponding estimates will be established for the following expression :

$$U_{n,p}^{(5)}(x,f) = \frac{1}{\pi A_n^{\alpha}}\int_{\frac{\pi}{n}}^{\pi} \varphi(x,t)\frac{\cos\left(\frac{1}{2}-\frac{1}{2p}\right)t}{\left(\sin\frac{t}{2}\right)^{1-\frac{1}{p}}}\sin nt\,dt \ (29)$$

$$U_{n,p}^{(6)}(x,f) = \frac{1}{\pi A_n^{\alpha}}\int_{\frac{\pi}{n}}^{\pi} \varphi(x,t)\frac{\cos\left(\frac{1}{2}-\frac{1}{2p}\right)t}{\left(\sin\frac{t}{2}\right)^{1-\frac{1}{p}}}\cos nt\,dt \ (30)$$

In dalneishem potato men that

$$V(t,p) \equiv \frac{\cos\left(\frac{1}{2}-\frac{1}{2p}\right)t}{\left(\sin\frac{t}{2}\right)^{1-\frac{1}{p}}}, \quad |t| \in]0;\pi]; \qquad (31)$$

Below we discuss in detail the issue of evaluation $U_n^{(6)}(x,f)$ expression. Under discussion will do vivod , that for him establishing $U_{n,p}^{(5)}(x,f)$ expression for the following expression

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$$U_{n,p}^{(7)}(x,f) = n^{\frac{1}{p}} \int_{\frac{\pi}{n}}^{\pi} \varphi(x,t) V(t,p) \cos nt\, dt \quad (32)$$

If in this expression t replace through $t+\frac{\pi}{n}$ and let's sum up the obtained two values $U_{n,p}^{(7)}(x,f)$-, let's have

$$2U_{n,p}^{(7)}(x,f) = n^{\frac{1}{p}} \int_{\frac{\pi}{n}}^{\pi} \varphi(x,t)V(t,p)\cos nt\, dt - n^{\frac{1}{p}} \int_{0}^{\pi-\frac{\pi}{n}} \varphi\left(x,t+\frac{\pi}{n}\right) V\left(t+\frac{\pi}{n},p\right)\cos nt\, dt$$

$$= n^{\frac{1}{p}} \left\{ \int_{\frac{\pi}{n}}^{\pi-\frac{\pi}{n}} \left[\varphi(x,t) - \varphi\left(x,t+\frac{\pi}{n}\right)\right] V(t,p)\cos nt\, dt \right.$$

$$+ n^{\frac{1}{p}} \int_{\frac{\pi}{n}}^{\pi-\frac{\pi}{n}} \varphi\left(x,t+\frac{\pi}{n}\right) \left[V(t,p) - V\left(t+\frac{\pi}{n},p\right)\right] \cos nt\, dt +$$

$$\left. + \int_{\frac{\pi}{n}}^{\pi-\frac{\pi}{n}} \varphi(x,t)V(t,p)\cos nt\, dt - \int_{0}^{\frac{\pi}{n}} \varphi\left(x,t+\frac{\pi}{n}\right) V\left(t+\frac{\pi}{n},p\right)\cos nt\, dt \right\}$$

$$\equiv \sum_{k=8}^{11} U_{n,p}^{(k)}(x,f) \qquad (33)$$

Estimate $U_{n,p}^{(k)}(x,f)(k=8-11)$ expressions from equality (33). from equality (33), Using the hölder inequality we obtain

$$\left\|U_{n,p}^{(8)}(x,f)\right\|_c \le n^{\frac{1}{p}} \left\{ \int_{\frac{\pi}{n}}^{\pi-\frac{\pi}{n}} \left|\varphi(x,t) - \varphi\left(x,t+\frac{\pi}{n}\right)\right|^p dt \right\}^{\frac{1}{p}} \left\{ \int_{\frac{\pi}{n}}^{\pi-\frac{\pi}{n}} |V(t,p)|^{\frac{p}{p-1}} dt \right\}^{\frac{p-1}{p}}$$

Hence, if we take into account the designation (24), then according to (31) we conclude that

$$\left\|U_{n,p}^{(8)}(x,f)\right\|_c \le A(p) n^{\frac{1}{p}} \left\{ \int_{T} \left|f\left(t+\frac{\pi}{n}\right) - f(t)\right|^p dt \right\}^{\frac{1}{p}} \left\{ \int_{\frac{\pi}{n}}^{\pi} \frac{dt}{t} \right\}$$

$$\le A(p) n^{\frac{1}{p}} \omega\left(\frac{1}{n}, f\right)_p (\ln n)^{1-\frac{1}{p}} \quad (34)$$

Consider $U_n^{(9)}(x,f)$ expression of (33) equality, which can be so rewritten

$$U_{n,p}^{(9)}(x,f) = n^{\frac{1}{p}} \int_{\frac{\pi}{n}}^{\frac{2\pi}{n}} \varphi\left(x, t+\frac{\pi}{n}\right)\left[V(t,p) - V\left(t+\frac{\pi}{n}, p\right)\right] \cos nt \, dt$$

$$+ n^{\frac{1}{p}} \int_{\frac{2\pi}{n}}^{\pi-\frac{\pi}{n}} \varphi\left(x, t+\frac{\pi}{n}\right)\left[V(t,p) - V\left(t+\frac{\pi}{n}, p\right)\right] \cos nt \, dt$$

Again, the transformation of the variable get

$$U_{n,p}^{(9)}(x,f) = n^{\frac{1}{p}} \int_{\frac{\pi}{n}}^{\frac{2\pi}{n}} \varphi\left(x, t+\frac{\pi}{n}\right)\left[V(t,p) - V\left(t+\frac{\pi}{n}, p\right)\right] \cos nt \, dt +$$

$$+ \frac{n^{\frac{1}{p}}}{2}\left\{ \int_{\frac{2\pi}{n}}^{\pi-\frac{2\pi}{n}} \left[\varphi\left(x, t+\frac{\pi}{n}\right) - \varphi\left(x, t+\frac{2\pi}{n}\right)\right]\left[V(t,p) - V\left(t+\frac{\pi}{n}, p\right)\right] \cos nt \, dt + \right.$$

$$+ \int_{\frac{2\pi}{n}}^{\pi-\frac{2\pi}{n}} \varphi\left(x, t+\frac{2\pi}{n}\right)\left[V(t,p) - 2V\left(t+\frac{\pi}{n}, p\right) + V\left(t+\frac{2\pi}{n}, p\right)\right] \cos nt \, dt$$

$$+ \int_{\pi-\frac{2\pi}{n}}^{\pi-\frac{\pi}{n}} \varphi\left(x, t+\frac{\pi}{n}\right)\left[V(t,p) - V\left(t+\frac{\pi}{n}, p\right)\right] \cos nt \, dt -$$

$$\left. - \int_{\frac{\pi}{n}}^{\frac{2\pi}{n}} \varphi\left(x, t+\frac{2\pi}{n}\right)\left[V\left(t+\frac{\pi}{n}, p\right) - V\left(t+\frac{2\pi}{n}, p\right)\right] \cos nt \, dt \right\} \equiv \sum_{k=12}^{16} U_{n,p}^{(k)}(x,f) \qquad (35)$$

Known (see Page 11). L. V. Zhizhiashvili ([2], chapters II-IV p. 67)

$$\left|V(t,p) - V\left(t+\frac{\pi}{n}, p\right)\right| \le \frac{A(p)}{n t^{2-\frac{1}{p}}}, \qquad \frac{\pi}{n} \le t \le \pi, \qquad (36)$$

And

$$\left|V(t,p) - 2V\left(t+\frac{\pi}{n}, p\right) + V\left(t+\frac{2\pi}{n}, p\right)\right| \le \frac{A(p)}{n^2 t^{3-\frac{1}{p}}}, \qquad \frac{\pi}{n} \le t \le \pi, \qquad (37)$$

From equation (35) , (36) the ratio obtained

$$\left\|U_{n,p}^{(12)}(x,f)\right\|_c \le A(p) n^{-1+\frac{1}{p}} \omega\left(\frac{1}{n}, f\right)_c \int_{\frac{\pi}{n}}^{\pi} \frac{dt}{t^{2-\frac{1}{p}}} \le A(p)\omega\left(\frac{1}{n}, f\right)_c \quad (38)$$

Similarly, we have

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$$\left\|U_{n,p}^{(13)}(x,f)\right\|_c \leq A(p)n^{-1+\frac{1}{p}}\omega\left(\frac{1}{n},f\right)_c \int_{\frac{\pi}{n}}^{\pi}\frac{dt}{t^{2-\frac{1}{p}}} \leq A(p)\omega\left(\frac{1}{n},f\right)_c \quad (39)$$

If we take into account (37) relations, we conclude from (35) equality

$$\left\|U_{n,p}^{(14)}(x,f)\right\|_c \leq A(p)n^{-2+\frac{1}{p}}\int_{\frac{\pi}{n}}^{\pi-\frac{\pi}{n}}\omega(t,f)_c\frac{dt}{t^{3-\frac{1}{p}}} \leq A(p)n^{-2+\frac{1}{p}}\omega\left(\frac{1}{n},f\right)_c\int_{\frac{\pi}{n}}^{\pi}\left[\frac{nt+1}{t^{3-\frac{1}{p}}}\right]dt$$

$$\leq A(p)\omega\left(\frac{1}{n},f\right)_c \quad (40)$$

(35) and (36) ratios give

$$\left\|U_{n,p}^{(15)}(x,f)\right\|_c \leq A(p)n^{-1+\frac{1}{p}}\int_{\pi-\frac{2\pi}{n}}^{\pi-\frac{\pi}{n}}\omega(t,f)_c\frac{dt}{t^{2-\frac{1}{p}}} \leq A(p)n^{-1+\frac{1}{p}}\int_{\pi-\frac{2\pi}{n}}^{\pi-\frac{\pi}{n}}\omega(t,f)_c dt$$

$$\leq A(p)n^{-1+\frac{1}{p}}\int_{\pi-\frac{2\pi}{n}}^{\pi-\frac{\pi}{n}}\omega\left(\frac{1}{n},f\right)_c(nt+1)dt \leq A(p)n^{-1+\frac{1}{p}}\omega\left(\frac{1}{n},f\right)_c \quad (41)$$

Again taking into account (35) and (36) ratios, we conclude

$$\left\|U_{n,p}^{(16)}(x,f)\right\|_c \leq A(p)n^{-1+\frac{1}{p}}\omega\left(\frac{1}{n},f\right)_c\int_{\frac{\pi}{n}}^{\frac{2\pi}{n}}\frac{dt}{t^{2-\frac{1}{p}}} \leq A(p)\omega\left(\frac{1}{n},f\right)_c \quad (42)$$

Therefore, by using the ratio (35), (38) - (42), enclose

$$\left\|U_{n,p}^{(9)}(x,f)\right\|_c \leq A(p)\omega\left(\frac{1}{n},f\right)_c \quad (43)$$

Forth, according to (33) equality write on

$$\left\|U_{n,p}^{(10)}(x,f)\right\|_c \leq A(p)n^{\frac{1}{p}}\int_{\pi-\frac{\pi}{n}}^{\pi}\omega(t,f)_c dt \leq A(p)n^{\frac{1}{p}}\int_{\frac{\pi}{n}}^{\pi}\omega\left(\frac{n^{1-\frac{1}{p}}}{n^{1-\frac{1}{p}}}t,f\right)_c dt$$

$$\leq A(p)n^{\frac{1}{p}}\omega\left(\frac{1}{n^{1-\frac{1}{p}}},f\right)_c\int_{\pi-\frac{\pi}{n}}^{\pi}\left(n^{1-\frac{1}{p}}t+1\right)dt \leq A(p)\omega\left(\frac{1}{n^{1-\frac{1}{p}}},f\right)_c \quad (44)$$

If we consider (31), then from (33) equality, we conclude

$$\left\|U_{n,p}^{(11)}(x,f)\right\|_c \leq A(p)\omega\left(\frac{1}{n},f\right)_c, \quad (45)$$

So, according to (33), (34), (43), (44) and (45) the relation for (37) expressions, i.e. as it was said, for $U_{n,p}^{(11)}(x,f)$ (k=5,6) integrals we have the following estimation

$$\left\|U_{n,p}^{(k)}(x,f)\right\|_c \leq A(p)\left\{\omega\left(\frac{1}{n^{1-\frac{1}{p}}},f\right)_c + n^{\frac{1}{p}}\omega\left(\frac{1}{n},f\right)_p(\ln n)^{1-\frac{1}{p}}\right\} \quad (k=5,6); \quad (46)$$

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then from (27) equality we conclude that

$$\left\|U_{n,p}^{(3)}(x,f)\right\|_c \le A(p)\left\{\omega\left(\frac{1}{n^{1-\frac{1}{p}}},f\right)_c + n^{\frac{1}{p}}\omega\left(\frac{1}{n},f\right)_p (\ln n)^{1-\frac{1}{p}}\right\} \qquad (47)$$

Whereas (13) rating power (27) equality will have

$$\le A(p)\int_{\frac{\pi}{n}}^{\pi}\omega(t,f)_c\frac{dt}{nt^2} = A(p)\int_{\frac{\pi}{n}}^{\pi}\omega\left(\frac{n^{1-\frac{1}{p}}}{n^{1-\frac{1}{p}}}t,f\right)_c\frac{dt}{nt^2} \le A(p)\omega\left(\frac{1}{n^{1-\frac{1}{p}}},f\right)_c\int_{\frac{\pi}{n}}^{\pi}\frac{n^{1-\frac{1}{p}}t+1}{nt^2}dt$$

$$= A(p)\omega\left(\frac{1}{n^{1-\frac{1}{p}}},f\right)_c\int_{\frac{\pi}{n}}^{\pi}\left[\frac{1}{n^{\frac{1}{p}}t}+\frac{1}{nt^2}\right]dt \le A(p)\omega\left(\frac{1}{n^{1-\frac{1}{p}}},f\right)_c \qquad (48)$$

Then from the equality (25) and given (26), (27), (47) and (48) estimates, we find:

$$\left\|\sigma_n^{-\frac{1}{p}}(x,f)-f\right\|_c \le A(p)\left\{\omega\left(\frac{1}{n^{1-\frac{1}{p}}},f\right)_c + n^{\frac{1}{p}}\omega\left(\frac{1}{n},f\right)_p (\ln n)^{1-\frac{1}{p}}\right\}$$

Theorem 1 is proved.

REFERENCES

[1] A.Zigmund. Trigonometric serias..T-I,M.Mir 1965 (in Russian).

[2] G.G.Xardi D.E.Littlvud.G.Polia. Inequalities. N., GIIL, 1948(in Russian).

[3] L.V.Zhizhiashvili.Some problems about trigonometric series and its conjugates. Tbilisi, TGU, 1993(in Russian).

[4] M.S.Bakuridze. some approximate properties of Cesaro mean Fourier series and their conjugates .International Journal of Engineering Science and Innovative Technology (IJESIT) (Volume 4 Issue 6, November 2015).

[5] M.S.Bakuridze. some approximate properties of Cesaro mean Fourier series and their conjugates .International Journal of Engineering Science and Innovative Technology (IJESIT) (Volume 5 Issue 4, July 2016).

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