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Properties of operations on predicates in the intelligence theory

Dr. Khudhair Abed Thamer [1], Shabanova-Kushnarenko L.V.[2]
[1]Assistant, Kuliyyah Al-Maaref University College, Republic Iraq
[2]Engineer, National Technical University "KhPI", Kharkiv, Ukraine

Abstract: ***Formal description of the laws of intellectual activity in the algebra of finite predicates language led to the need of the development of abstract equivalent of this algebra - algebra of concepts [1]. The choice of such title is caused by the fact that totality elements – carrier of the algebra concepts is naturally interpreted as the concept of intelligence (i.e. thoughts, concepts, in general – any subjective conditions of human) and algebra concepts operations over these elements – as actions of the intelligence over concepts. Formal description of intellectual human activity conformities is achieved within psychological concepts and laws of algebra concepts interpretation. The validity of such an interpretation will be substantiated in each specific case by an experimental study of the corresponding properties of the subject's behavior. Such experiments are set in accordance with the method of comparator identification. By the subject we mean that particular person whose intellectual activity is formalized. From the content point of vie the carrier of the algebra of concepts is the set of all concepts of the subject with the predicate of equality given on it. In the role of the prototype of the algebra of concepts, the algebra of one-place k-predicates of the first order is used. The predicate of equality, or the comparator realized by the subject in the process of establishing by him a coincidence or difference of concepts presented to him could be studied purely empirically without resorting to the formulation of any laws (axioms). It would be possible to study not the laws of the intellectual behavior of the subject, but only behavior. In order to do this, it would be sufficient simply to compile a table of the binary responses of the subject to all possible pairs of concepts. However, as the centuries-old experience in the development of research in physics has shown, such an empirical approach is less effective, it is usually used only at the initial stage of work in order to accumulate a sufficient number of initial facts necessary for the subsequent construction of the theory. Thus, for example, the formulation of the motion of celestial bodies in science was preceded by the compilation of tables of the planets location on the celestial sphere in different time moments. The axiomatic representation of the phenomena of nature (i.e., the description of the laws underlying them) usually turns out to be immeasurably more economical, convenient and better penetrating into the essence of the studied processes than a direct description of the processes themselves.***

Key words: **intelligence theory, algebra of finite predicates, comparative identification.**

I. INTRODUCTION

This work is the extension of the articles [2, 3], in which multidimensional predicate model of comparator identification, justified axiomatic of this model was proposed. Models and axiomatic of relations in the language of algebra of finite predicates (AFP), operations over predicates are considered.

The main predecessors of this article were the following works: monograph [4], in which the algebra of finite predicates was created - the mathematical basis of the intelligence theory; articles [5-7] in which some issues of the theory and practice of comparative identification, created in the frames of the intelligence theory for objective physical and study of psychological human conditions were considered. In this paper the properties of operations over predicates included to the model of comparator identification are considered. Formulas for calculating of the quantifiers are proposed, when in the sub-quantifier expression there are both objective and predicate variables.

II. OPERATIONS ON PREDICATES

Let $P(x_1, x_2, ..., x_n)$ be an arbitrary predicate posed on U^n.

The conclusion “The par $P(x_1, x_2, ..., x_i, ..., x_n) =$ is conducted for all x_i” binds the predicate through some unary relation, which can formally be set down as

$$\bigwedge_{x_i \in U} P(x_1, x_2, ..., x_i, ..., x_n) = \quad (1)$$

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Here $i=\overline{1,n}$. The expression given in the left part of the par (1) may be taken as an operation, reflecting a set of all the predicates set on U^n into itself. This operation is called generality quantifier in x_i and set down as $\forall x_i(P(x_1,x_2,...,x_i,...,x_n))$. The symbol $\forall$ is read "for all". Thus:

$$\forall x_i(P(x_1,x_2,...,x_i,...,x_n)) = \bigwedge_{x_i \in U} P(x_1,x_2,...,x_i,...,x_n) \quad (2)$$

The expression "There is x_i for which the par $P(x_1,x_2,...,x_i,...,x_n)=1$ is conducted" binds the predicate through another relation, which is formally expressed as

$$\bigvee_{x_i \in U} P(x_1,x_2,...,x_i,...,x_n)=1 \quad (3)$$

The operation given in the left part of the par (3) and reflecting a set of all the predicates set on U^n into itself is called an existential quantifier in x_i. It is written down as $\exists x_i(P(x_1,x_2,...,x_i,...,x_n))$. The symbol $\exists$ is read as "exist". We have:

$$\exists x_i(P(x_1,x_2,...,x_i,...,x_n)) = \bigvee_{x_i \in U} P(x_1,x_2,...,x_i,...,x_n)=1 \quad (4)$$

Quantifiers of generality and existing are used for formal transformations of different mathematical expressions. While transforming into a formal language, words "for all" (or "for each", "for any", etc.) are replaced with the symbol $\forall$, words "exist" ("there is") are replaced with the symbol; the relations figuring in an expression are replaced with predicates which are in accord with them; words "or", "and" (coma), ''not" ("it is illusory that", "it is invalid that"), "it results in" ("if…then", "in case that…then"), "matched" ("if and only if…then", "then and only then…when", "if") are changed into Boolean operations $\vee$, $\wedge$, $\overline{\ }$, $\supset$, ~.

Below there are examples of transforming math expressions into a formal language:

1) «Within any $x_1,x_2,...,x_n$ $(x_1,x_2,...,x_n)\in A_1\times A_2\times...\times A_n$ then and only then if $x_1\in A_1, x_2\in A_2,...,x_n\in A_n$»:

$$\forall x_1\forall x_2...\forall x_n((x_1,x_2,...,x_n)\in A_1\times A_2\times...\times A_n \sim (x_1\in A_1)\wedge(x_2\in A_2)\wedge...\wedge(x_n\in A_n))$$

2) «For each a $a=a$»: $\forall a\, D(a,a)$;

3) «$a=b$ matched $b=a$ for any a, b»: $\forall a\forall b(D(a,b)\sim D(b,a))$;

4) «aEb and bEc results in aEc for any a,b,c»:

$$\forall a\forall b\forall c(E(a,b)\wedge E(b,c)\supset E(a,c));$$

5) «For all a,b,c aFb and aFc results in $b=c$»: $\forall a\forall b\forall c(F(a,b)\wedge F(a,c)\supset D(b,c))$;

6) «There is x that is for any y xFy»: $\exists x\forall y F(x,y)$.

In order to reduce the number of brackets in formal expressions an operation $\wedge$ is considered to be dominant in relation to an operation $\vee$, and an operation $\vee$ dominant towards $\supset$ and ~. Amplitude of a quantifier is beyond brackets providing right understanding of formula structure.

In math texts there are often seen such expressions as "For all $x_i\in A$ $(x_1,x_2,...,x_n)\in F$" and "There exists such $x_i\in A$ that is $(x_1,x_2,...,x_n)\in F$". The first of them is transformed into a formal language as following: $\forall x_i(A(x_i)\supset P(x_1,x_2,...,x_n))$, the latter – $\exists x_i(A(x_i)\wedge P(x_1,x_2,...,x_n))$. Thus, the pars below are valid:

$$\forall x_i\in A\, P(x_1,x_2,...,x_n) = \forall x_i(A(x_i)\supset P(x_1,x_2,...,x_n)) \quad (5)$$

$$\exists x_i\in A\, P(x_1,x_2,...,x_n) = \exists x_i(A(x_i)\wedge P(x_1,x_2,...,x_n)) \quad (6)$$

1) "For all $a\in A$ and $b,c\in B$ aFb and aFc results in $b=c$":

$$\forall a\forall b\forall c(A(a)\wedge B(b)\wedge B(c)\wedge F(a,b)\wedge F(a,c)\supset D(b,c))$$

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2) “ aEa for all $a \in A$ ”:

$$\forall a(A(a) \to E(a,a))$$

3) “For any $a \in A$ there exists such $b \in B$ that aFb ”:

$$\exists a \forall b(A(a) \to B(b) \wedge F(a,b)).$$

Let’s list the most used identical equations for quantifiers at any A and B:

$$\forall x \forall x A(x) \equiv \forall x A(x) \quad (7)$$

$$\exists x \exists x A(x) \equiv \exists x A(x) \quad (8)$$

$$\exists x \forall x A(x) \equiv \forall x A(x) \quad (9)$$

$$\forall x \exists x A(x) \equiv \exists x A(x) \quad (10)$$

$$\forall x A(x) \equiv \forall y A(y), \quad (11)$$

$$\exists x A(x) \equiv \exists y A(y), \quad (12)$$

$$\forall x \forall y A(x,y) \equiv \forall y \forall x A(x,y) \quad (13)$$

$$\exists x \exists y A(x,y) \equiv \exists y \exists x A(x,y) \quad (14)$$

$$\forall x(A(x) \wedge B(x)) \equiv \forall x A(x) \wedge \forall x B(x) \quad (15)$$

$$\exists x(A(x) \vee B(x)) \equiv \exists x A(x) \vee \exists x B(x) \quad (16)$$

$$\overline{\forall x A(x)} \equiv \exists x \overline{A(x)}, \quad (17)$$

$$\overline{\exists x A(x)} \equiv \forall x \overline{A(x)}, \quad (18)$$

$$\forall x A(x) \equiv \overline{\exists x \overline{A(x)}}, \quad (19)$$

$$\exists x A(x) \equiv \overline{\forall x \overline{A(x)}}, \quad (20)$$

$$\forall x \forall y(A(x) \wedge B(y)) \equiv \forall x A(x) \wedge \forall y B(y) \quad (21)$$

$$\exists x \exists y(A(x) \vee B(y)) \equiv \exists x A(x) \vee \exists y B(y) \quad (22)$$

In these examples it means that the predicates A and B are n-ary and their value depends on the variables $x_1, x_2, ..., x_n$. In purpose of short writing down using symbols x and y from general list of variables, only variables x_i and y_i are used at arbitrary fixed $i, j = \overline{1, n}$. The variable x_i of the predicate $P(x_1, x_2, ..., x_i, ..., x_n)$ is called dummy or nonexistent variable, if the value of the predicate P doesn’t depend on values of this variable. The notion __ of a variable of the predicate P is determined as follows:

$$\forall x_1 \forall x_2 ... \forall x_{i-1} \forall x_{i+1} ... \forall x_n \forall x_i \forall x_i' \left(P(x_1, x_2, ..., x_{i-1}, x_i, x_{i+1}, ..., x_n) \equiv P(x_1, x_2, ..., x_{i-1}, x_i', x_{i+1}, ..., x_n) \right) \quad (23)$$

Operating with an existential or generality quantifier in x_i for any predicate $P(x_1, x_2, ..., x_n)$, we get $Q(x_1, x_2, ..., x_n)$ which x_i is artificial. If the predicate P has an artificial variable x_i, then this predicate doesn’t change supplement according to P of quantifiers $\forall x_i$ or $\exists x_i$. If the predicate B has an artificial variable x then for any A and B:

$$\forall x(A(x) \vee B) \equiv \forall x A(x) \vee B, \quad (24)$$

$$\exists x(A(x) \wedge B) \equiv \exists x A(x) \wedge B, \quad (25)$$

Existential or generality quantifiers may be taken not only according to objects but also to predicate variables. Such ability grounds on following reasons:

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When predicate P is variable than the par $P(x_1,x_2,...,x_m)\exists$ may be considered as $m+1$-ary relation $(x_1,x_2,...,x_m)\in F$, which bounds an object variable $x_1,x_2,...,x_m$, with variable n-ary relation corresponding to predicate P.

Here $x_1,x_2,...,x_m$ $(m<n)$ are essential variables of predicate P. Unessential variables are omitted in predicate P notes. To be more precisely, it's necessary to say that P is a variable posed in a set of names of of all kinds of m-ary relations. Let us denote a predicate corresponding the relation $(x_1,x_2,...,x_m)\in F$ as $\Pi(x_1,x_2,...,x_m,P)$. Here Π is a name of a constant predicate corresponding a frequent relation with name $\in$. If we replace everywhere expression $P(x_1,x_2,...,x_m)$ to expression $\Pi(x_1,x_2,...,x_m,P)$ in the formula containing a predicate which is before the sign of a quantum operation, then a variable predicate P will disappear in this formula and one more variable P will appear instead of it which is posed on a name set of all kinds of n-ary predicates. Thus, setting a quantifier in variable predicate P, in fact we set it in an object variable P that is set in a subset of universe. Predicate names are not included into a U set of all objects. That's why we may not consider the U set to be a universe of the problem. Now we need a wider universe which can be gained though sum of U set and set of names of all predicates. By this means it turns out that object variables are posed not on a universe, but on one of these subsets.

Let's write the formulas for calculating quantifiers in case of the presence of both subject and predicate variables in the sub quantifier expression. Let $x_1,x_2,...,x_n\in U$; $P_1,P_2,...,P_r\in U^n$; $R(x_1,x_2,...,x_n, P_1,P_2,...,P_r)$ - arbitrary predicate. Quarters by subject variable x_i are found by the formulas:

$$\forall x_i I(R(x_1,x_2,...,x_i,...,x_n,P_1,P_2,...,P_r)) = \bigwedge_{x_i\in U} I(R(x_1,x_2,...,x_i,...,x_n,P_1,P_2,...,P_r)) \quad (26)$$

$$\exists x_i I(R(x_1,x_2,...,x_i,...,x_n,P_1,P_2,...,P_r)) = \bigvee_{x_i\in U} I(R(x_1,x_2,...,x_i,...,x_n,P_1,P_2,...,P_r)) \quad (27)$$

Quantifiers by predicate variable P_j, $j=\overline{1,r}$ are calculated by the formulas:

$$\forall P_j I(R(x_1,x_2,...,x_n,P_1,P_2,...,P_j,...,P_r)) = \bigwedge_{P_j\in U^n} I(R(x_1,x_2,...,x_n,P_1,P_2,...,P_j,...,P_r)) \quad (28)$$

$$\exists P_j I(R(x_1,x_2,...,x_n,P_1,P_2,...,P_j,...,P_r)) = \bigvee_{P_j\in U^n} I(R(x_1,x_2,...,x_n,P_1,P_2,...,P_j,...,P_r)) \quad (29)$$

It is possible to find operation of taking quantifiers at predicate variables of the second and higher ranges by analogical method.

Give the examples of translation into formal language of mathematic predicating with quantifiers by predicate variables:

1) "For all a and b a=b only in the case, when $a\in A$ with any A" : $\forall a\forall b(I(a=b)\equiv\forall A(a\in A\equiv b\in A))$;

2) For all A and B A=B only in case, when $a\in A$ equally matched $a\in B$ with any A": $\forall A\forall B(I(A=B)\equiv\forall a(a\in A\equiv a\in B))$;

3) "There are many totalities M, which for all x $x\in M$: $\exists M\forall x M(x)$. Let's notice that predicate equality from first-order predicates is found by the formula

$$I(P_i,P_j)=\bigvee_{A\in U^n}\hat{P}_i,\hat{P}_j, \quad (30)$$

Analogical to expression (43)[2].

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We reviewed the line of operations over predicates, reflecting set of all predicates, defined on U^n into it. It is logical product, disjunction and predicate negation, as well as quantifiers of generality and existence with respect to variables $x_i \ (i=\overline{1,n})$. Let's introduce a family of all predicates corresponding to relations $A \subseteq U^n$, each of which is considered as constant operation A, accepting on all predicate sets the same meaning A. Let's also introduce operation of predicate A recognition on predicate variable $P_j \ (j=\overline{1,t})$, determined in the following way:

$$P_j^A = \begin{cases} 1, & if \ P_j = A, \\ 0, & if \ P_j \neq A. \end{cases}$$

Here symbols 1 and 0 mean identically true and identically false predicates.

Any t-local operation F over predicates $P_1, P_2, \dots, P_t$ can be expressed in the following way in the form of already introduces operations superposition:

$$F(P_1,P_2,\dots,P_t) = \bigvee_{(A_1,A_2,\dots,A_t) \subseteq U} \left(F(A_1,A_2,\dots,A_t) \, P_1^{A_1} P_2^{A_2} \dots P_t^{A_t} \right).$$

Here $F(A_1,A_2,\dots,A_t)$ - fixed predicate, representing meaning of F operations on the set of predicates $(A_1,A_2,\dots,A_t)$. In such a way, the system of operations consisting of disjunction, logic product, recognizing of all possible predicates on variable $P_1,P_2,\dots,P_t$ and all contrastive operations is full.

Recognizing of every predicate A by any predicate variables $P_j \ (j=\overline{1,t})$ is expressed in the form of the following superposition of generality quantifier, operations of predicate equivalence and fixed (individual) predicate A:

$$P_j^A = \forall x_1 \forall x_2 \dots \forall x_n \left(P_j(x_1,x_2,\dots,x_n) \sim A(x_1,x_2,\dots,x_n) \right) \quad (31)$$

Operation of predicate $P \sim Q = \overline{P}\,\overline{Q} \vee PQ$ equality (g) is expressed through disjunction, logical product and predicate negation. Let $x=(x_1,x_2,\dots,x_n)$ set of object variables. A common quantifier for x set is operation

$$\forall x P(x) = \forall x_1 \forall x_2 \dots \forall x_n P(x_1,x_2,\dots,x_n) \quad (32)$$

We see that any operation over predicates can be expressed by operations superposition $\forall x$, $\vee$, $\wedge$, $\bar{}$ and all constant operations (i.e. individual predicates). Such operations system is irreducible.

Operations over predicates, which accept meanings only from set 0, 1 are predicates of predicates. Each predicate T of predicate $P_1,P_2,\dots,P_t$ corresponds to some relation T, connecting relations of $P_1,P_2,\dots,P_t$. Any t – ary predicate T from predicates $P_1,P_2,\dots,P_t$ can be expressed by the formula:

$$T(P_1,P_2,\dots,P_t) = \bigvee_{(A_1,A_2,\dots,A_t) \in T} P_1^{A_1} P_2^{A_2} \dots P_t^{A_t} \quad (33)$$

We see that the system of all predicate recognition together with disjunction and logical product operations is full at expression of any predicate from predicates with its help. If desirable do without predicates recognition so for expression of any predicates from predicates it is possible to use the system described in previous paragraph, consisting of operations $\vee$, $\wedge$, $\bar{}$, $\forall x$ and all constant operations.

Predicates from predicates are used for formal record of mathematic predicating. If predicates from predicates take meaning 1, so true predicating corresponds to it, if 0 – false. Let's give the examples of true and false predicating in formal language:

1) "For any set M exists such element x, that $x \in M$": $\exists M \forall x M(x)$;
2) "For any relations A and B $A \cup B = B \cup A$: $\forall A \forall B Q(A \cup B, B \cup A)$.

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Statement 1) false, statement 2) true. If the formula, expressing pericardia operation includes quantifier $\forall x_i$ or quantifier $\exists x_i$, it is said that variable x_i in this formula is tied. Otherwise variable in the formula is free. Formula, which includes all variables, is isolated. Isolated formulas are always true or false.

Let us introduce a new variant of the notion model, determining it with such calculation so it satisfied the requests of intellectual theory to a greater extent than classic general model notion. Model over universe of letters $L=(a_1, a_2,\ldots,a_k)$ and universe of variables $V=(x_1,x_2,\ldots,x_n)$, name any pair $\langle M, P\rangle$ which has any subset M n-decartile set degree L, i.e. $M\subseteq L^n$ in the role of the first component, and the role of the second component is performed by any n-local predicate $P=(x_1,x_2,\ldots,x_n)$, defined for L^n. The first component M of the model $\langle M, P\rangle$ named as carrier or basic set of the model. The number of elements in the set M is named model power. The second component P is named predicate of the model $\langle M, P\rangle$. Set Ln named as universal space of dimensionality n.

It consists of an possible n-component letter sets, taken of the set L. Power of space L is k contained in it the set of letters. For discrimination of introduced by us notion model with classical, models of just described type will be named modified.

In the experiments over tested the properties of the concept equality predicate act as experimentally checked postulates and axioms. In order that tests had an evidential value the system of all checked axioms in the experiment should be complete. It is desirable to have minimal scope of factual performed experimental work, so the system of axioms are required also demand of economy. Economy of the system of axioms is achieved only in the case, if, firstly, the system does not contain excess axioms, secondly, each axiom is simplified to the limit. Excess axioms are axioms, exception of which does not deprive the system completeness. The simplicity of the axiom is understood in the sense that its experimental verification requires minimum efforts. The reduction of the number of axioms in the system to minimum with the preservation of its completeness property is achieved by an exception of the system of dependent axioms, i.e. such axioms, which can be logically derived from the set of the remaining axioms. After elimination of all dependent axioms from the system the remaining axioms become independent one from another and further reduction of the axioms number in the system becomes impossible. Such axioms system is called irreducible.

III. CONCLUSION

The finite predicates are used as main mathematic apparatus in the theory algebra of intelligence. This choice is based on the fact of algebra of finite predicates completeness. In the algebra of finite predicates language any relation and any finite function can be recorded. This means that with algebra of finite predicates language any intelligence law and any intellectual activity using ECM can be expressed. Structure of algebra of finite predicates express the very essence of intellectual processes and phenomena, they admit a direct interpretation in psychological terms.

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AUTHOR BIOGRAPHY

[1] Khudhair Abed Thamer . Kuliyyah Al-Maaref University College, Republic Iraq, Head of the Chair of Computer Science, 009647830023883, maaref_database@yahoo.com

[2] Shabanova-Kushnarenko L.V .Engineer, National Technical University "KhPI", Kharkiv, Ukraine