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A study on the various design issues of digital filters

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***Abstract*— In this paper, introduction to digital filters is described. The importance of digital filters compared to analog filters is discussed in detail. A flowchart of the digital filter design is explained. Various errors that exist in the literature are defined and revisited. FIR and IIR filters definitions and design methods such bilinear transformation method, Fourier series expansion method, and many such techniques are explained. Butterworth and Chebyshev filters design and comparison of the same is discussed. A comparative discussion is carried out.**

***Index Terms*—Signal processing, digital filter, Impulse response, differential equation, stability, phase angle, sample, ripple, pass band, stop band.**

I. INTRODUCTION

In signal processing, the most important step is filtering out the noise or unwanted signal from the original signal. Filters are used to separate those frequencies which contain useful information [3,4]. Two kinds of filters are employed in Signal Processing named as Analog Filters and Digital Filters. Analog Filters operate on actual signal and are described by linear differential equations. The electrical components such as resistors, inductors and capacitors are employed to implement the analog filters. Digital Filters operate on digital samples of the signal and are defined by linear differential equations. They are implemented using digital logic components such as adders, sub tractors and delays [4, 5, 6].

The following points depict the advantages of digital filters when compared to their analog counter parts[4],

- The digital filter coefficients can be programmed easily.
- The digital filters can be emulated using a computer and can then be tested using hardware.
- The stability of digital filters is high with respect to temperature and time, as compared to that of analog one.
- Dissimilar in their analog equivalents, digital filters can employ low frequency signals efficiently as the Digital Signal Processing (DSP) technology steady to grow exponentially. In the past, RF domain high frequency signals were dealt with analog technology, whereas now a day's digital filters are exercised for the same.
- The parallelism is possible with digital filters which enable them to be used for complex calculations as compared to analog filters.

With the above mentioned advantages, the design of the digital filters is a promising area of research. The purpose of writing the paper is to revise some of the existing methods of designing the digital filters. The aim also includes the study of classification of filters, their approximation procedures and different ways of design techniques.

The paper is organized as follows. In section II, the flowchart of the digital filter design is discussed. Various types of errors used in the design of digital filters is discussed in the section III. Section IV deals with the various types of digital filters and their design procedures. Different types of approximations used in the design of digital filters is discussed in Section V. Finally results and conclusions are drawn in Section VI.

II. DIGITAL FILTERS AND THEIR DESIGN

Digital filter design [6,7] involves usually the following basic steps which are shown flowchart of Fig.1

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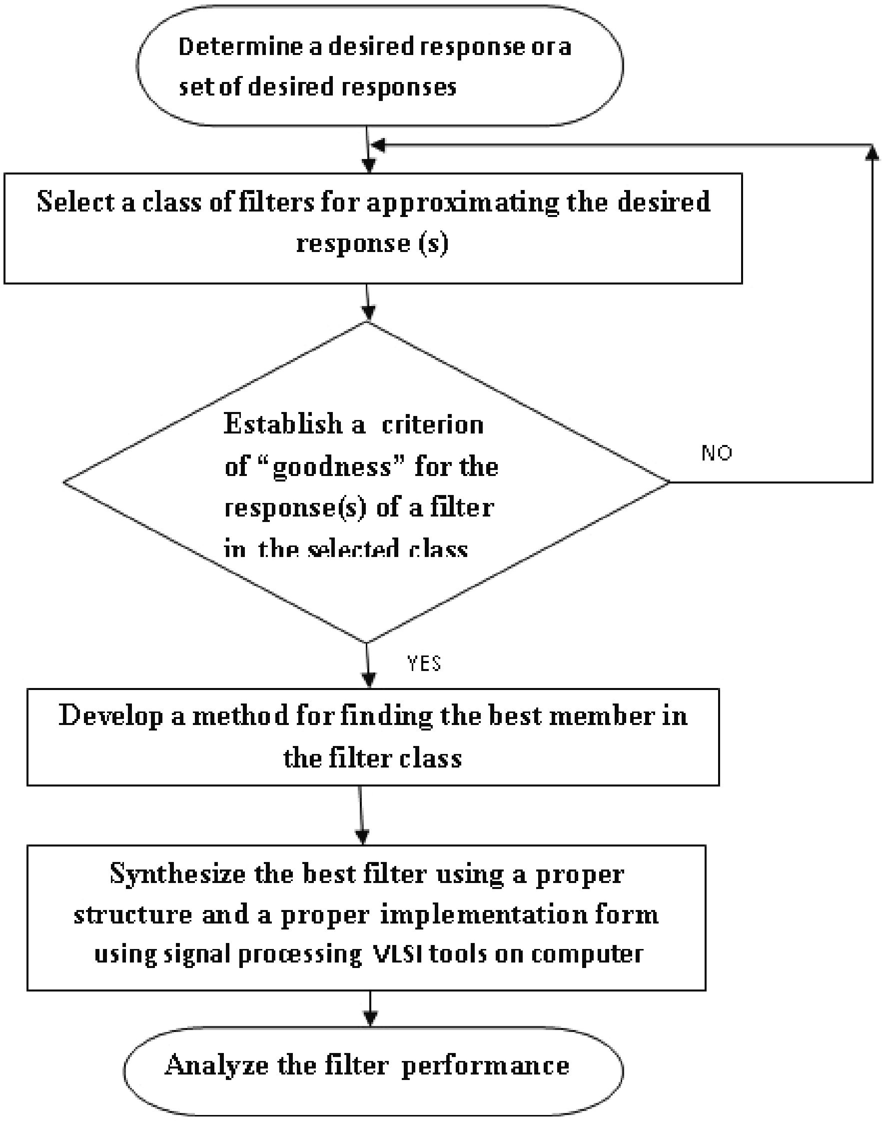

Fig.1 Digital filter design process flow chart

Step 1:- In most cases, determining the required response for either of magnitude or phase response of an incoming signal. The required magnitude response of the filter is generally described by ascertaining the frequencies region(s) where input signal components need to be filtered. The desired phase response is generally preferred to be linear with those frequency intervals where the signal components are retained. In some scenarios, time-domain conditions may be included. For example, Nyquist filter design requires zeroing restriction for some of impulse response values. In some applications, some constraints are imposed on step response of input signal component.

Step 2:- This step consists of choosing a proper class of filters to approximate the given response(s). First, it must be decided whether to use the filters whose impulse response is finite or infinite i.e, either FIR or IIR filters. After this, a proper class of FIR or IIR filters is selected. For many of the computationally efficient or low sensitivity FIR and IIR filter structures, there are constraints on filter transfer function. In these cases, the design of the transfer function and the filter implementation cannot be separated, and the desired filter structure determines the class of filters under consideration.

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Step 3:- In order to find the best member in the selected filter class, an error measure is needed by which the proximity of the approximating response(s) to the given response(s) is determined. There are several error measures. In many cases, the maximum permissible value of the error measure is specified, for example, the maximum permitted deviation from the given desired response. In this case, the problem is to first ascertain the minimum complexity of a filter (e.g., the minimum filter order) required to meet the criteria. The outstanding problem is to find the best member in the class of filters with this complexity to minimize the error measure.

Step 4:- The fourth step is to develop a method to find this best member. This chapter and the following chapter describe several design techniques for FIR and IIR digital filters.

Step 5:- The fifth step involves synthesizing the filter designed in the previous step.

Step 6:- The final step is to test whether the resulting filter meets all the given criteria. The above design process is often used iteratively. If the resulting filter does not possess all the desired properties, then the desired response(s), the filter class, or the error measure should be changed and the overall process is repeated until a filter is obtained with a satisfactory overall performance. The frequency domain specifications of a digital filter contain magnitude and phase responses. The low pass filter magnitude response is as shown below Fig.2.

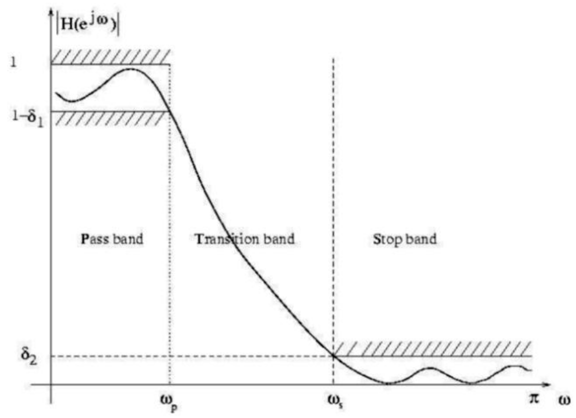

Fig.2: A ideal low pass filter approximation of tolerance limits

The desired magnitude response of low pass filter will be specified as[1,2,6],

$$D(\omega)=\begin{cases}1 & for\ \omega \in [0,\omega_p]\\ 0 & for\ \omega \in [\omega_s, 0]\end{cases} \quad (1)$$

and the specifications are given for magnitude response $|H(e^{j\omega})|$. Based on the specifications mentioned above, the filter needs to preserve the signal components in the region $[0, \omega_p]$ called the pass band of the filter. And it needs to reject the components in the region $[\omega_s, \pi]$ called the stop frequencies of the filter. Transition band has range of frequencies $[\omega_p, \omega_s]$, in which filter response is changing from non-zero in pass band region to zero in stop band region. Filter is approximated by the admissible errors in the pass band and stop band, δ_1 and δ_2 correspondingly. Specifications of this filter is given as [4,5]

$$1-\delta_1 \le |H(e^{jw})| \le 1 \qquad where\ |\omega| \le \omega_p \quad (2)$$

$$|H(e^{jw})| \le \delta_2 \qquad where\ \omega_s \le |\omega| \le \pi \quad (3)$$

The next move in the process is finding the expression and approximating the given specifications. That expression will be a polynomial for FIR type filters and is a rational approximation for an IIR filter.

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III. VARIOUS ERROR MEASURES IN DIGITAL FILTERS

Three different error measures are commonly used in designing digital filters.

Minimax Error Designs:- This method is based on minimizing the maximum error. The error under consideration will be the absolute difference between desired and approximating responses [1]. Optimization of transfer function coefficients is required to achieve these requirements, which is called as minmax or chebyshev approximation. For the low pass filter example, the weighted error function E(*ω)* is given by the equation,

$$E(\omega) = W(\omega)[|H(e^{jw})| - D(\omega)] \quad (4)$$

Where D(*ω)* called as desired function and W(ω) called as weighting function. To meet | $H(e^{jw})$ | given criteria, the maximal complete value of this function should be lesser than or equal to ϵ on X. Now transfer function coefficients need to be optimized to diminish the peak complete value of E(w) on X, i.e., the quantity

$$\epsilon = \max_{\omega\epsilon X}|E(\omega)| \quad (5)$$

Once the maximum permissible value of ϵ is specified, the approximation process is to identify the minimum order of a filter that meets the given criteria. As the minimum order of the filter is obtained, the transfer function coefficients of the filter need to be optimized in order to minimizeϵ. Elliptic (Cauer) IIR filters and Equiripple linear-phase FIR filters are few examples of minimax error design solutions.

Least-Squared Error Designs (L_p) : In some cases, instead of the minimax norm, the L_p norm is used. Here, it is used to minimize the function [2],

$$E_p = \int_X [W(w)[|H(e^{jw})| - D(w)]]^p \, dw \quad (6)$$

Where p is positive even integer. It can be shown that as p ->∞, the solution minimizing the above quantity approaches the minimax solution. This fact is exploited in some IIR filter design methods. For FIR filters, L_p error designs are of little practical use since there are efficient algorithms directly available for designing FIR filters with random specifications. The exception is the L_2 error or least-squared error designs, which can be found very effectively.

Maximally Flat Approximations. In the third approach, the approximating response is obtained based on a Taylor series approximation to the given desired response at a certain frequency point and the solution is called a maximally flat approximation. In some cases, such as in designing maximally flat (Butter-worth) IIR filters, there are two points, one in the pass band and one in the stop band, where a Taylor series approximation is applied. Most of the methods developed for designing digital filters use one of the above approximation criteria. In some synthesizing techniques, a combination of these criteria is used. For instance, consider the case of Chebyshev IIR filters, a Chebyshev approximation is used in pass band range frequencies and a maximally flat approximation is used in the stop band [7],

$$E_2 = \int_X [W(w)[|H(e^{jw})| - D(w)]]^2 \, dw \quad (7)$$

IV. VARIOUS TYPES OF DIGITAL FILTERS

A digital filter is said to be recursive, if the output at particular instant not only depends on the previous and current inputs but also depends on the previously obtained outputs. In the implementation of these filters, feedback is used. Example of recursive filters is a IIR Filter. A digital filter is said to be non-recursive, if the current output is computed only from the previous and current inputs and does not depends on previous outputs. In implementation of these filters, feedback is not required. FIR Filters falls under the category of non-recursive filters [3,4,5].

IIR Filters

The IIR filters will contain both zeros and poles and is defined by[1],

$$\sum_{m=0}^{N} a_m \, y[n-m] = \sum_{l=0}^{M} b_l \, x[n-l] \quad (8)$$

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From equation (8) the no. of poles and zeros will be equal for a stable IIR system and they lie within the unit circle. The conventional approach for designing analog filters contain many of the closed loop formulae, which makes the digital IIR filters design an easy process. The following condition is to satisfy the transfer of analog transfer function into a digital transfer function,

- The left half of S-plane is to be converted as unit circle of z-plane.
- Map s-plane imaginary axis into unit circle of z-plane.

IIR Filters can be designed by the following methods such as, Impulse Invariance Method, Approximation of Derivatives method etc.

IIR Filter Design by Impulse Invariance

This design is implemented by adopting the technique of equating the unit sample response of digital filters to unit impulse response of an analog filter, means, i.e[2].

$$h(l) = h_a(lT) \tag{9}$$

where T stands for sampling period

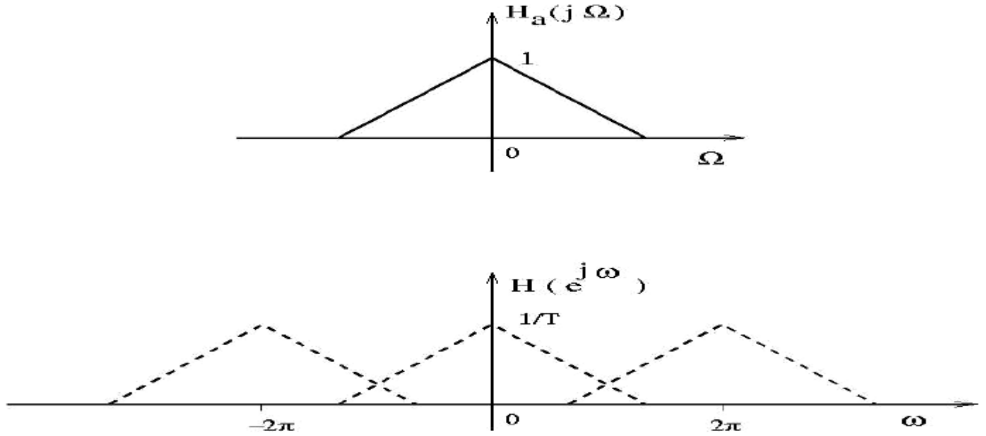

Fig.3 Aliasing effect

But, due to non-band limited nature of analog filter there is a problem of having an aliasing effect. To better understand the filter design approach, let us analyze the system function obtained from the analog filter given in terms of a partial fraction expansion.

$$H_a(s) = \sum_{k=1}^{N} \frac{A_k}{s-s_k} \tag{10}$$

The corresponding impulse response is

$$h_a(t) = \sum_{k=1}^{N} A_k e^{s_k t} U(t) \tag{11}$$

The digital filter system function H(z) is likely,

$$H(z) = \sum_{k=1}^{N} \frac{A_k}{(1-e^{s_k t} z^{-1})} \tag{12}$$

IIR Filter Design By Approximation Of Derivatives

In this method, an analog type filter is transformed into digital type filter by approximating the differential equation by an equivalent difference equation. The derivative term is going to be approximated using various techniques like rectangular rule, trapezoidal rule etc [4]. The corresponding methods of approximations will be backward approximation and bilinear transform methods which are explained as below.

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Backward approximation: An analog transfer function**,** H(s) expressed in rational form with constant coefficients [8]:

$$H(s) = \frac{\sum_{k=0}^{M} \beta_k s^k}{\sum_{k=0}^{N} \alpha_k s^k} \qquad (13)$$

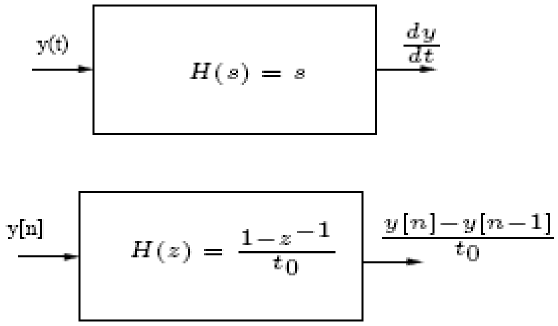
The corresponding time-domain equation will be,

$$\sum_{k=0}^{M} \alpha_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{N} \beta_k \frac{d^k x(t)}{dt^k} \qquad (14)$$

By using backward difference, we can approximate a derivative of equation 14 as

$$\frac{dy(t)}{dt} |t = nt_0 = \frac{y(nt_0) - y(nt_0 - t_0)}{t_0} = \frac{y(n) - y(n-1)}{t_0} \qquad (15)$$

LHS and RHS of equation (15) represent a continuous time and discrete time system which are supposed to be equivalent:



For both systems to be equivalent, we must have the following mapping

$$s = \frac{1 - z^{-1}}{t_0} \qquad (16)$$

This relationship between s and z holds for all the orders of the derivative, with s replaced by s^k and the first order difference replaced by the k^{th} order difference. Hence it holds for the system described by equation (14).Solving previous for z,

$$z = \frac{1}{1 - st_0} \qquad (17)$$

This is one mapping in the s and z planes. The complete left part of the s-plane is mapped with this onto a circle centered at z=[1/2,0] and radius 1/2. It is not a useful mapping if you want to create a digital filter with poles in other regions inside the unit circle. However, there is no aliasing.

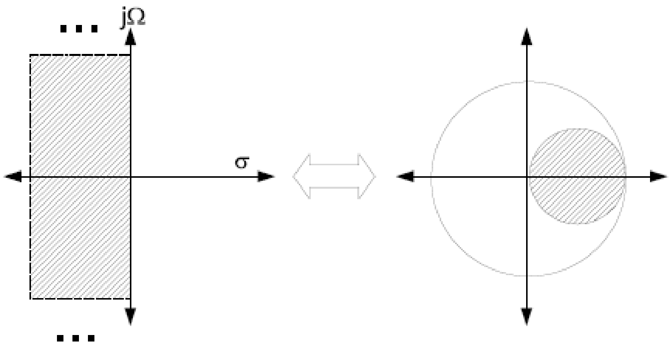


Fig.4: Usage of backward approximation to map the s-plane onto the z-plane

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Bilinear Transformation: It is based on an approximate solution of the continuous-time equation (15), but instead of approximating the derivative(s), it approximates integrals using the Trapezoidal Rule[8].

Consider the system: $c_1 y'_a(t) + c_0 y_a(t) = d_0 x(t)$, (18)

with system function H(s) given by: $H_a(s) = \frac{d_0}{c_1 s + c_0}$. (19)

Express $y_a(t)$ as an integral of $y'_a(t)$:

$$y_a(t) = \int_r^t y'_a(t) + y_a(r) \quad (20)$$

and let t = n t_0 and r = (n − 1) t_0. Then using the Trapezoidal Rule to approximate the integral, equation (20) can be written [8]

$$y_a(nt_0) = y_a((n-1)t_0) + \frac{t_0}{2}[y'_a(nt_0) + y'_a((n-1)t_0) \quad (21)$$

Substituting for $y'_a(nt_0)$ from equation (20) and using y[n]=y(nt_0) and applying z- transform , by knowing the fact that

$$Z\{y[n-1]\} = z^{-1}Y(z) \quad (22)$$

We get, $H(z) = \frac{Y(z)}{X(z)} = \frac{d_0}{c_1 \frac{2}{t_0}\frac{1-z^{-1}}{1+z^{-1}} + c_0}$ (23)

Comparing (23) to (19): $H(z) = H_a(s)|_{s=\frac{2}{t_0}\frac{1-z^{-1}}{1+z^{-1}}}$ i.e the discrete time transform will equal the continuous time transform if

$$s = \frac{2(1-z^{-1})}{t_0(1+z^{-1})} \quad (24)$$

Substituting s=jΩ and $z = e^{jw}$ and using the definition

$$\tan(x) = \frac{\sin(x)}{\cos(x)} = j\frac{e^{jx}-e^{-jx}}{e^{jx}+e^{-jx}} , \quad (25)$$

we get the following relation between Ω and ω

$$\Omega = \frac{2}{t_0}\tan\left(\frac{\omega}{2}\right) , \quad (26)$$

The relation between Ω and ω and the mapping between s- and z-planes are shown below.

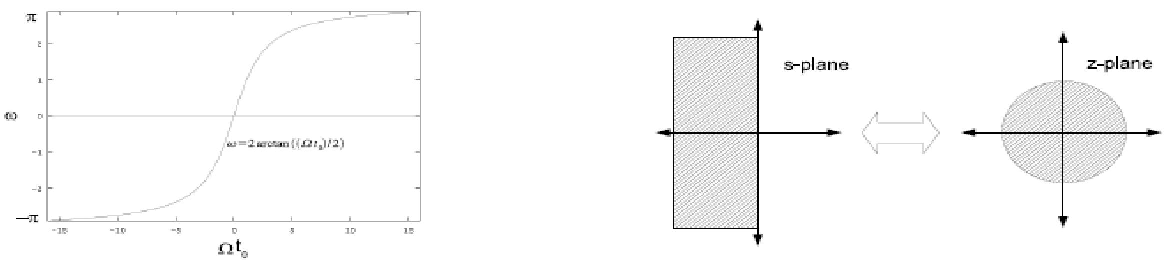

Fig.5: Usage of Bilinear transformation to map the s—plane onto the z-plane

Note that the bilinear transform maps the complete left part of the s-plane to the z-plane's unit circle interiors, and that higher frequencies along the jΩ axis are compressed compared to the frequencies near zero.

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FIR FILTERS

The governing equation for an FIR filters of length N is

$$y[l] = \sum_{k=0}^{N-1} b_k\, x(l-k) \tag{27}$$

FIR filter impulse response is given as ,

$$h[l] = \begin{cases} b_l, & l = 0, \ldots\ldots N-1 \\ 0, & otherwise \end{cases} \tag{28}$$

It is said to have linear phase if,

$$h[l] = h[N-1-l], \quad l = 0,1,2, \ldots\ldots N-1 \tag{29}$$

The advantages of FIR filters are, that they are naturally stable and have a linear phase. In determining the magnitude response of FIR filters have greater flexibility. They are simple and convenient to realize.

FIR Filters design: FIR Filters are designed by the techniques such as, Fourier series approach, Frequency Sampling method, and Window techniques. These techniques are as explained below,

The Fourier Series Approach : A system frequency response $H(e^{j\omega})$ is periodic with 2π period. From Fourier series analysis, a periodic function is given or represented as linear arrangement of complex exponentials. So the required frequency response of an FIR filter can be explained by the Fourier series.

$$H_d(e^{j\omega}) = \sum_{l=-\infty}^{\infty} h_d(l) e^{-j\omega l} \tag{30}$$

$h_d(l)$ represents the sequence of desired impulse response.

$$h_d(l) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega l} d\omega \tag{31}$$

The z -transform of the string is obtained from

$$H(z) = \sum_{l=-\infty}^{\infty} h_d(l) z^{-l} \tag{32}$$

The Eq.(1.32) will be of infinite duration and can be made finite by the process of truncation.

$$h(l) = h_d(l) \text{ for } |l| \le \frac{N-1}{2}$$
$$= 0 \quad \text{otherwise} \tag{33}$$

Then $$H(z) = \sum_{h=-(\frac{N-1}{2})}^{\frac{N-1}{2}} h_d(l) z^{-l} \tag{34}$$

Apply symmetry property on Equation(34) after its expansion

$$H(z) = h(0) + \sum_{l=1}^{\frac{N-1}{2}} h(l)[z^l + z^{-l}] \tag{35}$$

The above transfer function is difficult to realize physically. Multiply the Eq.(35) by $z^{-(N-1)/2}$ to realize it physically.

$$H'(z) = [h(0) + \sum_{l=1}^{\frac{N-1}{2}} h(l)[z^l + z^{-l}]] z^{-(N-1)/2} \tag{36}$$

Frequency sampling method: Let *h(l)* is the filter coefficients of an FIR filter and *H(k)* is the DFT of *h(l)*.Then

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$$h(l)=\frac{1}{N}\sum_{K=0}^{N-1} H(k)e^{j2\pi kl/N} \quad l=0,1,\ldots\ldots N\text{-}1 \tag{37}$$

$$H(k)=\sum_{k=0}^{N-1} h(l)e^{-j2\pi kl/N} \quad k=0,1,\ldots..N\text{-}1 \tag{38}$$

Based on DFT definition, $H(k)=H(z)\big|z = e^{j2\pi k/N}$ (39)

The transfer function *H(z)* of an FIR filter with impulse response is given by

$$H(z)=\sum_{l=0}^{N-1} h(l)\, z^{-l} \tag{40}$$

Substituting Equation (39) in Equation (40) , we get

$$H(z) = \sum_{l=0}^{N-1}[\frac{1}{N}\sum_{k=0}^{N-1} H(k)e^{j2\pi kl/N}]z^{-l}$$

$$= \sum_{k=0}^{N-1}\frac{H(k)}{N}\sum_{l=0}^{N-1} H(k)(e^{\frac{j2\pi k}{N}}z^{-1})^{l}$$

$$= \sum_{k=0}^{N-1}\frac{H(k)}{N}\frac{1-(e^{j2\pi k/N}z^{-1})^{N}}{1-e^{\frac{j2\pi k}{N}}z^{-1}}$$

$$= \frac{1-z^{-N}}{N}\sum_{k=0}^{N-1}\frac{H(K)}{1-e^{\frac{j2\pi k}{N}}z^{-1}} \tag{41}$$

We Know , $H(e^{j\omega})\big|\omega = \frac{2\pi k}{N}$=$H(e^{j2\pi kl/N})$=$H(k)$ (42)

i.e *H(k)* is the k^{th} DFT component obtained by sampling the frequency response of $H(e^{j\omega})$. As such, this approach for designing FIR filter is called the frequency sampling method.

Design of FIR filter employing windowing technique: The design process is as enumerated below

1. Calculate an ideal filter frequency response
2. Expand using Fourier series.
3. Truncate using window technique.

Summary of the windowed FIR filter design procedure

- Choose an appropriate window function
- Identify an ideal response $H_d(\omega)$
- Calculate the coefficients of the ideal filter $h_d(n)$
- Multiply the idyllic coefficients by the window function to provide the filter coefficients
- The frequency response of resulting filter is evaluated and repeated if necessary.

Comparison

- IIR filters have infinite Impulse Response with non-linear phase.
- FIR filters are preferred in linear phase applications.
- Hardware complexity is reduced with IIR compared to FIR.
- FIR filters are used in applications where narrow transition band requirements are required.
- FIR filters requires additional arithmetic operations and hardware components when compared to IIR filters.
-

V. VARIOUS APPROXIMATIONS USED IN DIGITAL FILTERS

In this section some of the approximations used in the design of the digital filters are discussed.

Butterworth Filter: This is also called as the maximally flat filter; this filter has flat frequency response in its pass band. The flatness is determined by the number of poles and closeness to ideal one is achieved for higher number of poles. The poles and zeros of the approximation form a circle. The Properties of Butterworth Approximation are[6,7,8],

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- The drawbacks with increasing the order are phase response become non-linear, increasing the overshot and ringing in step response.
- As the order increases, the roll-off rate is fast at cutoff frequency.
- In pass band and stop band the amplitude response is monotonic

Design Steps of Butterworth Filter

- Convert the filter specifications to their equivalents in the lowpass prototype .
- From pass band attenuation (α_p) determine the ripple factor ε
- From stop band attenuation (α_s) determine the filter order, *N.*
- Determine the left-hand poles, using the equations given.
- Construct the low pass prototype filter transfer function.
- Use the frequency conversion techniques to convert the low pass prototype filter to the given specifications

CHEBYSHEV FILTER: The transition region of chebyshev filters is thinner when compared to Butterworth filter of the same order. At a frequency of dc the attenuation is zero dB. The poles of a chebyshev approximation lies on an ellipse. Chebyshev filters are classified as, Type-1 chebyshev filter and Type-2 chebyshev filter

Type-1 Chebyshev filter: They illustrate equiripple performance in pass band and monotonic characteristic in stop band. It is a all pole filter. In s-plane, all its poles lie on an ellipse. Type-I Chebyshev filter frequency response characteristics are magnitude squared and is given as,

$$|H(\Omega)|^2 = \frac{1}{1+\varepsilon^2 T_N{}^2(\Omega/\Omega_p)} \qquad (43)$$

Where ε pass band ripple and $T_N(x)$ is the Nth order Chebyshev polynomial

$$T_N(x) = \begin{cases} \cos(N\cos^{-1}x) & |x| \le 1 \\ \cosh(N\cosh^{-1}x) & |x| > 1 \end{cases} \qquad (44)$$

The chebyshev polynomials can be developed by the recursive equations. The Properties of chebyshev approximation are,

- $|T_N(x)| \le 1 \; for\ all |x| \le 1$
- $|T_N(1)|$=1 for all N.
- All roots for the polynomial $|T_N(x)|$ occur in the range of -1$\le x \le 1$

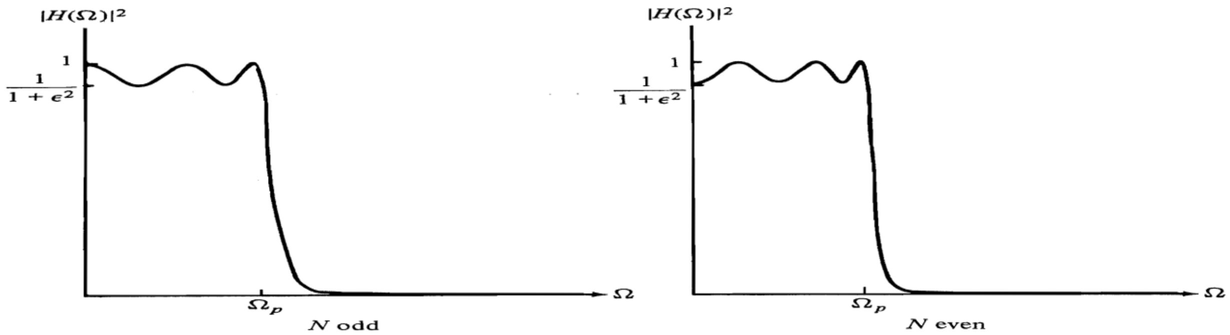

Fig.6 Type I Chebyshev filter characteristic for N odd and even

The design equations are as follows:

$$A_p = 10\log(1+\varepsilon^2) \qquad (45)$$

$$\varepsilon = \sqrt{10^{0.1A_p} - 1} \qquad (46)$$

$$A_r = 10\log(1+\varepsilon^2 C_n{}^2\left[\frac{\omega_r}{\omega_p}\right]) \qquad (47)$$

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OR $A_r = 10\log\left(1+\varepsilon^2 cosh^2\left[N\, cosh^{-1}\left[\frac{\omega_r}{\omega_p}\right]\right]\right)$ (48)

$$N \geq \frac{cosh^{-1}[10^{0.1A_r-1}/\varepsilon^2]^{1/2}}{cosh^{-1}\left[\frac{\omega_r}{\omega_p}\right]} \quad (49)$$

Transfer function left hand poles can be given as,

$$S_K = \sinh\left[\frac{1}{n} sinh^{-1}\frac{1}{\varepsilon}\right] sin\left[(2K+1)\frac{\pi}{2n}\right] + j\cosh\left[\frac{1}{n} sinh^{-1}\frac{1}{\varepsilon}\right]\cos[(2k+1)\frac{\pi}{2n}] \quad (50)$$

$$H(s) = -\prod_{K=0}^{n-1}\frac{S_K}{S-S_K} \qquad n \;\; odd \quad (51)$$

$$H(s) = \frac{1}{\sqrt{1+\varepsilon^2}}\prod_{K=0}^{n-1}\frac{S_K}{S-S_K} \qquad n \;\; even \quad (52)$$

Chebyshev 1 filter characteristics

i) Having ripples in the pass band.
ii) Transition band is sharper when compared to Butterworth.
iii) Group delay is poor comparatively.
iv) Having added ripples in pass band indicates shoddier phase response, narrower transition band, sharper cut-off and high pole Q.
v) It contains all poles and are located on an ellipse inside the unit circle

Type-2 Chebyshev filter: This filter transfer function contains equiripple performance in stop band and monotonic performance in the pass band. In s-plane, the zeros lie on the imaginary axis. The frequency response of this filter is magnitude squared and is specified as

$$|H(\Omega)|^2 = \frac{1}{1+\varepsilon^2[T_N{}^2(\Omega_s/\Omega_p)/T_N{}^2(\Omega_s/\Omega)} \quad (53)$$

Where $T_N(x)$ is the Chebyshev polynomial for order N and Ω_s is the stop band frequency. On imaginary axis, the zeros are located and the points are at

$$S_K = j\frac{\Omega_s}{sin\,\emptyset_k}, \qquad K = 0,1,2,\ldots,N-1 \quad (54)$$

The poles are located at the points (v_k, ω_k) where

$$v_k = \frac{\Omega_s x_k}{\sqrt{x_k{}^2+y_k{}^2}}, \quad k = 0,1,2,\ldots\ldots,N-1 \quad (55)$$

$$\omega_k = \frac{\Omega_s y_k}{\sqrt{x_k{}^2+y_k{}^2}}, \quad k = 0,1,2,\ldots\ldots,N-1 \quad (56)$$

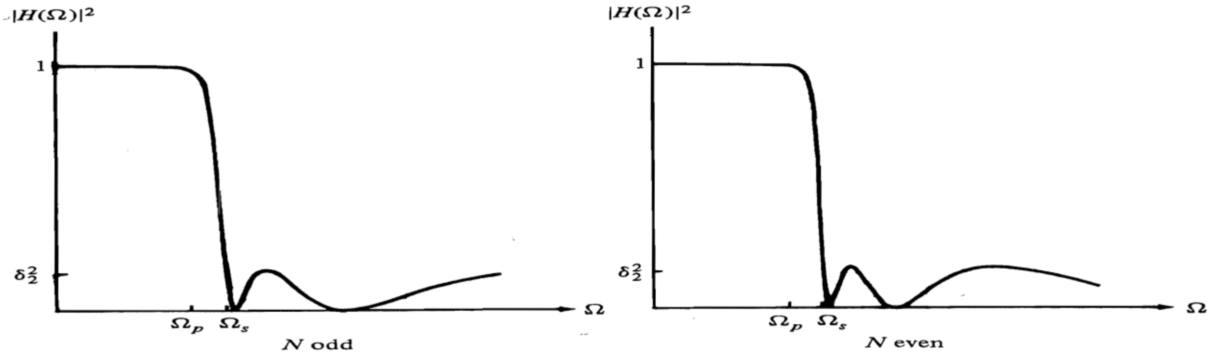

Fig.7 Type 2 Chebyshev filters frequency response

$$N = \frac{\log\left[\left(\sqrt{1-\delta_2^{\,2}} + \sqrt{1-\delta_2^{\,2}(1+\varepsilon^2)}\right)/\varepsilon\delta_2\right]}{\log\left[\Omega_s/\Omega_p\right] + \sqrt{(\Omega_s/\Omega_p)^2-1}\ ]} \tag{57}$$

Where $\delta_2 = 1/\sqrt{1+\delta^2}$ or $N = \frac{\cosh^{-1}(\delta/\varepsilon)}{\cosh^{-1}(\Omega_s/\Omega_p)}$ (58)

Chebyshev 2 filter Characteristics

i) Pass band has no ripple
ii) Stop band has Nulls or notches
iii) In comparison with Butterworth filter, it has sharper transition band.
iv) Further linear phase in pass band is compared to Chebyshev I filter.
v) The order of the filter where the order is denoted with 'n')
vi) No. of zeros are equal to n-1
vii) Complex conjugate zeros located on $j\omega$ axis and Poles located both inside & outside of the unit Circle.
viii) Nulls are created in stop band by zeros

VI. CONCLUSION

This paper deals with the digital filters and their design procedures. Basic definition of the digital filters and their advantages is studied. The digital filter can be either recursive or non-recursive type. Various design procedures for the design of IIR and FIR filters is discussed. Various types of approximation such as chebyshev, Butterworth are studied and their comparison is discussed.

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