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Investigating the unsteady MHD mixed convective flow with Hall Effect of a viscous incompressible fluid past a vertical porous plate with heat source

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Abstract: This research has discussed an analysis of unsteady MHD mixed convective flow with Hall Effect of a viscous incompressible fluid past an infinite vertical porous flat plate in the presence of a heat source. The governing equations describing the flow along with the boundary conditions have been solved by Finite Difference Method using the MATLAB computer generated program. A numerical investigation is performed to estimate the effects of varying the Prandtl number, the Schmidt number, Hall parameter, magnetic field parameter and the heat source parameter on fluid velocity and temperature distribution on MHD mixed convective flow. The effects of changing these values on Temperature distribution and velocity flow are shown and discussed in tables and displayed graphs. In this study we have looked at the effects of varying Schmidt number on fluid concentration for MHD mixed convective flow. We notice that the effect of increasing values of Sc leads to decrease the concentration profile in the flow field. Physically, the increase of Sc means decrease of molecular diffusivity D. This results in a decrease of concentration boundary layer. Hence, the concentration of the species is higher for small values of Sc and lower for larger values of Sc. The values of the concentration increases gradually near the plate and then decrease slowly for away from the plate. Obtained results are displayed both graphically and in tabular form to illustrate the effect of different parameters on the velocity, temperature and concentration profiles.

Index terms: **Prandtl number, hall parameter, magnetic field parameter and heat source.**

I. INTRODUCTION

Mixed convection flow is the combination of free and forced convection flow. In the recent years, considerable work has been done in this area. It has a lot of applications such as cooling of electronic devices, lubrication techniques, drying techniques, and etc. Magneto hydrodynamic (MHD) mixed convection heat transfer has a significant role in the collection of solar energy and different chemical procedures. The effect of heat source on heat transfer is another significant aspect in view of many physical problems. Heat generation or absorption may change the heat distribution in the fluid which consequently affects the particle deposition rate in the system such as semiconductors, electronic devices and nuclear reactors. Heat source may be considered constant, space dependent or temperature dependent. The influence of variable thermal conductivity on the flow and heat transfer have become more significant in engineering applications such as geothermal systems, crude oil extraction and machinery lubrication, etc. Especially in machineries, the moving part like bearings gets heated due to friction which needs lubrication for smooth functioning. Thermal conductivity of such lubricants may be affected by that frictional temperature. Combined heat and mass transfer problems are important in many processes and have therefore received a considerable amount of attention. In many mass transfer processes, heat transfer considerations arise due to chemical reaction and often due to the very nature of the process. In processes, such as drying, evaporation at the surface of a water body, energy transfer in a wet cooling tower and the flow in a desert cooler, heat and mass transfer occur simultaneously. Further, the effect of Hall current on the fluid flow with variable concentration has many applications in MHD power generation, in several astrophysical and meteorological studies as well as in plasma flow through MHD power generators. From the point of application, model studies on the Hall Effect on free and forced convection flows have been made by several investigators. The hydro magnetic convection with heat and mass transfer in porous medium has been studied due to its importance in the design of MHD generators and accelerators in geophysics, in design of underground water energy storage system, soil-sciences, astrophysics, nuclear power reactors and so on. Magneto hydrodynamics is currently undergoing a period of great enlargement and differentiation of subject matter. The interest in these new problems generates from their importance in liquid metals, electrolytes and ionized gases. On account of their varied importance, these flows have been studied by several authors – notable amongst them are Shercliff [22], Ferraro and Plumpton [10] and Crammer and Pai[7] Aboeldahab [1], Datta *et al* [8] , Acharya *et al* [2] and Biswal *et al* [6] have studied the Hall Effect on the MHD free and forced

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convection heat and mass transfer over a vertical surface. In the above mentioned studies the heat source effect is ignored. Due to its great applicability to ceramic tiles production problems, the study of heat transfer in the presence of a heat source has acquired newer dimensions. A number of analytical studies have been carried made of various forms of heat generation (Ostrach [17], Raptis [19] Bhupendra *et al* [5] presented unsteady MHD mixed convective flow of a viscous incompressible fluid past an infinite vertical porous flat plate in the presence of a heat source with Hall Effect by complex variable method. The solution was found to be dependent on the governing parameters including the Hartman number, the Prandtl number, the Grashof number, the Schmidt number, the Hall parameter, rotating parameter and the heat source parameter. It was noticed that an increase in Hall parameter (*m*) leads to an increase in the velocity for air, while the reverse phenomenon is observed for water. The concentration of the species is higher for small values of heat source parameter and lower for larger values of heat source parameter. The values of the concentration increases gradually near the plate and then decrease slowly far away from the plate. The problem investigated here is the study of the Hall Effects on the combined heat and mass transfer unsteady flow which occur due to buoyancy forces caused by thermal diffusion (temperature differences) and mass diffusion (concentration differences) of comparable magnitude past a vertical porous plate which is immersed in porous medium with a constant magnetic field and heat source applied perpendicular to the plate. It was noticed that; Velocity decreases with an increase in Magnetic field while it increases with increase in Grashof number, modified Grashof number, Permeability parameter. Temperature increases with increase in heat absorption and it shows reverse effect in the case of heat generation and Prandtl number. Finally concentration decreases with increase in Schmidt number and chemical reaction parameter. Acharya et al. [3] have studied free convection and mass transfer flow through a porous medium bounded by vertical infinite surface with constant suction and heat flux. But in those studies they considered the flow to be steady. In this study, the following conclusions are set out; In case of cooling of the plate ($Gr>0$) the velocity decreases with an increase of magnetic parameter, heat source parameter, suction parameter, Schmidt number and Prandtl number. On the other hand, it increases with an increase in Soret number. In case of heating of the plate ($Gr<0$) the velocity increases with an increase in magnetic parameter, heat source parameter, suction parameter, Schmidt number and Prandtl number. On the other hand, it decreases with an increase in Soret number. The temperature increase with increase of heat source parameter and suction parameter. And for increase of Prandtl number it is vice versa. The concentration increases with an increase of Soret number and Schmidt number whereas it decreases with an increase of suction parameter. Unsteady oscillatory free convection flow plays an important role in chemical engineering, turbo machines, and aerospace technology. Such flows arise due to unusual motion of boundary or boundary temperature. Sahoo et al. [20] have analyzed MHD unsteady free convective flow past an infinite vertical plate with constant suction and heat source. Extension to this problem has been done by Muthucumaraswamy and Atul Kumar [4]. In this study thermal radiation effect on moving infinite vertical plate in presence of variable temperature and mass diffusion is considered. Mohammed [12] analyzed the effect of radiation on a steady two-dimension magneto-hydrodynamic mixed convection flow from a vertical plate embedded in a saturated porous media with melting .The non-linear partial differential equations, governing the flow field under consideration, were transformed by a similarity transformation into a system of non-linear ordinary differential equations and then solved numerically by applying Nachtsheim-swigert shooting iteration technique together with sixth order Runge-Kutta integration schemes. The resulting non-dimensional velocity and temperature profiles were presented graphically for different values of the parameters of physical and engineering interest. It was observed that the Nusselt number decreases with increase in melting parameter, while it increases with increase in radiation parameter. Otieno *et al* [18] studied the effects of magneto hydrodynamics (MHD) fluid flow on a two dimensional boundary layer flow of a steady free convection heat and mass transfer on an inclined plate in which the angle of inclination is varied. The PDEs were transformed into ordinary differential equations (ODEs) by some similarity transformation. The ODEs were solved using the shooting method with the fourth order Runge-Kutta numerical method together with the Secant technique of root finding to determine their solutions. The study established that the flow field and other quantities of physical interest are significantly influenced by these parameters. In particular, it was found that the velocity increases with an increase in the thermal and solutal Grashof numbers. The velocity and concentration of the fluid decreases with an increase in the Schmidt number. Vedavathi *et al* [25] presented a paper that dealt with the effects of heat and mass transfer on two-dimensional unsteady MHD free convection flow past a vertical porous plate in a porous medium in the presence of thermal radiation under the influence of Dufour and Soret effects. The governing nonlinear partial differential equations were reduced to the coupled nonlinear ordinary differential equations by the similarity transformations. The resulting equations were solved numerically using shooting method along with Runge–

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Kutta fourth order integration scheme. It was found that an increase in thermal radiation parameter, the velocity and temperature profiles increase. Suction stabilizes the hydrodynamic, thermal as well as concentration boundary layers growth. Magnetic field retards the motion of the fluid. The velocity and concentration profiles decrease as an increase in Schmidt number. An increase of the absorption of radiation parameter, both velocity and temperature profiles increases. It is noted that negligible effect of Soret and Dufour numbers on temperature profile. The heat transfer rate is reduced by increasing thermal radiation parameter, the absorption of radiation parameter. The mass transfer rate increases as an increase of Schmidt number. Mohammad [13] numerically investigated the influence of the Hall current and constant heat flux on the Magneto hydrodynamic (MHD) natural convection boundary layer viscous incompressible fluid flow in the manifestation of transverse magnetic field near an inclined vertical permeable flat plate. The governing boundary layer equations were transferred into non-similar model by implementing similarity approaches. The physical dimensionless parameter were set up into the model as Prandtl number, Eckert number, Magnetic parameter, Schmidt number, local Grashof number and local modified Grashof number. The numerical method of Nactsheim- Swigert shooting iteration technique together with Runge-Kutta six order iteration scheme were used to solve the system of governing non-similar equations. The physical effects of the various parameters on dimensionless primary velocity profile, secondary velocity profile, and temperature and concentration profile are discussed graphically. Moreover, the local skin friction coefficient, the local Nusselt number and Sherwood number are shown in tabular form for various values of the parameters. It is noted that the velocity and temperature profiles as well as the heat transfer coefficients are significantly affected by the radiation parameter in the medium. It was found out that the heat transfer coefficients are reduced with increasing melting parameter and grows with increasing radiation parameter. Kwanza *et al* [11] presented the work on MHD Stokes free convection flow past an infinite vertical porous plate subjected to a constant heat flux with ion slip and radiation absorption. The concentration velocity and temperature distributions were discussed and results presented in graphs and tables. Nyabuto *et al* [15], investigated MHD stokes free convection of an incompressible, electrically conducting fluid between two horizontal parallel infinite plates subjected to a constant heat flux and pressure gradient. Analysis of velocity profiles and temperature distribution were obtained and the effect of the Eckert, Prandtl and Hartmann numbers on velocity profiles and temperature distribution investigated. The resulting non-linear differential equations obtained were solved using finite difference method. The results obtained indicated that an increase in Hartmann number leads to an increase in velocity profiles and temperature distribution while an increase in values of Prandtl number results in a fall in temperature distribution. Okelo [16] investigated, unsteady free convection incompressible fluid past a semi-infinite vertical porous plate in the presence of a strong magnetic field inclined at angle to the plate with Hall and ion-slip current effects. The effects of modified Grashof number, suction velocity, the angle of inclination, time, Hall current, ion-slip current, Eckert number, Schmidt number and heat source parameter on the convectively cooled or convectively heated plate restricted to laminar boundary were studied. He found that an increase in mass diffusion parameter causes a decrease in concentration profiles, absence of suction velocity or an increase of it causes an increase in concentration profiles, an increase of Eckert number causes an increase in temperature profiles and also an increase of an angle of inclination leads to an increase in primary velocity profiles but a decrease in secondary velocity profiles. The results were presented in tables and graphs. Sigey *et al* [24]) carried out a study of magnetic hydrodynamic free convection flow past an infinite porous plate in an incompressible electrically conducting fluid. The investigation of the effect of viscous dissipation on the velocity profiles and temperature distribution of the fluid in the presence of a transverse magnetic field subject to a constant suction velocity was conducted. The ordinary differential equations governing the flows were analyzed using an explicit finite. The numerical results of the study showed that an increase in the viscous dissipation causes an increase in the velocity profiles and temperature distribution of the fluid. This study finally asserted that an increase in the viscous dissipation parameter or term leads to an increase in velocity and temperature profiles. This increase in the velocity profiles and temperature profile occurred at a distance away from the porous plate. Sib *et al* [23] studied the effects of radiation on unsteady MHD free convective flow of a viscous incompressible electrically conducting fluid past an oscillating vertical porous plate embedded in a porous medium with an oscillatory heat flux in the presence of a uniform transverse magnetic field. The governing equations describing the flow were solved analytically. It was observed that increase in radiation parameter leads to decrease in fluid velocity near the plate and to increase away from the plate. The fluid velocity increases near the plate and it decreases away from the plate with an increase in suction parameter. The solution exists for both suction and blowing at the plate. The fluid velocity increases near the plate and it decreases away from the plate with an increase in Darcy number. Also the fluid temperature was seen to decrease near the plate and it increases away from the plate with an increase in either radiation parameter

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or Prandtl number or suction parameter. Sankar *et al* [21] studied the combined effects of Hall current and radiation on the unsteady MHD free convective flow in a vertical channel with an oscillatory wall temperature. They two different cases one being the flow due to the impulsive motion of one of the channel walls and the other was flow due to the accelerated motion of one of the channel walls. The governing equations were solved analytically and it was found that the primary velocity and the magnitude of the secondary velocity increased with an increase in Hall parameter for the impulsive as well as the accelerated motions of one of the channel walls. The fluid temperature also decreases with an increase in radiation parameter. Further, the shear stresses at the left wall reduced with an increase in either radiation parameter or frequency parameter for the impulsive as well as the accelerated motions of one of the channel wall.

II. MATHEMATICAL FORMULATION

Consider the unsteady flow of an electrically conducting fluid past an infinite vertical porous flat plate coinciding with the plane $y = 0$ such that the x-axis is along the plate and y-axis is normal to it. A uniform magnetic field B_O is applied in the direction y-axis and the plate is taken as electrically non-conducting. Initially the plate and the fluid are at same temperature T_1 in a stationary condition with concentration level C_1 at all points. For t > 0, the plate starts moving impulsively in its own plane with a velocity U_o, its temperature is raised to T_w and the concentration level at the plate is raised to C_w. The flow configuration and coordinate system are shown in the following Fig 1.

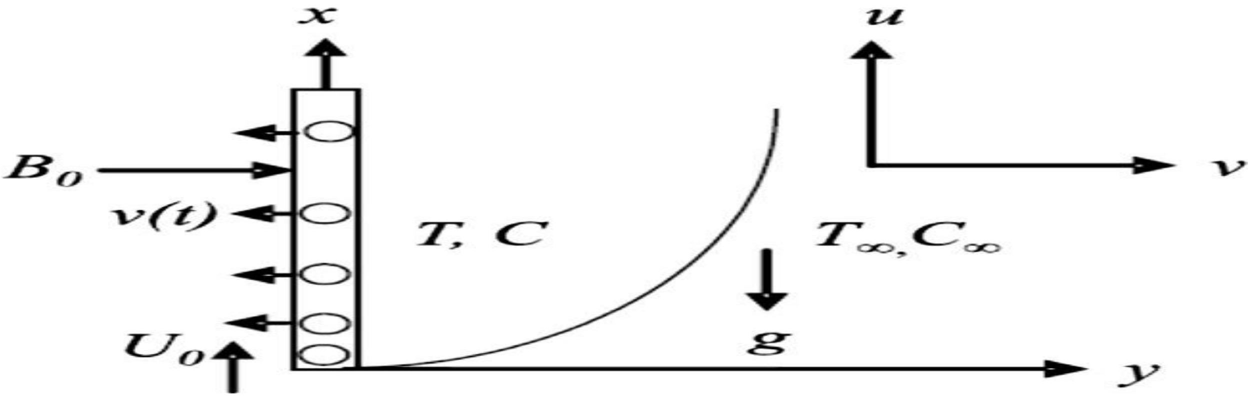

Fig 1: Flow configuration and coordinate system

Taking z-axis normal to xy-plane and assuming that the velocity $\vec{V}$ and the magnetic field $\vec{H}$ have components (u, v, w) and (H_x, H_y, H_z) respectively, the equation of continuity

$$\nabla\cdot\vec{V} = 0 \qquad (1)$$

And solenoidal relation

$$\nabla\cdot\vec{H} = 0$$

Given v = $-V_o$ constant, $V_o>0$. From Maxwell's electromagnetic field equations,

$$\frac{\partial H_y}{\partial y} = 0 \qquad (2)$$

If the magnetic Reynold number is small, induced magnetic field is negligible in comparison with the applied magnetic field, Cowling (1957), so that Hx= H_z= 0 and H_y= B_o (constant). $\frac{\partial v}{\partial y} = 0$ .If (J_x, J_y, J_z) are the components of electric current density $\vec{J}$. The equation of conservation of electric charge $\nabla.\vec{J}$ = 0 gives, J_y= constant. Since the plate is non-conducting, J_y= 0 at the plate and hence zero everywhere in the flow. Neglecting polarization effect, we get $\vec{J}$ =0. Hence

$$\vec{J} = (J_x, 0, J_z),\ \vec{H} = (0, B_o, 0),\ \vec{V} = (u, V_o, w) \qquad (3)$$

The generalized Ohm's law, taking Hall Effect into account, is given by

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$$\vec{J}+\frac{\omega_e\tau_e}{B_O}\left(\vec{J}\times\vec{H}\right)=\sigma\left(\vec{E}+\vec{V}\times\vec{B}+\frac{1}{e\eta_e}\nabla p_e\right) \quad (4)$$

In writing equation (4) ion slip, thermoelectric effects and polarization effects are neglected. Further, it is also assumed that $\omega_e\tau_e \sim 0$ and $\omega_i\tau_i \leq 1$, Equations (3) and (4) yield,

$$\left.\begin{aligned} J_x &= \frac{\sigma B_O}{1+m^2}(mu-w) \\ J_z &= \frac{\sigma B_O}{1+m^2}(u+mu) \end{aligned}\right\} \quad (5)$$

Where u and w are the x-component and z- component of $\vec{V}$ and $m = \omega_e\tau_e$ is the Hall parameter.

The equations of motion, energy and concentration governing the flow under the usual Boussinesq approximation are

$$\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_O(u+mw)}{\rho\left(1+m^2\right)}+g\beta(T-T_\infty)+g\beta^*(C-C_\infty)-\frac{\nu u}{\kappa} \quad (6)$$

$$\frac{\partial w}{\partial t}+v\frac{\partial w}{\partial y}=\frac{\partial^2 w}{\partial y^2}+\frac{\sigma B_O(mu-w)}{\rho\left(1+m^2\right)}-\frac{\nu w}{\kappa} \quad (7)$$

$$\frac{\partial(T-T_\infty)}{\partial t}+v\frac{\partial(T-T_\infty)}{\partial y}=\frac{k_O\partial^2(T-T_\infty)}{\rho C_p\partial y^2}+S(T-T_\infty) \quad (8)$$

$$\frac{\partial(C-C_\infty)}{\partial t}+v\frac{\partial(C-C_\infty)}{\partial y}=D\frac{\partial^2(C-C_\infty)}{\partial y^2} \quad (9)$$

In equation (8) the viscous dissipation and Ohmic dissipation are neglected and in equation (9), the term due to chemical reaction is assumed to be absent.

Now using

$$v=-V_0,\ T(y,t)-T_\infty=\theta(y,t) \text{ and } C(y,t)-C_\infty=C^*(y,t)$$

Subjecting to the initial and boundary conditions

$t\leq u(y,t)=w(y,t)=0$, $\theta=0$, $C^*=0$ for all y

$$u(0,t)=w(0,t)=0,\theta=(0,t)=ae^{i\omega t},C^*(0,t)=be^{i\omega t} \text{ at } y=0$$

$$u(\infty,t)=w(\infty,t)=0,\theta=(\infty,t)=0,C^*(\infty,t)=0 \text{ at } y=0,t>0 \quad (10)$$

Introducing the following non-dimensional quantities

$$\eta=\frac{V_o y}{v},t'=\frac{V_o^2 T}{4v},u'=\frac{u}{V_o},w'=\frac{w}{V_o},\theta'=\frac{\theta}{a},c'=\frac{c^*}{b}$$

$$G=\frac{4g\beta vb}{V_o^3},G_c=\frac{4g\beta^* vb}{V_o^3},M=\frac{4g\beta_o^2\sigma v}{V_o^3},\Pr=\frac{v\rho c_p}{k_o} \quad (11)$$

$$S_c=\frac{v}{D},k'=\frac{V_o^2 k}{4v^2},S'=\frac{S_v}{V_o^2}$$

Equations (6) to (9) are transformed to their corresponding non-dimensional forms (dropping the dashes).

With $\theta=(T-T_\infty)$, $C_O=(C-C_\infty)$ and $M=\sigma B_O/\rho$

$$\frac{\partial u}{\partial t}-4\frac{\partial u}{\partial y}=4\frac{\partial^2 u}{\partial y^2}-\frac{M(u+mw)}{\left(1+m^2\right)}+G\theta+G_C C_O-\frac{u}{\kappa} \quad (12)$$

$$\frac{\partial w}{\partial t}-4\frac{\partial w}{\partial y}=4\frac{\partial^2 w}{\partial y^2}+\frac{M(mu-w)}{\left(1+m^2\right)}-\frac{w}{\kappa} \quad (13)$$

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.
$$\frac{\partial\theta}{\partial t}-4\frac{\partial\theta}{\partial y}=4\frac{k_o\partial^2\theta}{\rho C_p\partial y^2}+S\theta \tag{14}$$

$$\frac{\partial C_o}{\partial t}-4\frac{\partial C_o}{\partial y}=4D\frac{\partial^2 C_o}{\partial y^2} \tag{15}$$

Methods of Solution

Many real life problems generally do have numerical solutions but not analytical solutions. Mathematics being one of the scientific research disciplines that lead to real life situations requires numerical techniques to accomplish non-analytical solutions. The part of numerical analysis which has been most changed so far, is the solution of partial differential equations by difference methods. This is owing to the fact that second-order partial differential equations govern many of the real-life physical phenomena. Such equations include Maxwell's equations, heat and momentum equations and Newton's laws of motion. A very powerful and quite a general method of dealing with most second-order (partial) differential equations is the finite difference method

Discretization of the Motion equation

We investigate the equation of motion for velocity in both horizontal directions. The momentum equation (12) is discretized using the hybrid difference scheme as follows

$$\frac{U_{i,j+1}-U_{i,j}}{\Delta t}-4\left(\frac{U_{i+1,j}-U_{i-1,j}}{2\Delta y}\right)=4\left[\frac{U_{i,j+1}-2U_{i,j}+U_{i,j-1}}{(\Delta y)^2}\right]-M\left[\frac{mw+u}{1+m^2}\right]+(G\theta)+(G_cC)-\left[\frac{u}{\kappa}\right] \tag{16}$$

Where M and G are the magnetic field and Grash of number respectively. The equation is solved subject to the boundary conditions $U_{i,0}=1$.We investigate the effect of M on the fluid velocity. Taking $\Delta t=0.01, \Delta y=0.25$,M=5, m=1.0, u=$C_0$= $\theta=\kappa=1$ and S =G=5, with i=1 and j=1, we get the scheme

$$5.25U_{i,j+1}-4.25U_{i,j}-1.125U_{i,j-1}=6.5 \tag{17}$$

Taking i=1, j=1, 2, 3……6, we get the matrix equation

$$\begin{bmatrix} -4.25 & 5.375 & 0 & 0 & 0 & 0 \\ -1.125 & -4.25 & 5.375 & 0 & 0 & 0 \\ 0 & -1.125 & -4.25 & 5.375 & 0 & 0 \\ 0 & 0 & -1.125 & -4.25 & 5.375 & 0 \\ 0 & 0 & 0 & -1.125 & -4.25 & 5.375 \\ 0 & 0 & 0 & 0 & -1.125 & -4.25 \end{bmatrix}\begin{bmatrix} U_{1,1} \\ U_{1,2} \\ U_{1,3} \\ U_{1,4} \\ U_{1,5} \\ U_{1,6} \end{bmatrix}=\begin{bmatrix} 7.625 \\ 7.625 \\ 7.625 \\ 7.625 \\ 7.625 \\ 7.625 \end{bmatrix} \tag{18}$$

Energy equation

The equation (14) is discretized using the hybrid difference scheme as follows

$$\frac{\theta_{i,j+1}-\theta_{i,j}}{\Delta t}-4\left(\frac{\theta_{i+1,j}-\theta_{i-1,j}}{2\Delta y}\right)=4\left[\frac{\theta_{i,j+1}-2\theta_{i,j}+\theta_{i,j-1}}{(\Delta y)^2}\right]+S\left[\frac{\theta_{i,j+1}+\theta_{i,j}}{2}\right] \tag{19}$$

We investigate the effects of S= -1, 1 and 2on the fluid temperature. Taking $\Delta t=0.01, \Delta y=0.25$,we get the scheme

$$0.4385\theta_{i,j}-0.8\theta_{i+1,j}-0.64\theta_{i-1,j}=-0.9985\theta_{i,j+1} \tag{20}$$

Taking i=1,2,3…….6 and j=1 with $\theta_{0,j}=\theta_{1,j}=20^0$ we form the following systems of linear algebraic equations

which can be written in matrix-vector form as

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$$\begin{bmatrix} 0.435 & -0.8 & 0 & 0 & 0 & 0 \\ -0.64 & 0.435 & -0.8 & 0 & 0 & 0 \\ 0 & -0.64 & 0.435 & -0.8 & 0 & 0 \\ 0 & 0 & -0.64 & 0.435 & -0.8 & 0 \\ 0 & 0 & 0 & -0.64 & 0.435 & -0.8 \\ 0 & 0 & 0 & 0 & -0.64 & 0.435 \end{bmatrix} \begin{bmatrix} \theta_{1,1} \\ \theta_{2,1} \\ \theta_{3,1} \\ \theta_{4,1} \\ \theta_{5,1} \\ \theta_{6,1} \end{bmatrix} = \begin{bmatrix} -24.88 \\ -24.88 \\ -24.88 \\ -24.88 \\ -24.88 \\ -24.88 \end{bmatrix} \quad (21)$$

Concentration equation

The concentration equation (15) is discretized using the central difference scheme as follows

$$\frac{C_{i,j+1} - C_{i,j}}{\Delta t} - 4\left(\frac{C_{i+1,j} - C_{i-1,j}}{2\Delta y}\right) = \frac{4}{S_c}\left[\frac{C_{i,j+1} - 2C_{i,j} + C_{i,j-1}}{(\Delta y)^2}\right] \quad (22)$$

We investigate the effects of Sc=0.1, 0.4, 0.7on the fluid concentration. Taking $\Delta t = 0.01, \Delta\eta = 0.25$,we get the scheme

$$11.96C_{i,j} - 6.6C_{i+1,j} - 6.4C_{i-1,j} = -C_{i,j+1} \quad (23)$$

Taking and i=1, 2, 3…….6 and j=1we form the following systems of linear algebraic equations

$$\begin{bmatrix} 11.96 & -6.6 & 0 & 0 & 0 & 0 \\ -6.4 & 11.96 & -6.6 & 0 & 0 & 0 \\ 0 & -6.4 & 11.96 & -6.6 & 0 & 0 \\ 0 & 0 & -6.4 & 11.96 & -6.6 & 0 \\ 0 & 0 & 0 & -6.4 & 11.96 & -6.6 \\ 0 & 0 & 0 & 0 & -6.4 & 11.96 \end{bmatrix} \begin{bmatrix} C_{1,1} \\ C_{2,1} \\ C_{3,1} \\ C_{4,1} \\ C_{5,1} \\ C_{6,1} \end{bmatrix} = \begin{bmatrix} 3.2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (24)$$

III. NUMERICAL RESULTS AND DISCUSSION

Effects of Magnetic field number on fluid velocity

Solving equation (18) the values of $U_{i,j}$ for M = 5, 10 and 15are presented in the Fig 1.

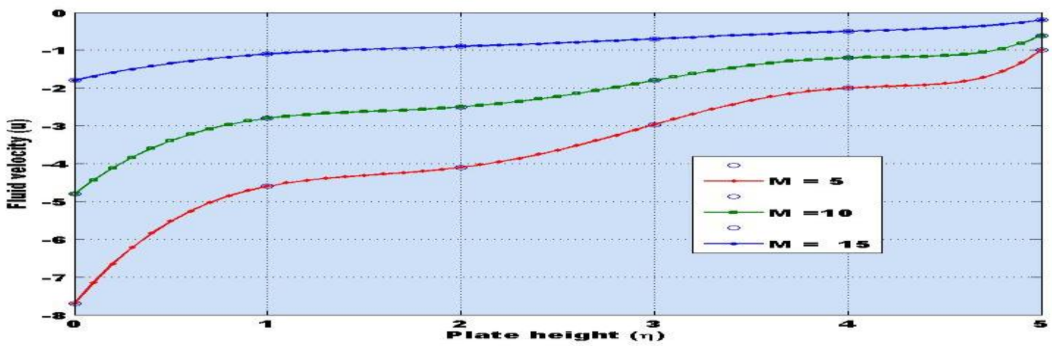

Fig 2: Graph of velocity against plate vertical height at varying magnetic field numbers

From Fig 2 it can be found that the values of velocity increase as the values of *M also* increases. Velocity also increases with increase in plate height. With an increase in M there is an increase in velocity i.e. the flow is accelerated. Physically it meets the logic that the applied magnetic field is therefore moving effectively with the free stream. The resulting L**o**rentzian body force therefore not acting as a drag force as in convectional MHD flow but as an aiding body force. The maximum velocity is achieved at intermediate distances from the plate. Moving away from the wall, the velocity profiles converge in accordance with the boundary condition imposed.

Effects of Schmidt number on fluid concentration

Solving equation (24) the values of $C_{i,j}$ for S_c= 0.1, 0.4 and 0.7 are presented in the Fig 3.

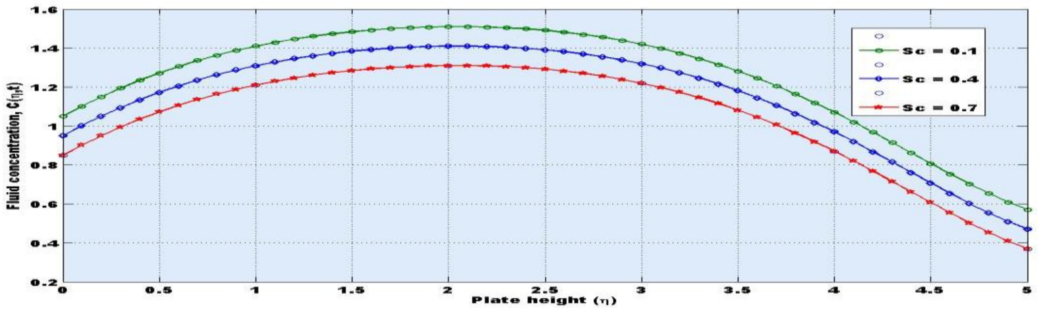

Fig 3: Graph of fluid concentration against plate height at varying Schmidt numbers

We notice that the effect of increasing values of Sc in figure 3 leads to decrease the concentration profile in the flow field. Physically, the increase of Sc means decrease of molecular diffusivity. This results in a decrease of concentration boundary layer. Hence, the concentration of the species is higher for small values of Sc and lower for larger values of Sc. The values of the concentration increases gradually near the plate and then decrease slowly for away from the plate.

Effects of sink parameter on fluid temperature

Solving equation (14) the values of $\theta_{i,j}$ for S= -1, 1 and 2are presented in the Fig 4.

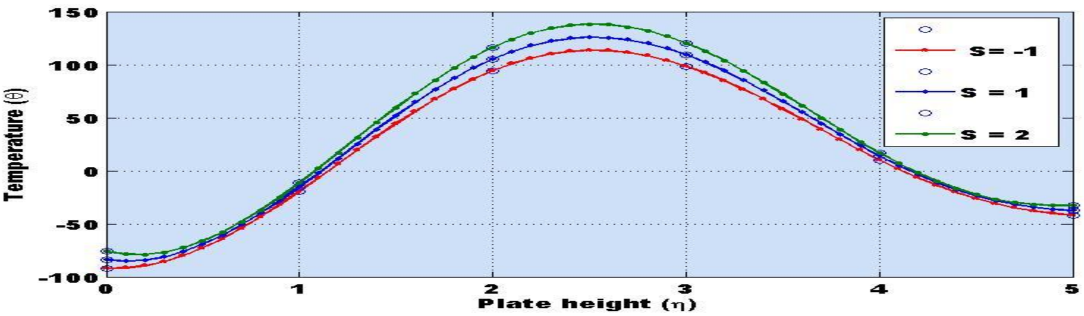

Fig 4: Graph of temperature against plate vertical height at varying sink parameter

The temperature profiles are shown in Fig. 4. The effects Son the temperature profiles are drawn from Fig 4 as; Temperature increases with increase in S parameter. As the plate height increases, the temperature increases till a point on the plate where it starts decreasing with increase in plate height. From the figure it is evident that the heat is generated for increasing values of sink parameter and this causes an increase in temperature. At the same time heat is absorbed for increasing values of S, as a result the temperature is decreased.

V. CONCLUSION

We have examined and solved the governing equations for the MHD mixed convective flow with Hall Effect of a viscous incompressible fluid past a vertical plate with heat source. In order to point out the effect of physical parameters namely; Prandtl number, hall parameter, magnetic field parameter and heat source on fluid velocity and temperature distribution on MHD mixed convective flow; the following conclusions are drawn.

(i) Velocity increases as the values of M also increases.
(ii) Concentration is higher for small values of SC and lower for larger values of SC
(iii) Temperature decreases with increase in Pr number.

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NOMENCLATURE

Pr	Prandtl number
Gr	Grashof number
ω_e	Electron frequency [Hz]
τ_e	Electron collision time [s]
E	Electron charge [c]
η_e	Number density of electron
P_e,	Electron pressure
Σ	Electric conductivity [Ω-1 m-1]
$\vec{E}$	Electric field [v]
B_0	A uniform magnetic field [wbm^{-2}]
$\vec{V}$	Velocity [ms^{-1}]
$\vec{H}$	Magnetic field [Am^{-1}]
T	Temperature of the fluid within the boundary layer [K]
C	Species concentration [kmol/m3]
P	Fluid density of boundary layer [$Kg\ m^{-3}$]
ν	Kinematic viscosity [m2 s-1]
K_O	Thermal conductivity [W/ Mk]
C_p	Specific heat at constant pressure [J/ kg K]
D	Chemical and molecular diffusivity [$m^2\ s^{-1}$]
k	Permeability of porous medium
S	Heat source parameter

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