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Modeling circular closed channels for sewer lines

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Abstract: ***The study models closed channel with circular cross-section to allow smooth flow of sewage without blocking and spilling. The model considers the effects of the flow depth, cross-sectional area of tunnel, and tunnel friction slope and tunnel angle of inclination on sewer velocity. The continuity and momentum governing equations describing the flow along sewer tunnel with the boundary conditions are solved by Finite Difference Method using the MATLAB computer generated program. A numerical investigation is performed to estimate the effects of varying tunnel friction slope, tunnel angle of inclination, and tunnel cross-sectional area on sewer velocity along the tunnel. The effects of changing these values on sewer depth and velocity of flow are shown in the graphical and tabular forms. It was seen that an increase in the cross sectional area of sewer flow results to a decrease in the sewer depth. We observe that a reduction in the friction slope leads to an increase in the velocity of sewer flow. It is seen that an increase in tunnel angle of inclination results in an increase in sewer velocity.***

Index terms: **Closed channel, velocity, depth, angle of inclination area of pipe and channel slope.**

I. INTRODUCTION

For conducive environment, there is need for pipes of the sewer line to be well modelled for easy flow of sewage. Designing and sizing of sewers considers the population to be served over the anticipated life of the sewer, per capita wastewater production and flow peaking from timing of daily routines. Minimum sewer diameters are often specified to prevent blockage by solid materials flushed down toilets and gradients are selected to maintain flow velocities generating sufficient turbulence to minimize solids deposition within the sewer. Fluid flow is classified as external and internal, depending on whether the fluid is forced to flow on a surface or in a conduit. Internal flow where the conduit is completely filled with the fluid is driven primarily by a pressure difference. Most fluids, especially liquids, are transported in circular pipes. This is because pipes with a circular cross section can withstand large pressure differences between the inside and the outside without undergoing significant distortion. If the flow is smooth, such that neighbouring layers of the fluid slide by each other smoothly, the flow is said to be streamline or laminar flow. The fluid velocity in a pipe changes from zero at the surface because of the no-slip condition to a maximum at the pipe center. In fluid flow, it is convenient to work with an average velocity, which remains constant in incompressible flow when the cross-sectional area of the pipe is constant. The value of the average velocity at some streamwise cross-section is determined from the requirement that the conservation of mass principle be satisfied. Friction is considered in the flow since it is directly related to the pressure drop during flow through pipes. A typical sewer line involves pipes of different diameters connected to each other by various fittings or elbows to route the fluid, valves to control the flow rate, and gravity to give rise to pressure in the fluid. Pumps may be necessary where gravity sewers serve areas at lower elevations than the sewage treatment plant, or distant areas at similar elevations. The pump may discharge to another gravity sewer at that location or may discharge through a pressurized force main to some distant location. Tsombe [15] modeled fluid flow in an open channel with circular cross-section and found that for a given flow area, the velocity of flow increases with increasing depth and that the velocity is maximum slightly below the free surface. Moreover, increase in the slope of the channel and energy coefficient leads to an increase in flow velocity whereas increase in roughness coefficient, radius of the conduit and area of flow leads to a decrease in flow velocity. Junke and Pierre [6] studied shear stress in smooth rectangular open channel flows. He discovered that the shear stresses are function of three components gravitational, secondary flows and interfacial shear stress. Jomba *et al* [5] modeled fluid flow in open channel with horseshoe cross – section and found that for a given flow area the velocity of flow increases with increasing depth. Moreover, increase in the slope of the channel leads to an increase in flow velocity whereas increase in manning constant, radius of the channel and lateral inflow leads to a decrease in flow velocity. Tachie *et al*. [14] conducted an experimental study to investigate the effects of roughness on the structure of turbulent boundary layers in open channels. The study was carried out using the laser-Doppler anemometer in shallow flows for three different types of rough surfaces and also a hydraulically smooth surface. The flow Reynolds number based on the boundary layer momentum thickness ranged from 1400 to 4000. The boundary layer thickness was comparable to the depth of the flow and the turbulence intensity in the channel flow varied from 2 to 4%. The defect velocity profile was correlated using an approach which allowed the skin friction to vary. Wall roughness also led to higher

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turbulence levels in the outer region of the boundary layer. The profound effect of surface roughness on the outer region and the effect of channel turbulence on the main flow indicates a strong interaction, which must be accounted for in turbulence models. Antonia and Pimentel *et al.* [10] presented a low computational cost semi-analytical procedure for the solution of incompressible fully developed turbulent flow through symmetric and asymmetric ducts with rough walls. Using a modified algebraic turbulence model to represent the influence of rough surfaces, they solved the flow field in ducts with different wall roughness's. They presented the velocity distribution and the friction factor in two different geometries, circular ducts and parallel plates, and compared them with experimental and numerical data available in the literature, and obtained satisfactory agreement. From the investigation on the development of the laminar flow of a viscous incompressible fluid from the entry to the fully developed situation in a straight circular pipe by Sinha and Meena [12]. The velocity increases more rapidly during the initial development of the flow in comparison to the downstream flow. During the initial stages of the development of the flow, the rate of increase in stream wise velocity is larger and consequently the pressure drop is larger in comparison with their values further downstream. This is due to the retarded fluid particles in the boundary layer which are pushed towards the core more rapidly near the entry where the boundary layer is thinner as compared to the downstream region.Ojiambo *et al* [9] studied a mathematical model of the fluid flow in circular cross-sectional open channels. They established that increase in the cross-sectional area leads to decrease in the flow velocity, an increase in the radius of the channel leads to decrease in the flow velocity, an increase in the roughness coefficient leads to decrease in the flow velocity, an increase in the flow depth leads to decrease in the flow velocity and a decrease in the lateral inflow rate per unit length of the channel leads to an increase in the flow velocity. Darcy [4] introduced the concept of relative roughness, where the ratio of the internal roughness of a pipe to the internal diameter of a pipe will affect the friction factor for turbulent flow. In a relatively smoother pipe the turbulence along the pipe walls has less overall effect, hence a lower friction factor is applied. The Darcy Friction factor (which is 4 times greater than the Fanning Friction factor) used with Weisbach equation has now become the standard head loss equation for calculating head loss in pipes where the flow is turbulent. Darcy noted from his data that the discharge was dependent on the type of surface, on the diameter of the pipe and its inclination. According to his data, the resistance factor for a given relative roughness varies only slightly with the Reynolds number. The friction factor decreases with increasing the Reynolds number and the rate of decrease becomes slower with greater relative roughness. For certain roughness, his data indicated that the friction factor is independent of the Reynolds number. For a constant Reynolds number, friction factor increases markedly with increasing the relative roughness. Capart *et al* [2] and Streeter *et al* [13] worked on the phenomenon of transition from free surface to pressurized flow which occurs in many situations as storm sewers, waste or supply pipes in hydroelectric installations. This is caused by sudden changes in the boundary conditions (failure of a pumping station, rapid change of the discharge, blockage of the line etc.). During the transition, the excess pressure rise may damage the pipe and cause related problem. Cunge and Wegner [3] studied the pressurized flow in a pipe as if it were a free-surface flow by assuming a narrow slot to exist in the upper part of the tunnel, the width of the slot being calculated to provide the correct sonic speed. On the other hand, the commonly used model to describe pressurized flows in pipes is the system of the Allievi equations, Streeter *et al* [13]. Unified modelisation with a common set of conservative variables is of a great interest for the coupling between free-surface and pressurised flows and its numerical simulation is more effective. Nalluri and Adepoju [7] analysed experimental data on resistance to flow in smooth channels of circular cross-section. The results of the tests showed that the measured friction factors are larger than those for a pipe of equivalent diameter (D - 4R). Through the analysis of the data for the range of flow depths $0 < y < 0.85D$, the geometric parameter y/P was found more appropriately representing the shape effects on resistance to flow in circular channels than using a single parameter like the hydraulic mean radius, R. Fanning did much experimentation to provide data for friction factors, however the head loss calculation using the Fanning Friction factors has to be applied using the hydraulic radius equation (not the pipe diameter). The hydraulic radius calculation involves dividing the cross sectional area of flow by the wetted perimeter. For a round pipe with full flow the hydraulic radius is equal to ¼ of the pipe diameter. The Colebrook-White equation which provides a mathematical method for calculation of the friction factor (for pipes that are neither totally smooth nor wholly rough) has the friction factor term on both sides of the formula and is difficult to solve without trial and error i.e. mathematical iteration is normally required to find friction factor. Nikuradse [8] investigated the effects of Reynolds number and relative roughness on friction factor and velocity distribution in pipe flow. He used circular pipes covered on the inside, as tightly as possible, with sand of a definite grain size glued on the wall. By choosing pipes of varying diameters and by changing the grain size, he was able to vary the relative roughness, from about 1/500 to 1/15. He obtained experimental data for six different degrees of

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relative roughness with Reynolds numbers ranging from 10^4 to 10^6. Powe and Townes [11] investigated the turbulence structure for fully developed flow in rough pipes. The method used to determine the turbulence structure involved examination of the fluctuating velocity spectra in all three coordinate directions. An important conclusion of this work was that in the central region of the pipe, the flow was relatively independent of the nature of the solid boundary. In contrast, the flow near the wall presents a marked dependence on the nature of the solid boundary. Several deferent ways of representing the effects of rough surfaces on turbulent flows have been proposed previously. This study is on investigation of effects of the flow depth, cross-sectional area of the conduit, and tunnel friction slope and tunnel angle of inclination on sewer velocity. The results gotten will be applied in the design of sewers that will accommodate the capacity of the growing population and infrastructure in the county. The objectives of this study are to determine the effects of tunnel cross-sectional area, of tunnel friction slope and on of tunnel friction slope sewer depth along the tunnel.

II. MATHEMATICAL ANALYSIS

Mathematical Representation of Sewer Flow in a Pipe

The open-channel phase of the sewer flow can be represented mathematically by a pair of partial differential equations of hyperbolic type Yen [15,16]

$$\frac{\partial A}{\partial t}+\frac{\partial Q}{\partial x}=0 \qquad (1)$$

$$\frac{1}{gA}\frac{\partial Q}{\partial t}+\frac{1}{gA}\frac{\partial}{\partial x}\left(\frac{\beta}{A}Q^2\right)+\cos\theta\frac{\partial}{\partial x}(Kh)+\left(K-K'\right)h\cos\theta\frac{1}{A}\frac{\partial A}{\partial x}=S_o-S_f+\frac{1}{\gamma A}\frac{\partial T}{\partial x} \qquad (2)$$

Equation (1) is the continuity equation and Equation (2) is the momentum equation. They are derived from the principle of conservation of mass and Newton's second law, respectively. Because of the difficulties in solving Equations (1) and (2), in practice they are simplified by assuming $\beta = 1$ (uniform velocity distribution over A), hydrostatic pressure distribution (K = K' = l), and neglecting the last term in Equation (2) containing T. The result is the well known complete (but not exact) dynamic wave or Saint Venant equations,

$$\frac{1}{gA}\frac{\partial Q}{\partial t}+\frac{1}{gA}\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right)+\cos\theta\frac{\partial h}{\partial x}-(S_o-S_f)=0 \qquad (3)$$

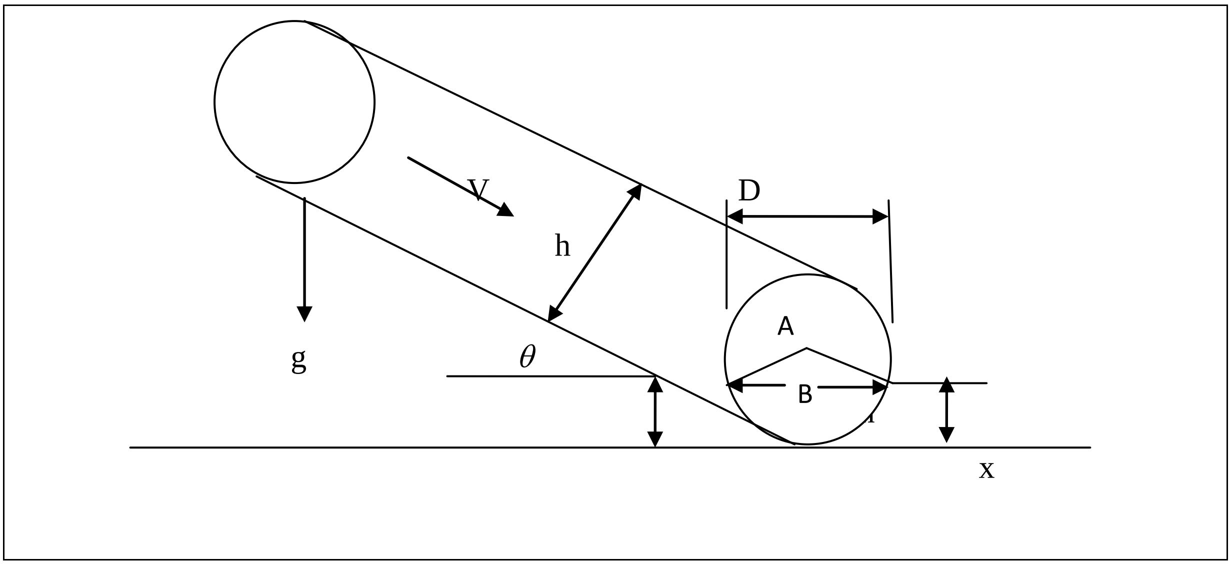

Fig 1: Open channel flow in a sewer

Alternatively, Equation (1) and (3) can be expressed in terms of the average velocity, V, over the cross sectional area A, i.e.

$$\frac{\partial h}{\partial t}+\frac{A}{B}\frac{\partial V}{\partial x}+V\frac{\partial h}{\partial x}=0 \qquad (4)$$

$$\frac{\partial V}{\partial t}+V\frac{\partial V}{\partial x}+g\cos\theta\frac{\partial h}{\partial x}-g(S_o-S_f)=0 \qquad (5)$$

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Discretization of the governing equations

We consider the continuity and momentum equations in (4) and (5) respectively.

Sewer tunnel depth

We investigate tunnel depth of the sewer line using equation (4). For the Central Difference scheme (CDS), the values u_t and u_x are replaced by the forward and central difference approximation respectively. When these values are substituted into Equation (4), we get

$$\left[\frac{h_{i,j+1}-h_{i,j}}{\Delta t}\right]+\frac{A}{B}\left[\frac{h_{i+1,j}-h_{i-1,j}}{2\Delta x}\right]+\left[\frac{V_{i+1,j}-V_{i-1,j}}{2\Delta x}\right]+V\left[\frac{h_{i+1,j}-h_{i-1,j}}{2\Delta x}\right]=0 \tag{6}$$

We investigate the effect of sewer pipe cross-sectional area A, on the tunnel sewer depth. Taking A = 3m^2 and V = 5ms^{-1}, B = 0.4m and multiplying equation (6) by Δt with $r=\dfrac{\Delta t}{2(\Delta x)}$ we get the scheme

$$5rh_{i+1,j}-h_{i,j}-5rh_{i-1,j}=-2h_{i,j+1}-7.5rV_{i-1,j}-7.5rV_{i+1,j} \tag{7}$$

Taking i = 1,2,3…….6 and j = 1 we form the following systems of linear algebraic equations

$$\left.\begin{aligned}
5rh_{2,1}-h_{1,1}-5rh_{0,1}&=-2h_{1,2}-7.5rV_{0,1}-7.5rV_{2,1}\\
5rh_{3,1}-h_{2,1}-5rh_{1,1}&=-2h_{2,2}-7.5rV_{1,1}-7.5rV_{3,1}\\
5rh_{4,1}-h_{3,1}-5rh_{2,1}&=-2h_{3,2}-7.5rV_{2,1}-7.5rV_{4,1}\\
5rh_{5,1}-h_{4,1}-5rh_{3,1}&=-2h_{4,2}-7.5rV_{3,1}-7.5rV_{5,1}\\
5rh_{6,1}-h_{5,1}-5rh_{4,1}&=-2h_{5,2}-7.5rV_{4,1}-7.5rV_{6,1}\\
5rh_{7,1}-h_{6,1}-5rh_{5,1}&=-2h_{6,2}-7.5rV_{5,1}-7.5rV_{7,1}
\end{aligned}\right\} \tag{8}$$

The above algebraic equations can be written in matrix form as

$$\begin{bmatrix}-1&5r&0&0&0&0\\-5r&-1&5r&0&0&0\\0&-5r&-1&5r&0&0\\0&0&-5r&-1&5r&0\\0&0&0&-5r&-1&5r\\0&0&0&0&-5r&-1\end{bmatrix}\begin{bmatrix}h_{1,1}\\h_{2,1}\\h_{3,1}\\h_{4,1}\\h_{5,1}\\h_{6,1}\end{bmatrix}=\begin{bmatrix}-2h_{1,2}-7.5rV_{0,1}-7.5rV_{2,1}\\-2h_{2,2}-7.5rV_{1,1}-7.5rV_{3,1}\\-2h_{3,2}-7.5rV_{2,1}-7.5rV_{4,1}\\-2h_{4,2}-7.5rV_{3,1}-7.5rV_{5,1}\\-2h_{5,2}-7.5rV_{4,1}-7.5rV_{6,1}\\-2h_{6,2}-7.5rV_{5,1}-7.5rV_{7,1}\end{bmatrix} \tag{9}$$

Taking $\Delta x = 0.25$ and $\Delta t = 0.01$, gives r = 0.02 and with initial and boundary conditions as

V (0,t) = 5, h (0,t) = 0.5 for all t > 0 (10)

V(x,t) = 5 and h(x,t) = 0.5 for all t > 0 (11)

We get the matrix-vector equation

$$\begin{bmatrix}-1&0.1&0&0&0&0\\-0.1&-1&0.1&0&0&0\\0&-0.1&-1&0.1&0&0\\0&0&-0.1&-1&0.1&0\\0&0&0&-0.1&-1&0.1\\0&0&0&0&-0.1&-1\end{bmatrix}\begin{bmatrix}h_{1,1}\\h_{2,1}\\h_{3,1}\\h_{4,1}\\h_{5,1}\\h_{6,1}\end{bmatrix}=\begin{bmatrix}0.185\\0.185\\0.185\\0.185\\0.185\\0.185\end{bmatrix} \tag{12}$$

If A is varied to 3.5m^2 and 4.0m^2 we get matrix-vector equation

$$\begin{bmatrix}-1&0.1&0&0&0&0\\-0.1&-1&0.1&0&0&0\\0&-0.1&-1&0.1&0&0\\0&0&-0.1&-1&0.1&0\\0&0&0&-0.1&-1&0.1\\0&0&0&0&-0.1&-1\end{bmatrix}\begin{bmatrix}h_{1,1}\\h_{2,1}\\h_{3,1}\\h_{4,1}\\h_{5,1}\\h_{6,1}\end{bmatrix}=\begin{bmatrix}1.075\\1.075\\1.075\\1.075\\1.075\\1.075\end{bmatrix} \tag{13}$$

and

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$$\begin{bmatrix} -1 & 0.1 & 0 & 0 & 0 & 0 \\ -0.1 & -1 & 0.1 & 0 & 0 & 0 \\ 0 & -0.1 & -1 & 0.1 & 0 & 0 \\ 0 & 0 & -0.1 & -1 & 0.1 & 0 \\ 0 & 0 & 0 & -0.1 & -1 & 0.1 \\ 0 & 0 & 0 & 0 & -0.1 & -1 \end{bmatrix} \begin{bmatrix} h_{1,1} \\ h_{2,1} \\ h_{3,1} \\ h_{4,1} \\ h_{5,1} \\ h_{6,1} \end{bmatrix} = \begin{bmatrix} 1.975 \\ 1.975 \\ 1.975 \\ 1.975 \\ 1.975 \\ 1.975 \end{bmatrix} \quad (14)$$

respectively. Solving the above matrix equation (12), (13) and (14), we get the solutions;

A = 3.0m^2	A = 3.5m^2	A = 4.0m^2
$h_{1,1} = -0.2143086$	$h_{1,1} = -0.2252988$	$h_{1,1} = -0.236289$
$h_{2,1} = -0.1930862$	$h_{2,1} = -0.2029881$	$h_{2,1} = -0.2128899$
$h_{3,1} = -0.1951706$	$h_{3,1} = -0.2051793$	$h_{3,1} = -0.2151881$
$h_{4,1} = -0.1947919$	$h_{4,1} = -0.2047813$	$h_{4,1} = -0.2147706$
$h_{5,1} = -0.1930899$	$h_{5,1} = -0.2029919$	$h_{5,1} = -0.212894$
$h_{6,1} = -0.175691$	$h_{6,1} = -0.1847008$	$h_{6,1} = -0.1937106$

Sewer flow velocity

We investigate sewer flow velocity along the tunnel using equation (6). For the Central Difference scheme (CDS), the values V_t and h_x are replaced by the forward and central difference approximation respectively. When these values are substituted into Equation (6), we get

$$\left[\frac{V_{i,j+1}-V_{i,j}}{\Delta t}\right]+V\left[\frac{V_{i+1,j}-V_{i-1,j}}{2\Delta x}\right]+g\cos\theta\left[\frac{h_{i+1,j}-h_{i-1,j}}{2\Delta x}\right]-g\left(S_o-S_f\right)=0 \quad (15)$$

Effects of tunnel slope on sewer flow velocity

We investigate the effect of tunnel slope on the tunnel sewer flow velocity. Taking $\Delta x = 0.25$ and $\Delta t = 0.01$, S_o = 0.02mm^{-1}, θ =0^o and S_f = 0.015, g = 10m/s^2 and multiplying equation (15) by Δt with $r = \frac{\Delta t}{2(\Delta x)}$ we get the scheme

$$5rV_{i+1,j} - V_{i,j} - 5rV_{i-1,j} = 0.05 - V_{i,j+1} - 8.7rh_{i-1,j} - 8.7rh_{i+1,j} \quad (16)$$

Taking and i = 1,2,3…….6 and j = 1 we form the following systems of linear algebraic equations

$$\left.\begin{aligned} 5rV_{2,1} - V_{1,1} - 5rV_{0,1} &= 0.05 - V_{1,2} - 8.7rh_{0,1} - 8.7rh_{2,1} \\ 5rV_{3,1} - V_{2,1} - 5rV_{1,1} &= 0.05 - V_{2,2} - 8.7rh_{1,1} - 8.7rh_{3,1} \\ 5rV_{4,1} - V_{3,1} - 5rV_{2,1} &= 0.05 - V_{3,2} - 8.7rh_{2,1} - 8.7rh_{4,1} \\ 5rV_{5,1} - V_{4,1} - 5rV_{3,1} &= 0.05 - V_{4,2} - 8.7rh_{3,1} - 8.7rh_{5,1} \\ 5rV_{6,1} - V_{5,1} - 5rV_{4,1} &= 0.05 - V_{5,2} - 8.7rh_{4,1} - 8.7rh_{6,1} \\ 5rV_{7,1} - V_{6,1} - 5rV_{5,1} &= 0.05 - V_{6,2} - 8.7rh_{5,1} - 8.7rh_{7,1} \end{aligned}\right\} \quad (17)$$

The above algebraic equations can be written in matrix form as

$$\begin{bmatrix} -1 & 5r & 0 & 0 & 0 & 0 \\ -5r & -1 & 5r & 0 & 0 & 0 \\ 0 & -5r & -1 & 5r & 0 & 0 \\ 0 & 0 & -5r & -1 & 5r & 0 \\ 0 & 0 & 0 & -5r & -1 & 5r \\ 0 & 0 & 0 & 0 & -5r & -1 \end{bmatrix} \begin{bmatrix} V_{1,1} \\ V_{2,1} \\ V_{3,1} \\ V_{4,1} \\ V_{5,1} \\ V_{6,1} \end{bmatrix} = \begin{bmatrix} -2V_{1,2} - 7.5rh_{0,1} - 7.5rh_{2,1} \\ -2V_{2,2} - 7.5rh_{1,1} - 7.5rh_{3,1} \\ -2V_{3,2} - 7.5rh_{2,1} - 7.5rh_{4,1} \\ -2V_{4,2} - 7.5rh_{3,1} - 7.5rh_{5,1} \\ -2V_{5,2} - 7.5rh_{4,1} - 7.5rh_{6,1} \\ -2V_{6,2} - 7.5rh_{5,1} - 7.5rh_{7,1} \end{bmatrix} \quad (18)$$

Taking $\Delta x = 0.25$ and $\Delta t = 0.01$, gives r = 0.02 and with initial and boundary conditions as

V (0,t) = 5, h (0,t) = 0.5 for all t > 0 (19)

V(x,t) = 5 and h(x,t) = 0.5 for all t > 0 (20)

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We get the matrix-vector equation

$$\begin{bmatrix} -1 & 0.1 & 0 & 0 & 0 & 0 \\ -0.1 & -1 & 0.1 & 0 & 0 & 0 \\ 0 & -0.1 & -1 & 0.1 & 0 & 0 \\ 0 & 0 & -0.1 & -1 & 0.1 & 0 \\ 0 & 0 & 0 & -0.1 & -1 & 0.1 \\ 0 & 0 & 0 & 0 & -0.1 & -1 \end{bmatrix} \begin{bmatrix} V_{1,1} \\ V_{2,1} \\ V_{3,1} \\ V_{4,1} \\ V_{5,1} \\ V_{6,1} \end{bmatrix} = \begin{bmatrix} -4.8633 \\ -4.8633 \\ -4.8633 \\ -4.8633 \\ -4.8633 \\ -4.8633 \end{bmatrix} \quad (21)$$

If S_f is varied from $S_f = 0.015$ to $S_f = 0.017$ and $S_f = 0.019$ and Solving the above matrix equation (21), we get the solutions;

$S_f = 0.015$	$S_f = 0.017$	$S_f = 0.019$
$V_{1,1} = 5.344857$	$V_{1,1} = 5.366838$	$V_{1,1} = 5.388818$
$V_{2,1} = 4.81557$	$V_{2,1} = 4.835373$	$V_{2,1} = 4.855177$
$V_{3,1} = 4.867554$	$V_{3,1} = 4.887572$	$V_{3,1} = 4.907589$
$V_{4,1} = 4.858111$	$V_{4,1} = 4.878089$	$V_{4,1} = 4.898068$
$V_{5,1} = 4.815662$	$V_{5,1} = 4.835466$	$V_{5,1} = 4.85527$
$V_{6,1} = 4.381734$	$V_{6,1} = 4.399754$	$V_{6,1} = 4.417773$

Effects of tunnel angle with horizontal plane on sewer flow velocity

We investigate the effect of the angle that the tunnel makes with horizontal plane on the sewer flow velocity. When the angles 30^o, 60^o and 80^o are substituted into equation (6) while keeping other parameters constant, we obtain the following results.

$\theta = 30^0$	$\theta = 60^0$	$\theta = 80^0$
$V_{1,1} = 5.344857$	$V_{1,1} = 5.385191$	$V_{1,1} = 5.407171$
$V_{2,1} = 4.81557$	$V_{2,1} = 4.85191$	$V_{2,1} = 4.871713$
$V_{3,1} = 4.867554$	$V_{3,1} = 4.904286$	$V_{3,1} = 4.924304$
$V_{4,1} = 4.858111$	$V_{4,1} = 4.894772$	$V_{4,1} = 4.91475$
$V_{5,1} = 4.815662$	$V_{5,1} = 4.852003$	$V_{5,1} = 4.871807$
$V_{6,1} = 4.381734$	$V_{6,1} = 4.4148$	$V_{6,1} = 4.432819$

III. RESULTS AND DISCUSSION

In this section, the effects of tunnel cross-sectional area (A), friction slope (s) and tunnel angle of inclination (θ) on sewer velocity and depth are discussed. We consider and solve the continuity and momentum equations in (4) and (5) respectively

Effects of tunnel cross-sectional area on sewer depth

Solving the equation (4) , the values of $h_{i,j}$ for A = 3.0m^2, 3.5m^2 and 4.0m^2 are presented in the fig 2.

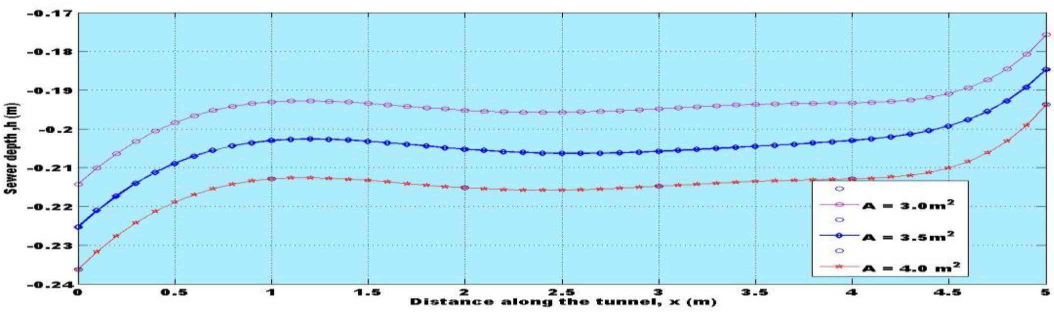

Fig 2: Sewer depth against tunnel length at varying tunnel cross-sectional area

From fig 2, we observe that an increase in the cross sectional area of sewer flow from 3.0m^2 to 3.5m^2 and 4.0m^2 results to a decrease in the sewer depth as shown by the curves. An increase in the distance along the

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tunnel leads to an increase in the sewer depth between 0-1 and 2.5-5. For a constant cross sectional area, the sewer depth increases along the tunnel due to a decreasing velocity of the flow. Since the initial velocity is high as compared to the subsequent motion of the particles, there is accumulation of more particles along the channel. This means that the cross sectional area of the tunnel should be adjusted slightly as the length increases.

Effects of tunnel friction slope on sewer flow velocity

Solving equation (5) the values of $V_{i,j}$ for S_f = 0.015, 0.017 and 0.019 are presented in the fig 3 below

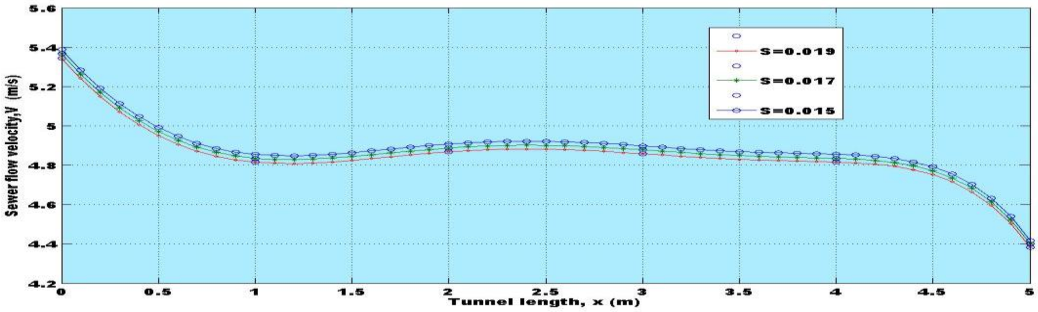

Fig 3: Sewer flow velocity against tunnel length at varying tunnel friction slope

From fig 3, we observe that a reduction in the friction slope from 0.019 m/m, 0.017m/m and 0.015 m/m leads to an increase in the sewer flow velocity as shown from curves. As the tunnel length increases, the flow velocity generally decreases. Increasing the roughness of the channel results to large shear stress at its sides. This leads to the decrease in motion of the fluid particles at or near the surface of the conduit. The velocity of the neighbouring molecules is also lowered due to constant bombardment with the slow moving molecules leading to an overall reduction to the flow velocity. To maintain the velocity of the fluid along the channel, there is need to minimise the roughness of the channel as the length increases.

Effects of tunnel angle of inclination on sewer flow velocity

Solving equation (5) the values of $V_{i,j}$ for $\theta = 30^o$, 60^o and 80^o are presented in the fig 4

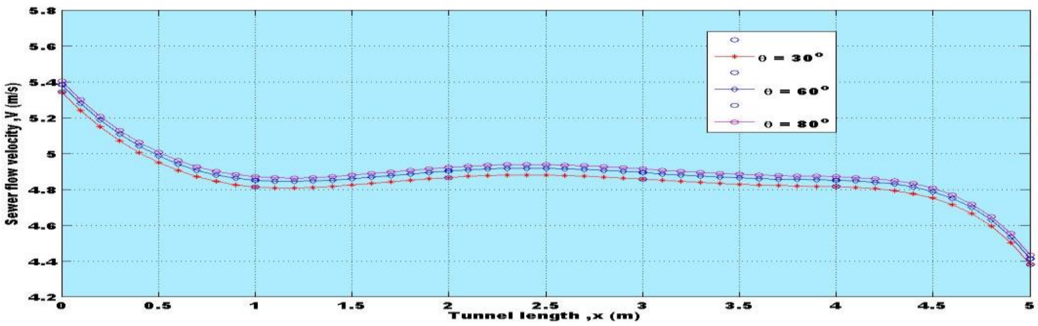

Fig 4: Sewer flow velocity against tunnel length at varying tunnel angle of inclination

It is seen from fig 4 that an increase in tunnel angle of inclination results in increase in sewer velocity. This is because of risen centre of gravity which causes instability in the sewage particles. The sewer flow velocity decreases for both angles of inclination between 0-1m and 2.5-5m as the tunnel length increased. The slight increase in velocity in between 1-2.5m is due to accumulation of the flowing particles along the channel. This shows that increase in flow depth increases the flow rate of the sewage. For a constant cross section area, at a specific angle of inclination, the sewer flow velocity decreases due to shear stress along the tunnel. To maintain some specific velocities, angle of inclination of the channel should be varied to give the required velocities along the tunnel.

IV. CONCLUSION

Numerical study has been conducted over the effects of tunnel friction slope and tunnel angle of inclination on sewer velocity in the tunnel as well as effect tunnel cross-sectional area on sewer depth. The following conclusions are drawn from the results gotten. An increase in the cross-sectional area of the tunnel results to a decrease in the sewer depth. Also, as the distance along the tunnel increases there is a general increase in sewer depth. An increase in the cross sectional area of flow results to a decrease in the flow velocity due to an increased wetted perimeter. A large wetted perimeter results to high shear stresses at the sides of the channel which results to a reduction in the flow velocity. An increase in friction slope results to a decrease in the velocity of sewer flow. As the tunnel length increases, the flow velocity generally decreases. An increase in the roughness results to large shear stresses at the sides of the channel. This means that the motion of fluid particles at or near the surface of the conduit will be reduced. The velocity of the neighboring molecules will also be lowered due to constant bombardment with the slow moving molecules leading to an overall reduction in the flow velocity. An increase in the angle of inclination of the tunnel with horizontal plane results to an increase in sewer flow velocity hence angle inclination is directly proportion to the sewer velocity. At a given angle of inclination, there is general decrease in sewer velocity as the length of the tunnel increases.

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NOMENCLATURE

A	Cross- sectional area of flow (m^2)
D	Hydraulic diameter (m)
x	Length of the channel (m)
Q	Discharge (m^3s^{-1})
S	Slope of the channel bottom
g	Acceleration due to gravity (ms^{-2})
r	Radius of the conduit (m)
t	Time (s)
p	Pressure force (Nm^{-2})
s_f	Friction of slope
s_o	Sewer tunnel slope (^{o})
h	Flow depth(m)
T	Force due to internal stresses (N)
K and k’	Correction factors for non-hydrostatic pressure distribution
v	Velocity (ms^{-1})
B	Sewage surface width (m)
q_l	Volumetric rate of lateral inflow per unit length of the channel ($m^3s^{-1}m^{-1}$)
m	Mass (Kg)

GREEK SYMBOLS

β	Momentum Flux Correction Factor
ϴ	Angle between the sewer axis and a horizontal plane
γ	Specific weight of the liquid

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