ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 6, Issue 6, November 2017

Development of a Computational Tool for the Simulation of Refractive Elements in the Human Eye

Gerardo Diaz-Gonzalez, Agustin Santiago-Alvarado, Jorge Gonzalez-Garcia, Felipe Trujillo-Romero

***Abstract*— In this paper we present the development of a computational tool based on geometrical optics principles in order to simulate the optical performance of refractive elements, in particular, optical elements with similar characteristics to those used in the study of the human eye. The theoretical model is used as the basis for the implementation of a computational algorithm that is programmed in an imperative language. Then an intuitive graphic user interface is made for easily modify input parameters and visualize the ray tracing graphics, particularly for the optical system of the eye.**

***Index Terms*—Applied computing, geometrical optics, optometry, ray tracing, visual optics.**

I. INTRODUCTION

At the present time, the development of software created for the treatment of scientific issues has grown steadily. It is because the theories of the so called exact sciences have been used to develop computer programs for solving problems at a given field. In the field of optics, there are many kinds of programs for optical simulation. However, there is the problem, on the one hand, that some of them have expensive licenses that provide access only through security keys; on the other hand, these programs are made for people highly specialized in optics. For this reason, it is important to develop a computational tool for optical simulation, oriented to be of free access and easy to use at the same time, especially for people not familiarized with optics, for example science or engineering students. This tool need to have a user interface to introduce the data easily and visualize the graphics.

In this work, we present the development of a computational tool able to simulate the ray propagation through optical systems composed by two or more spherical lenses, considering variations on the main characteristics of their components, such as refractive indexes, thickness and curvature radii. The development of the computational tool is based on the matrix optics theory, where the so called ray-transfer matrices or ABCD matrices are used. These matrices are known for be used to describe the trajectories of the rays passing by optical systems formed by spherical lenses and/or mirrors in the paraxial regime [1], [2]. Then, to avoid complications with the mathematical calculations, the theoretical model is used to implement a computational algorithm that is programmed in an imperative language.

As a particular case, we propose to analyze and simulate the optical system of the human eye, for which we consider its most important elements as an imaging system: cornea, aqueous, lens and vitreous [3]–[5]. By considering this case, the application of the computational tool can be extended to fields of study as optometry, ophthalmology and ophthalmic lens design. We develop a graphic user interface in order to facilitate the task of entering data and visualize the results. In this way, the use of the tool can be oriented to people not specialized in optics or students.

Given the well-known nature of the human lens [6], [7], the theoretical model is extended to gradient index media. For this, a refractive index distribution with its corresponding ABCD matrix [8] are used. The aim is to extend the usefulness of the tool in the field of visual optics.

II. MODELING REFRACTIVE OPTICAL ELEMENTS FOR RAY TRACING

Matrix optics is a very useful method for paraxial ray tracing through optical systems composed by spherical surfaces with a certain medium between them. The parameters that can be calculated with this method are the height and slope of the rays [1], [2]. Basically, only two matrices are needed to describe the optical elements that constitute the eye, considering them as spherical interfaces. The simplest matrix that we use is that describing the propagation of a ray in a slab of material with refractive index *n* and length *d*, given as:

$$\begin{bmatrix} 1 & \frac{d}{n} \\ 0 & 1 \end{bmatrix}. \tag{1}$$

The second one is the matrix corresponding to a surface of curvature radius *R* between two media with refractive indexes n_1 and n_2:

$$\begin{bmatrix} 1 & 0 \\ \frac{-(n_2-n_1)}{n_2R} & \frac{n_1}{n_2} \end{bmatrix}. \tag{2}$$

These matrices are enough to calculate the heights and slopes of the rays in the paraxial regime, going through an optical system composed by refractive spherical surfaces and homogeneous media. As an example, consider a biconvex spherical lens and the corresponding matrices, assigned as in Fig. 1.

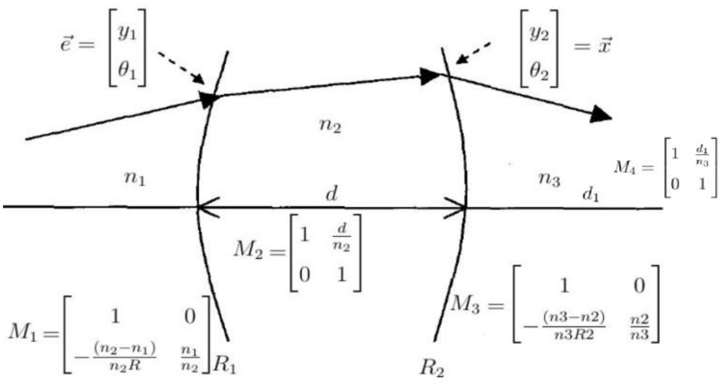


Fig. 1. ABCD matrices and the corresponding parameters assigned to each of the refractive elements conforming a convergent lens. Vector components y_1 and θ_1 are the input height and angle of the ray, and y_2 and θ_2 are the output height and angle.

To find the matrix associated to this lens, we just multiply the matrices corresponding to each medium in order from right to left [1]. The matrix obtained has the form

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix},$$

Where

$$C = \frac{n_2-n_3}{n_3R_2} + \frac{n_1-n_2}{n_3R_1} + \frac{d(n_1-n_2)(n_2-n_3)}{n_2n_3R_1R_2} = \frac{1}{f}, \tag{3}$$

being f the effective focal length of the lens.

With the correct substitutions, (3) becomes the lens maker formula. It is also well known that the inverse of the focal length is defined as the refractive power (*P*) of the lens. If the focal length is measured in meters (m), the refractive power will be expressed in diopters (D).

By using the formality of the matrix theory, we be able to model any optical system formed by spherical interfaces and homogeneous media, although we are not limited in this last aspect as we will show later. The mathematical difficulties involved in increasing the number of elements contained in an optical system, can be easily addressed

ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 6, Issue 6, November 2017

computationally.

Since in this work we will focus on the optical system of the human eye, then we will show the way to perform simulations of this optical system.

III. SIMULATION OF THE OPTICAL SYSTEM OF THE HUMAN EYE

The human eye is an organ capable of capturing light and perform the imaging process. When this process is correct, it is said that the eye is emmetropic, which means that the optical system of the eye produces well-focused images on the retina, as shown in Fig. 2 [3–5]. The easiest way to analyze the proper functionality of the eye is by considering that the eye is relaxed; that is, without accommodation, which is achieved by assuming that the light comes from a distant point source. For practical cases the source is positioned at a distance of 20 feet (or 6 meters) from the first surface of the cornea [3]. In this way, light rays come nearly parallel to the optical axis, and in the retina plane an image of the point source is formed. When the optical system does not focus on the retina, it is said that the eye has refractive errors known as ametropies (Fig. 3) [3], [5].

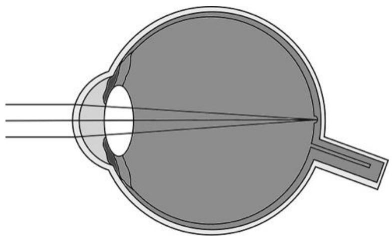
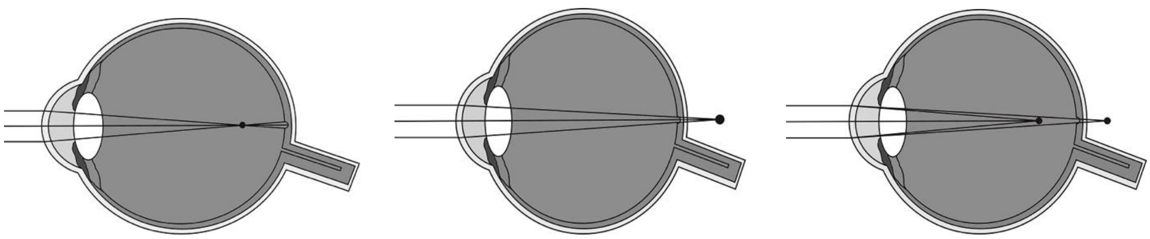


Fig. 2. Emmetropic eye: parallel incident rays coming from a distant point object are focused on the retina.



(a) (b) (c)

Fig. 3. Human eye ametropies. (a) Myopic eye: parallel incident rays are focused before the retina. (b) Hyperopic eye: parallel incident rays are focused after the retina. (c) Astigmatic eye: parallel incident rays are focused in different regions due to the curvature differences on the corneal surface.

A. Programming method

In Fig. 4, each of the blocks represents an optical media or a lens. The ABCD matrix defining each of the elements produces an output vector for a given input vector, this output vector works as a new input vector for the following component, and so on. Thus, we can obtain the ray propagation through the complete optical system. Our program calculates numerically the equivalent matrix that defines the optical system, and the vectors (y_{m+1}, θ_{m+1}) from (y_m, θ_m).

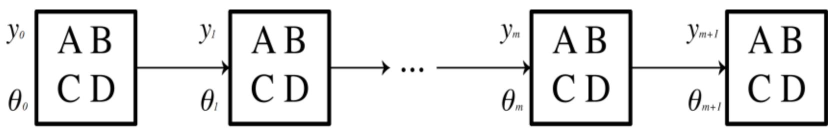


Fig. 4. Block diagram representing each of the elements that make up an arbitrary optical system and their respective ABCD matrix.

ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 6, Issue 6, November 2017

Modeling the human eye by means of matrix optics is a good way to calculate the refractive power and the focal length of the optical system, as well as to implement the ray tracing through the optical system by computer. The equivalent matrix associated to the complete optical system of the eye is obtained by multiplying the matrices of each ocular medium, beginning with the matrix of the last element to the matrix of the first one. The model programming is made by means of a block routine, assuming that the rays are generated by a far punctual source, so that they arrive to the first corneal surface in a parallel way to the optical axis. We have to consider the typical parameters of the human eye in relaxed state, so we take the parameters given by Navarro *et al*. [9] for refractive indexes of the ocular media and curvature radii of the surfaces, which are given in Table I and Table II, respectively. We take the thickness of the ocular media as the given in Table III [10].

Table I. Refractive indexes of the ocular media.

Element	Refractive index
Cornea	1.376
Aqueous	1.3374
Lens	1.42
Vitreous	1.3360

Table II. Curvature radii for an emmetropic eye in relaxed state.

Element	Value
Curvature radius of the anterior cornea	7.72 mm
Curvature radius of the posterior cornea	6.5 mm
Curvature radius of the anterior lens	10.2 mm
Curvature radius of the posterior lens	-6 mm

Table III. Thickness of the media for an emmetropic eye in relaxed state.

Element	Value
Cornea thickness	0.5 mm
Aqueous thickness	3 mm
Lens thickness	4 mm

B. Graphical user interface

For facilitate the task of enter the data, we programmed a graphical user interface (Fig. 5). This permits to visualize the resulting ray tracing through the optical system of the eye, as well as the calculated focal length in millimeters and refractive power in diopters. As we can see in Fig. 5, the focus is a well-defined point, due that paraxial optics and homogeneous media do not induce aberration. However, this kind of focus is very useful in optometry, where theoretical studies are usually made by analyzing punctual images from punctual sources [11].

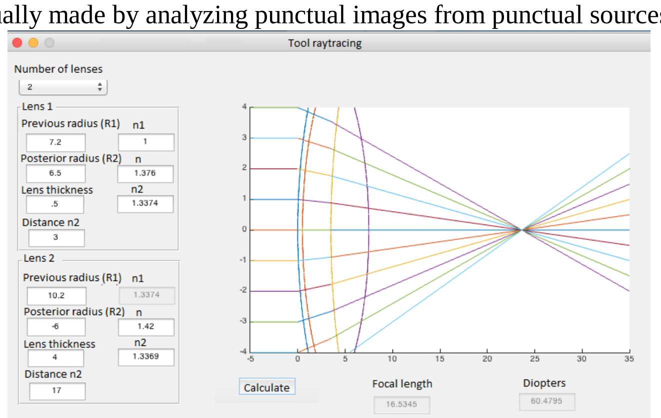


Fig. 5. Graphic user interface to insert data and visualize the result.

ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 6, Issue 6, November 2017

In this case we have introduced two lenses, the first one corresponding to the cornea and the other one corresponding to the crystalline lens.

IV. RAY TRACING FOR REFRACTIVE CORRECTION OF HUMAN EYE AMETROPIES

The ametropies of the eye are usually linked to anomalies in the surfaces and thicknesses of its optical elements, or the position of the retina. Below, we describe the most common ametropies: myopia, hyperopia, and astigmatism [3], [5].

A. Myopia (nearsightedness)

Commonly, the myopia is an ametropia caused by a larger eyeball, which means that the position of the retina regarding the optical system of the eye is larger than the normal one. In this way, a distant point object is focused before the retina by the eye in relaxed state [see Fig. 3(a)], which leads to a visual deficiency.

B. Hyperopia (farsightedness)

Contrary to nearsightedness, the most common hyperopia is the one caused by a smaller eyeball, which means that the position of the retina regarding the optical system of the eye is smaller than normal. Thereby, a distant point object is focused after the retina by the eye in relaxed state, as diagrammed in Fig. 3(b).

C. Astigmatism

Astigmatism is usually produced by the cornea, when it has distinct curvatures on its surface. This causes different refraction between two or more ocular meridians. Therefore, the refracting symmetry of the rays becomes a cylindrical and inconsistent component. In this way, the objects will not be well-focused on the retina. In other words, when an eye has astigmatism, at least two focal points are formed in different regions near the retina, as shown in Fig. 3(c).

D. Correction of ametropies

Whatever the cause of refractive error, this can be corrected with appropriate lenses placed on eyeglasses, contact lenses or modern techniques such as refractive surgery with UV pulsed lasers [3]. It is essential to know the length at which the focal point is formed by the optical system of the eye, since this is the way how the refractive power is calculated. Both parameters are used by optometrists and ophthalmologists to know the type and degree of correction that will be given to the patient who is suffering from visual defects [3], [11]. As previously mentioned, the focal length and the refractive power are related by $P = 1/f$. It is important to know how each optical element of the eye affects both parameters, with the aim of finding the best correction for a specific problem. Here, we calculate the refractive power of the eye, and therefore its focal length, by means of the computational tool developed with matrix optics, which allows us to introduce variations in the parameters that cause the most common ametropies; for example, the length between the lens and the retina, as well as the curvature radius of the cornea on its horizontal and vertical planes.

E. Simulations

By considering the typical parameters of the human eye presented in Tables I–III, the calculations give a refractive power equal to 60.47 diopters and a focal length of 16.53 mm for an emmetropic eye without accommodation. Therefore, the ocular globe of the emmetropic eye would have a size of 24 mm over the optical axis, from the first surface of the cornea to the retina plane, as it is shown in Fig. 6.

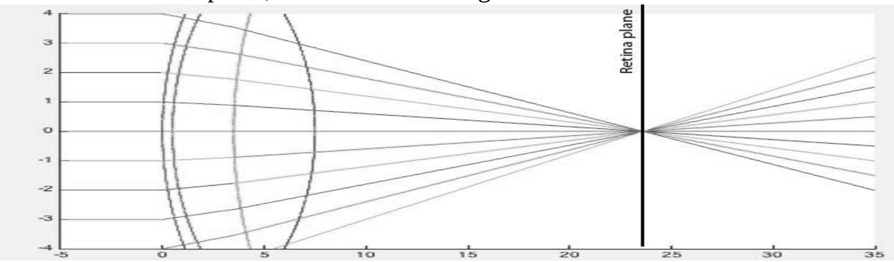


Fig. 6. Simulation of an emmetropic eye. The optical system has a refractive power equal to 60.47 diopters, and a focal length of 16.53 mm.

ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 6, Issue 6, November 2017

As it was mention previously, when an eye suffers myopia, the ocular globe is larger than the normal size. According to our simulations, an eye whose myopia is 4 diopters, would have a size of 25.2 mm, but the focusing region would be at 24 mm, as it is shown in Fig. 7. To correct this grade of myopia, the anterior corneal radius, that initially was 7.72 mm as given in Table I, would have to change to 8.49 mm. In this way, the optical system would focus at the retinal plane of the myopic eye, as it is shown in Fig. 8.

Now, when an eye suffers hyperopia, the ocular globe is smaller than the normal size. According to the simulation, an eye whose hyperopia is 4 diopters, would have a size of 23 mm, but the focusing region would be at 24 mm, as it is shown in Fig. 9. To correct this grade of hyperopia, the anterior corneal radius, initially of 7.72 mm, would have to change to 7.08 mm. In this way, the optical system would focus at the retinal plane of the hyperopic eye, as it is shown in Fig. 10.

As previously mentioned, astigmatism is produced by different curvature radii on the corneal surface. Here are going to consider only two meridians. Thereby, when the curvature radius in one meridian is, for example, 7.23 mm, the optical system would focus on 23.25 mm, and if other meridian has a radius of 8.49 mm, the system would focus on 25.2 mm. This example is shown in Fig. 11.

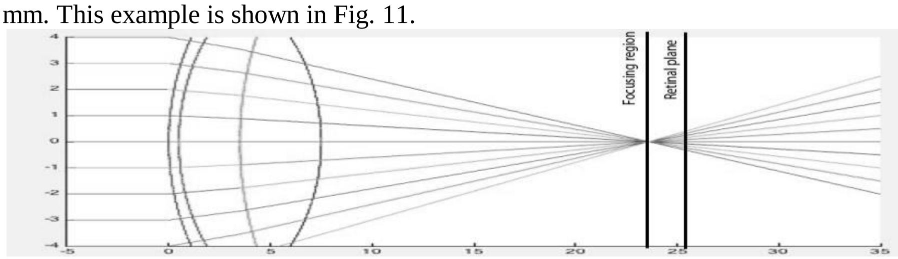


Fig. 7. Positions of the retinal plane and focusing region for an eye with a myopia of 4 diopters.

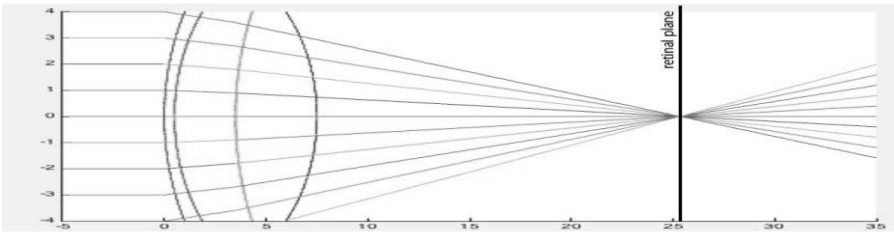


Fig. 8. Correction of the myopia of 4 diopters by modifying the corneal shape. The curvature radius of the cornea must be 8.49 mm according to our simulation.

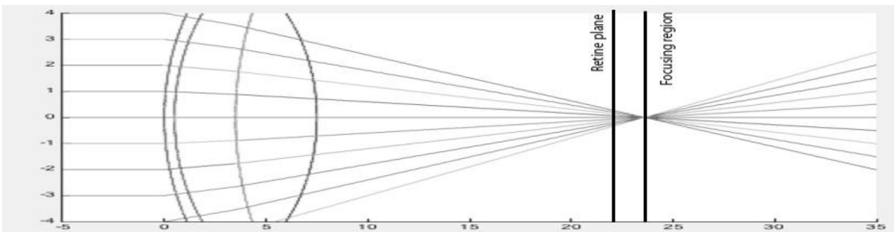


Fig. 9. Simulation of an eye with a hyperopia of 4 diopters.

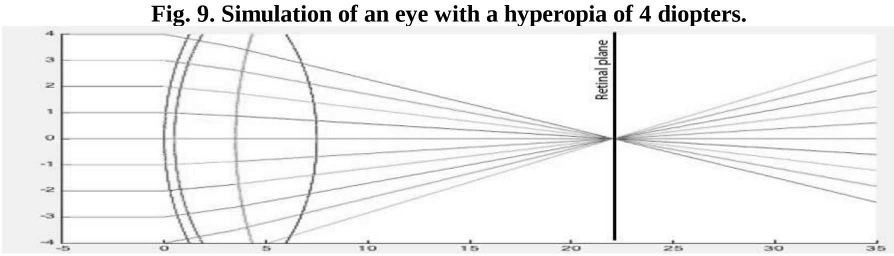


Fig. 10. Correction of the hyperopia of 4 diopters by modifying the corneal shape. The curvature radius of the cornea must be 7.08 mm according to our simulation.

ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 6, Issue 6, November 2017

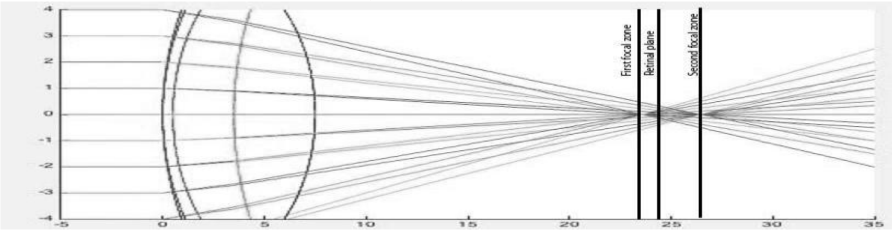


Fig. 11. Simulation of an eye with astigmatism.

V. EXTENSION TO GRADIENT INDEX MEDIA

We cannot limit our study to homogenous materials since the crystalline lens of the human eye is composed by a gradient index medium. The developed computational tool can be extended to simulate gradient index media by assigning a refractive index distribution and its associated matrix. For this, we use a method for the computation of the ray propagation in an inhomogeneous medium [8]. In this method, a slab of inhomogeneous material with refractive index *n(x,y,z)* and thickness *t* is divided in *m* parallel slices. In each slice the refractive index is considered with a refractive index of the form:

$$n_i(r) = n_{ih}\left(1 - \frac{1}{2L^2}r^2\right), \tag{4}$$

where $r = \left(x^2 + y^2\right)^{1/2}$ is the radial distance from the optical axis, n_0 is the central refractive index, and L is the rate of change of such index to the border obtained from $L = \left(n_{ih}^2 r_b^2 / 2\left(n_{ih} - n_{ib}\right)\right)^{1/2}$, where n_{ib} is the refractive index at some specific height r_b. The ABCD matrix for such media is well known, and has the form [12]:

$$\begin{bmatrix} \cos\left(\frac{d}{L}\right) & L\sin\left(\frac{d}{L}\right) \\ -\frac{1}{L}\sin\left(\frac{d}{L}\right) & \cos\left(\frac{d}{L}\right) \end{bmatrix}, \tag{5}$$

Where $d=t/m$. Then for a giving ray arriving to the *i*–th slice with some height and slope, the trajectory of such ray can be calculated using the ABCD matrix and the Snell law. It is important to note that we are considering materials with higher refractive index near the optical axis and lower one at the borders of the lens.

The simplest inhomogeneous lens is obtained when the refractive index changes quadratically with the radial distance but does not change with the axial distance, as expressed mathematically by (4). To simulate this lens, we consider a refractive index of n_{lh}= 1.4063 at the center, and n_{ll}= 1.3863 at the distance r_b= 4 mm from the center. The result is shown in Fig. 12, where we can see that the gradient index medium introduce spherical aberration, in difference with the paraxial model with homogeneous media, where the focusing region is a well-defined point.

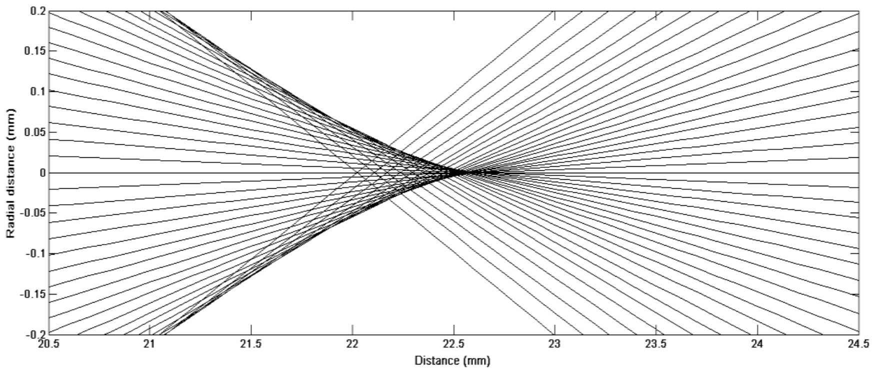


Fig. 12. Focusing properties of an eye with a quadratic refractive index of the lens.

VI. CONCLUSION

A friendly computational tool to perform ray tracing through the optical system of the human eye has been presented. This allows to calculate fast and easily two very important quantities in optometry and ophthalmology: the refractive power and focal length of the optical system. This calculations are important to know the grade of correction to be made in an eye presenting refractive errors. Moreover, simulations allow visualizing how the focusing regions change regarding the variations in one or more parameters of the eye; for example, the anterior curvature radius of the cornea and the distances between the elements. The simulations are easily configured because of the developed graphical user interface.

Axial myopia, axial hyperopia and mixed astigmatism were the ametropia types addressed in this work. However, the simulations could be extended to other ametropia kinds such as the refractive ametropies or myopic and hyperopic astigmatism.

It is worth mentioning that the developed software works for up to ten lenses. Therefore, this computational tool can be useful as a medium of instruction in the field of optics, optometry, ophthalmology, and numerical computing for optical engineering.

Although spherical surfaces give good results for several studies in optics and optometry, we want to consider spherical surfaces as a future improvement of this work because of physiological characteristics of the cornea. On the other hand, exact ray tracing is desirable to obtain a better approximation of the light propagation, as well as the analysis and application of other functions to describe the gradient refractive index of the lens; all this included within the environment of the graphical user interface.

ACKNOWLEDGMENT

GDG would like to thank to PRODEP (Programa para el Desarrollo Professional Docente, SEP México) for the scholarship given through the Cuerpo Académico de Óptica Aplicada UTMIX-CA-22.

REFERENCES

[1] B. E. A. Saleh and M. C. Teich, Fundamentals of Photonics, 2nd. Ed., John Wiley & Sons, USA, (2007) 24-29.

[2] R. Guenther, Modern Optics, John Wiley & Sons, (1990) 182-190.

[3] J. Schwiegerling, Field Guide to Visual and Ophthalmic Optics, SPIE Press, Bellingham, Washington (2004).

[4] D. A. Atchison and G. Smith, Optics of the Human Eye, Butterworth-Heinemann Ltd., England (2002).

[5] D. Malacara, Óptica Básica, Fondo de Cultura Económica, México (2004) 149-156.

[6] L. Matthiessen, Untersuchungen ber den aplanatismus und die periscopie der krystalllinsen in den augen der fische, Pflueg. Arch. Gesamte Physiol. Men-schen Tiere 231 (1980) 287.

[7] L. Matthiessen, Untersuchungen ber den aplanatismus und die periscopie der krystalllinsen in den augen der fische, Pflueg. Arch. Gesamte Physiol. Men-schen Tiere 27 (1982) 510

[8] J. F. Aguilar-Gutiérrez, M. L. Arroyo-Carrasco and M. D. Iturbe-Castillo, “Slices method to describe ray propagation in inhomogeneous media,” Opt. Commun. 383, 208-214 (2017).

[9] R. Navarro, J. Santamaría and J. Bescós, “Accommodation-dependent model of the human eye with aspherics,” J. Opt. Soc. Am. A 2, 1273-1281 (1985).

[10] A. C. Kooijman, “Light distribution on the retina of a wide-angle theoretical eye,” J. Opt. Soc. Am. 73, 1544-1550 (1983).

[11] W. D. Furlan, J. García Monreal and L. Muñoz Escrivá, Fundamentos de optometría: Refracción ocular, 2ª Ed. Universitat de València, Spain (2011).

[12] A. Yariv, Quantum Electronics, 3rd. Ed., John Wiley & Sons, Inc., USA (1989).

AUTHOR BIOGRAPHY

Gerardo Diaz-Gonzalez. Posdoctoral fellow at Universidad Tecnológica de la Mixteca, Oaxaca, Mexico. He received his MS degree and PhD degree in optics in 2008 and 2013, respectively, both from Instituto Nacional de Astrofísica, Óptica y Electrónica (INAOE), in Puebla,

ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 6, Issue 6, November 2017

Mexico. In 2005 he received his BSc degree in Electronics from Universidad Autónoma de Puebla, Mexico. He is a national researcher for the Mexican National Research System. His research interests include adaptive optics, visual optics and applied computational optics.

Agustin Santiago-Alvarado. Professor and researcher since 1998 at Universidad Tecnológica de la Mixteca, Oaxaca, Mexico. He received his Physics degree in 1993 from Benemérita Universidad Autónoma de Puebla (BUAP), Mexico. He received his MS degree and PhD degree in optics in 1995 and 2001, respectively, both from Instituto Nacional de Astrofísica, Óptica y Electrónica (INAOE), in Puebla, Mexico. He is a national researcher for the Mexican National Research System. His research interests include optical testing, optical design and adaptive optics.