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Control of Two-Wheeled Mobile Robot Based On an Integral Sliding Mode Controller for Tracking Desired Trajectory

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***Abstract*—This paper proposes a control scheme that makes the combination of a kinematic controller (KC) and an integral sliding mode controller (ISMC) for a two wheeled mobile robot (TWMR) to track a desired trajectory with a constant velocity. First, the kinematic controller is designed to make the position error vector go to zero asymptotically. Then, the integral sliding mode controller is designed to make velocity error vector go to zero asymptotically. The stability of system is proved based on the Lyapunov stability theory. The simulation results are presented to illustrate effectiveness of the proposed control scheme.**

***Index Terms*—Kinematic Controller (KC), Integral Sliding Mode controller (ISMC), Two-Wheeled Mobile Robot (TWMR), Lyapunov Stability Theory.**

I. INTRODUCTION

Wheeled mobile robots (WMRs) have been used extensively in various industrial and service applications such as transportation, security, inspection, and planetary exploration, etc, with various mobility configurations (wheel number and type, their location and actuation, single or multibody vehicle structure). A detailed analytical study of the structure of the kinematic and dynamic models of the wheeled mobile robots can be found in [1].

In the trajectory-tracking problem, the TWMR is to track a reference trajectory. Using the kinematic model of TWMRs, the trajectory-tracking problem was solved in [2]. In [3], dynamic feedback linearization has been used for reference trajectory tracking and posture stabilization of mobile robot systems. All the above controllers consider only the kinematic model which ignored the mechanical system dynamic and considered only velocity as the system input. Recently, many control schemes have been proposed to dealing with the tracking control problem including the mobile robot dynamics. Most of them, they used a control scheme of integrating a kinematic modeling into a dynamic modeling [4]-[7]. However, the simulation and experimental results show that the linear velocity of mobile robot were not keep constant velocity smoothly as desired. Chung, et al. [8] also proposed a new sliding mode controller for spot bead welding mobile robot (SWMR). The mobile robot is considered in terms of dynamic model with known parameters in the presence of bounded disturbances. However, although this controller makes sliding surface go to zero but posture error and velocity tracking error does go to zero and their values are bounded from ideal sliding surface. Besides that in [9], Hung et al. proposes a new tracking controller that combines a kinematic controller and an integral sliding mode dynamic controller for an omnidirectional mobile platform to track a desired trajectory at a desired velocity under disturbance and surface friction. It guarantees that the omnidirectional mobile robot has a good trajectory taking performance.

To solve the trajectory tracking problem of TWMR, this paper proposes a new control scheme that makes the integration of a kinematic controller and an integral sliding mode dynamic controller with bounded external disturbances for a TWMR to track a desired trajectory at a desired velocity. The above controllers are obtained by back stepping method. The system stability is proved using the Lyapunov stability theory. The simulation results are presented to illustrate effectiveness of the proposed controller for TWMR.

II. SYSTEM DESCRIPTION AND MODELING

In this section, the system description, the kinematic and dynamic models of a two-wheeled mobile robot (TWMR) are presented.

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A. System description

The Fig. 1 shows geometric model of the TWMR in the Cartesian space. It consist of frame, two driving wheels, one passive casters. The two driving wheels are independently driven by two servo motors to achieve the motion and orientation. Both driving wheels have the same radius denoted by r and are separated by $2b$. The center of mass of the TWMR is located at C; point M is the intersection of a straight line passing through the middle of the vehicle and an axis of the two driving wheels and is rotation center of TWMR. The distance between the two points is denoted by d. A camera is located at tracking point M of TWMR. The body length of the TWMR is l. The posture of the TWMR in the global coordinate frame OXY is specified by the vector $\mathrm{q}=[x,\ y,\ \varphi]^T$, where x and y are the coordinate of point M in the global coordinate frame and ϕ is the orientation of the local frame MX_0Y_0 attached on the TWMR's platform. The TWMR is modeled under the following assumptions:

(1) The radius of reference curve is sufficiently larger than the turning radius of the TWMR.
(2) The TWMR has two driving wheels for body motion, and those are mounted on an axis passed through the robot in the rear of the TWMR.
(3) One passive caster is mounted in front of the TWMR for its balance, and their motion can be ignored in the dynamics.
(4) The velocity at the point contacted with the ground in the plane of the wheel is zero.
(5) The mass center and the geometric center of the TWMR are assumed to coincide.
(6) The uncertainties and external disturbance are assumed to be unknown and bounded, and also their derivatives are assumed to be zero.

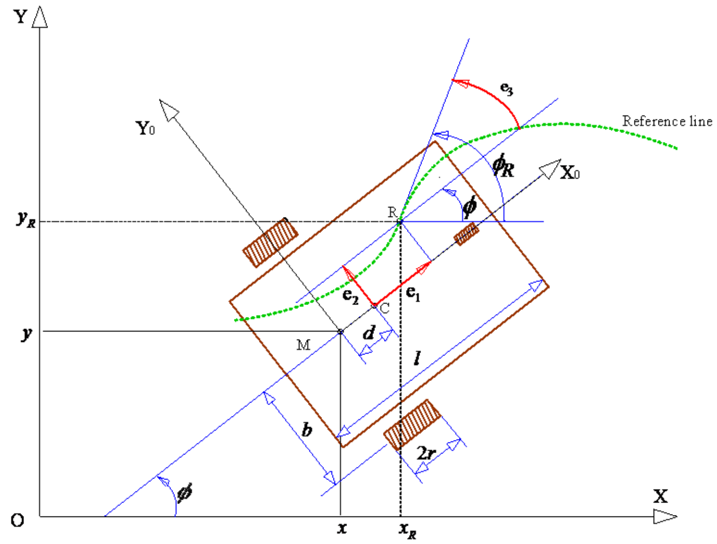

Fig. 1. Configuration for geometric model of the TWMR in the Cartesian coordinates

B. Kinematic Modeling [4]-[5]

Consider a robot system having an n-dimensional configuration space with a generalized coordinate vector $\mathrm{q}=[\mathrm{x},\mathrm{y},\varphi]^{\mathrm{T}}$ and the robot is subjected to m constraints of the following form:

$$\mathrm{A}(\mathrm{q})\dot{\mathrm{q}}=0 \tag{1}$$

where $\mathrm{A}(\mathrm{q})\in \mathrm{R}^{m\times n}$ is the matrix associated with the nonholonomic constraints.

The kinematic model under the nonholonomic constraints in Eq. (1) can be derived as follows:

$$\dot{\mathrm{q}}=\mathrm{H}(\mathrm{q})\mathrm{z} \tag{2}$$

where $\mathrm{H}(\mathrm{q})$ is a $n\times(n-m)$ full rank matrix satisfying $\mathrm{H}^T(\mathrm{q})\mathrm{A}^T(\mathrm{q})=0$, and $\mathbf{z}\in R^{n-m}$ is a velocity vector.

For the TWMR with nonholonomic constraint, $\mathrm{A}(\mathrm{q})$ in Eq. (1) can be rewritten as follows:

$$\mathrm{A}(\mathrm{q})=\begin{bmatrix}-\sin\phi & \cos\phi & 0\end{bmatrix} \tag{3}$$

From Eq. (3), this system has $n=3$ and $m=1$.

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The velocity vector in Eq. (2) is defined as

$$z = [v \quad \omega]^T \tag{4}$$

where v and ω are linear velocity and angular velocity of the TWMR, respectively.
In the kinematic model of Eq. (2), H(q) is given by

$$H(q) = \begin{bmatrix} \cos\phi & 0 \\ \sin\phi & 0 \\ 0 & 1 \end{bmatrix} \tag{5}$$

The relationship between v, ω and the angular velocities of two driving wheels is given as follows:

$$\begin{bmatrix} \omega_{rw} \\ \omega_{lw} \end{bmatrix} = \begin{bmatrix} 1/r & b/r \\ 1/r & -b/r \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \tag{6}$$

where ω_{rw}, ω_{lw} are angular velocities of the right and the left wheels, respectively.

C. Dynamic Modeling

The dynamic equations of the mechanical system under nonholonomic constraints in Eq.(1) can be described by Euler-Lagrange formulation as follows [4]-[5]:

$$M(q)\ddot{q} + V(q,\dot{q})\dot{q} = D(q)\tau - A^T(q)\lambda \tag{7}$$

where $M(q) \in R^{n\times n}$ is a symmetric positive definite inertia matrix; $V(q,\dot{q}) \in R^{n\times n}$ is a centripetal and Coriolis matrix; $D(q) \in R^{n\times r}$ is an input transformation matrix; $A(q) \in R^{m\times n}$ is a matrix of nonholonomic constraints; $\tau \in R^{r\times 1}$ is a control input vector; and $\lambda \in R^{m\times 1}$ is a constraint force vector, $r = n - m$.
Differentiating Eq. (2), substituting this result in Eq. (7), dynamics equation of WMR is as follows [4]-[5]:

$$H^T M H \dot{z} + H^T (M\dot{H} + VH) z = H^T D \tau \tag{8}$$

Multiplying by $(H^T D)^{-1}$, dynamic equation Eq. (8) of WMR with the external disturbances can be rewritten as follows:

$$\bar{M}(q)\dot{z} + \bar{V}(q,\dot{q})z + \tau_d = \tau \tag{9}$$

where

$\bar{M}(q) = (H^T D)^{-1} H^T M H \in R^{r\times(n-m)}$,

$\bar{V}(q,z) = (H^T D)^{-1} H^T (M\dot{H} + VH) \in R^{n\times(n-m)}$,

τ is a control input vector.

$\tau_d = \bar{M}(q) f \in R^{r\times 1}$ is a external disturbance vector

$f \in R^{(n-m)\times 1}$ is a external disturbance vector

First, a control vector $u \in R^{(n-m)\times 1}$ is defined by computed-torque method as follows [5],[8]:

$$\tau = \bar{M}(q)\dot{z}_d + \bar{V}(q,\dot{q})z + \bar{M}(q)u \tag{10}$$

where $z_d \in R^{(n-m)\times 1}$ is a control velocity vector and $\mathbf{z} \in R^{(n-m)\times 1}$ is an actual velocity vector.
From Eqs. (9)-(10), the following is obtained as

$$f = u + (\dot{z}_d - \dot{z}) \tag{11}$$

In this system, when $q = [x, y, \phi]^T$ is taken, $n = 3$, $m = 1$ and $r = 2$. The followings are obtained from Eqs. (9)-(11).

$$\bar{M}(q) = \begin{bmatrix} \frac{r^2}{4b^2}(mb^2 + I) + I_w & \frac{r^2}{4b^2}(mb^2 - I) \\ \frac{r^2}{4b^2}(mb^2 - I) & \frac{r^2}{4b^2}(mb^2 + I) + I_w \end{bmatrix} \bar{V}(q,\dot{q}) = \begin{bmatrix} 0 & \frac{r^2}{2b} m_c d\dot{\phi} \\ -\frac{r^2}{2b} m_c d\dot{\phi} & 0 \end{bmatrix} \tag{12}$$

$I = m_c d^2 + 2m_w b^2 + I_c + 2I_m$,

$m = m_c + 2m_w$, $\mathbf{z} = [v \quad \omega]^T$, $z_d = [v_d \quad \omega_d]^T$,

$\tau = [\tau_{rw} \quad \tau_{lw}]^T$, $f = [f_1 \quad f_2]^T$.

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where m_c is the mass of the body, m_w is the mass of each driving wheel, I_w is the moment of inertia of each driving wheel about the wheel axis, I_m is the moment of inertia of each driving wheel the wheel diameter axis, I_c is the moment of inertia of the body about a vertical axis through the intersection of the axis of symmetry with the driving wheel axis, and τ_{rw}, τ_{lw} are torques of the motors acting on the right and the left wheels.

III. INTEGRAL SLIDING CONTROLLER DESIGN FOR TWMR

Problem Statement: The objective is to design a nonlinear controller so that the tracking point $M(x_M, y_M, \phi_M)$ tracks to the reference point $R(x_R, y_R, \phi_R)$ moving on a reference line at a constant velocity v_R.

The reference point $R(x_r, y_r, \phi_r)$ is moving on the desired trajectory with the constant velocity. In Fig. 1, the tracking error vector $\mathrm{e} = [e_1, e_2, e_3]^T$ is defined as the difference between the tracking point M and the reference point $R(x_r, y_r, \phi_r)$ as follows:

$$\mathrm{e} = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x_M \\ y_r - y_M \\ \phi_r - \phi_M \end{bmatrix} \tag{13}$$

The first derivative of e yields

$$\dot{\mathrm{e}} = \begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{bmatrix} = \begin{bmatrix} -1 & e_2 \\ 0 & -e_1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} + \begin{bmatrix} v_r \cos e_3 \\ v_r \sin e_3 \\ \omega_r \end{bmatrix} \tag{14}$$

The Lyapunov function candidate is defined as follows:

$$V = V_1 + V_2 \geq 0 \tag{15}$$

where $$V_1 = \frac{1}{2}(e_1^2 + e_2^2) + \frac{1}{C_2}(1 - \cos e_3) \geq 0 \tag{16}$$

$$V_2 = \frac{1}{2}\mathrm{S}^{\mathrm{T}}\mathrm{S} \geq 0 \tag{17}$$

C_2 is a positive value.

$\mathrm{S}_v = [S_{v1} \quad S_{v2}]^T$ is a sliding surface vector

The kinematic controller based on kinematic modeling is designed as

$$z_d = \begin{bmatrix} v_d \\ \omega_d \end{bmatrix} = \begin{bmatrix} v_R \cos e_3 + C_1 e_1 \\ \omega_R + C_2 v_R e_2 + C_3 \sin e_3 \end{bmatrix} \tag{18}$$

where C_1, C_2, C_3 is a positive values.

With the velocity control input Eq. (18), the $\dot{V}_1$ becomes

$$\dot{V}_1 = -C_1 e_1^2 - \frac{C_3}{C_2} \sin^2 e_3 \leq 0 \tag{19}$$

The velocity error vector e_v is defined as

$$e_v = z_d - z = [e_{v1} \quad e_{v2}]^T \in \Re^{2\times1} \tag{20}$$

The integral sliding surface vector S_v is defined as

$$S_v = e_v + K_v \int e_v dt \tag{21}$$

where K_v is a positive diagonal matrix.

The derivative of S is as the following

$$\dot{\mathrm{S}}_v = \dot{e}_v + \mathrm{K}_v e_v = (\dot{\mathrm{z}}_\mathrm{d} - \dot{\mathrm{z}}) + \mathrm{K}_v e_v \tag{22}$$

Substituting Eq. (11) into Eq. (22), it is reduced to the following

$$\dot{\mathrm{S}}_v = \mathrm{f} - \mathrm{u} + \mathrm{K}_v e_v \tag{23}$$

The control law $\mathrm{u} = [u_1 \quad u_2]^T$ is chosen as

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$$u = QS_v + Psign(S_v) + K_v e_v \tag{24}$$

where

$$S_v = \begin{bmatrix} S_{v1} \\ S_{v2} \end{bmatrix}; Q = \begin{bmatrix} Q_1 & 0 \\ 0 & Q_2 \end{bmatrix}; \ P = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}; \ f = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} \le \begin{bmatrix} f_1^m \\ f_2^m \end{bmatrix}.$$

Q_i and P_i, $i = 1,2$ are constant positive values. f_i^m, $i = 1,2$ is the upper bounded value of f_i.

With the control law given by Eq. (24), Eq. (23) becomes

$$\dot{S}_v = -QS_v - Psign(S_v) + f \tag{25}$$

The first derivative of V_2 in Eq. (17) is derived

$$\dot{V}_2 = S_v^T \dot{S}_v \le -S_v^T QS_v - |S_{v1}|(P_1 - f_1^m) - |S_{v2}|(P_2 - f_2^m) \tag{26}$$

If $Q_i \ge 0$ and $P_i \ge f_i^m$, $i = 1,2$, $\dot{V}_2 \le 0$. By Barbalat's lemma [10] , $S_v \to 0$ as $t \to \infty$. That is, there exists the control law u stabilizing integral sliding surfaces Eq. (20).

From Eqs. (15)-(17) and Eq. (19), Eq. (26), we get derivative of Lyapunov fuction $\dot{V} \le 0$. That mean both $e \to 0$ and $e_v \to 0$, the M point of the TWMR tracks a reference point R which is moving on a desired trajectory at a constant velocity as desired. Block diagram for the proposed nonlinear controller is shown in Fig. 2.

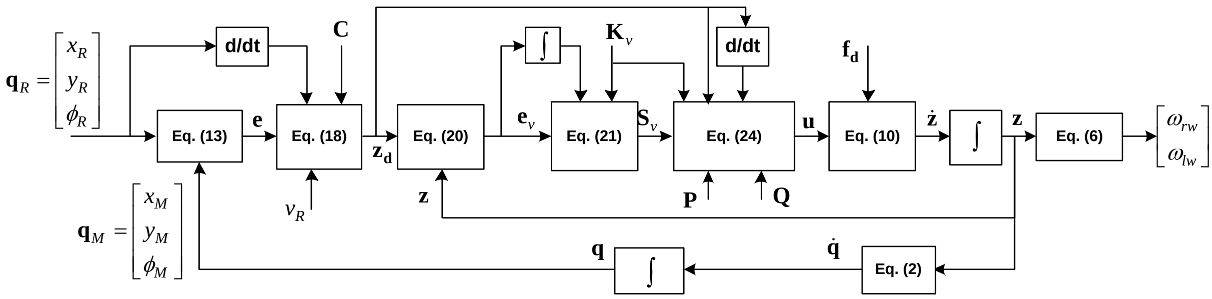

Fig.2. Block diagram for the proposed nonlinear controller

IV. SIMULATION RESULTS

To verify the effectiveness of the proposed controller, simulations have been done for a TWMR tracking a desire d trajectory. Fig. 3 shows the desired trajectory with straight line of $L_1 = 2m$, arc curve line of $(R_1 = 2m, 90^0)$, straight line of $L_2 = 2m$, arc curve line of $(R_2 = 2m, 90^0)$, and straight line of $L_3 = 2m$. The designed parameters of the controller are as follows: $P_1 = 1.5$, $P_2 = 1.7$, $Q_1 = 15$, $Q_2 = 20$; and $C_1 = 10$, $C_2 = 16$ and $C_3 = 0.9$, $K_v = [1.5 \ \ 0; 0 \ \ 1.5]$. The numerical parameter values and the initial values for simulation are given in Table 1 and Table 2.

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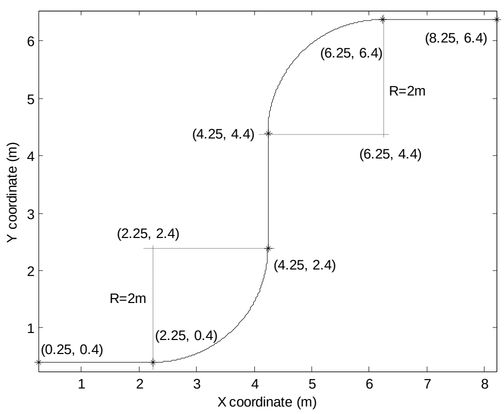

Fig.3. The desired trajectory

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Table 1. The numerical parameters values for simulation

Parameters	Values	Units
b	0.39	m
r	0.16	m
d	0.45	m
l	1.2	m
m_c	32.67	kg
m_w	2.75	kg
I_c	17.85	kgm^2
I_w	0.0135	kgm^2
I_m	0.0068	kgm^2

Table 2. The initial values for simulation

Parameters	Values	Units
x_r	0.25	m
y_r	0.40	m
ϕ	30	deg
x_p	0.22	m
y_p	0.35	m
v_r	50	mm/s
ω_r	0	rad/s

The simulation results for line tracking are shown in Figs. 4 - 8. Figs. 4 and 5 show the movement of the TWMR along the desired trajectory for the time beginning and full time 245 seconds. The blue line is the reference desired trajectory and the red line is the actual trajectory of TWMR. The simulation results for error tracking vector during 6 seconds at beginning and 245 seconds of full time are shown in Figs. 6 and 7, respectively. The errors go to zero from 4.5 seconds. The linear velocity of TWMR is shown in Fig. 8; its is shown that the linear velocity at the point M of the TWMR is at the vicinity of $50\,mm/s$ as desired. The simulation results are shown that the TWMR has good trajectory tracking performance.

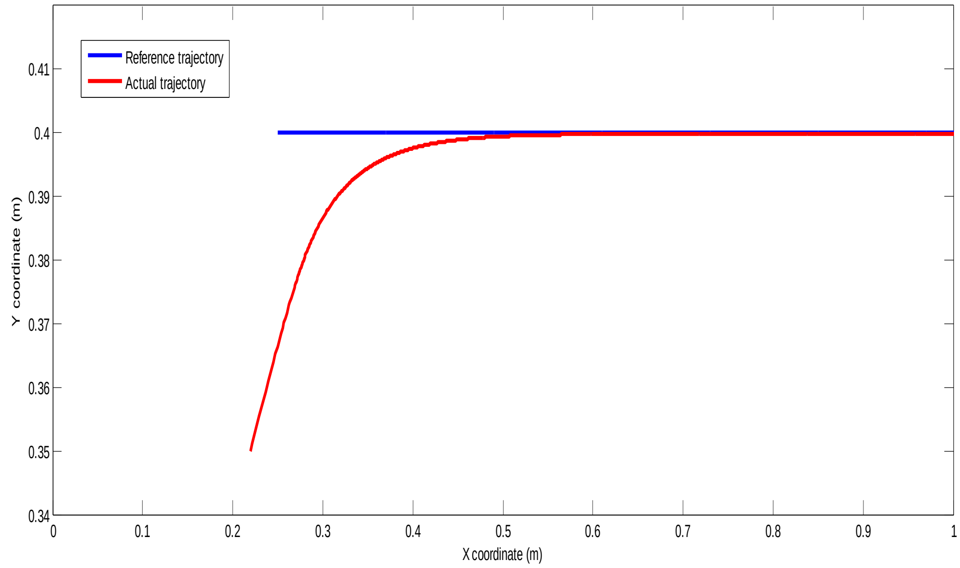

Fig. 4. Movement of the TWMR at the beginning time

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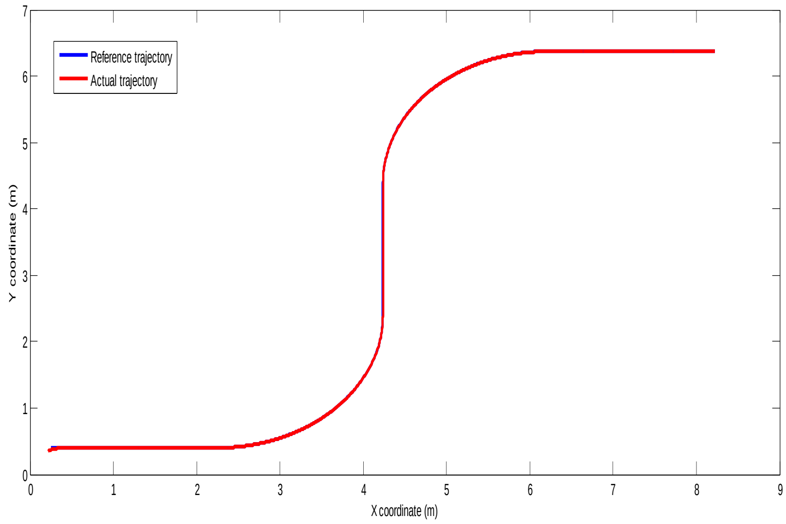

Fig. 5. Movement of the TWMR for full time 245 seconds

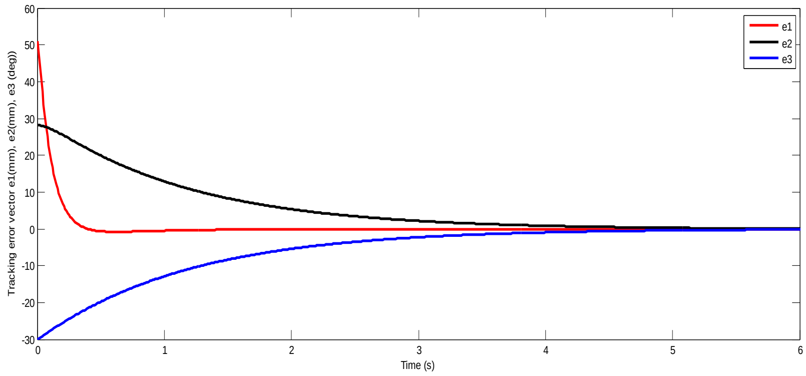

Fig. 6. Tracking errors at beginning time about 6 seconds

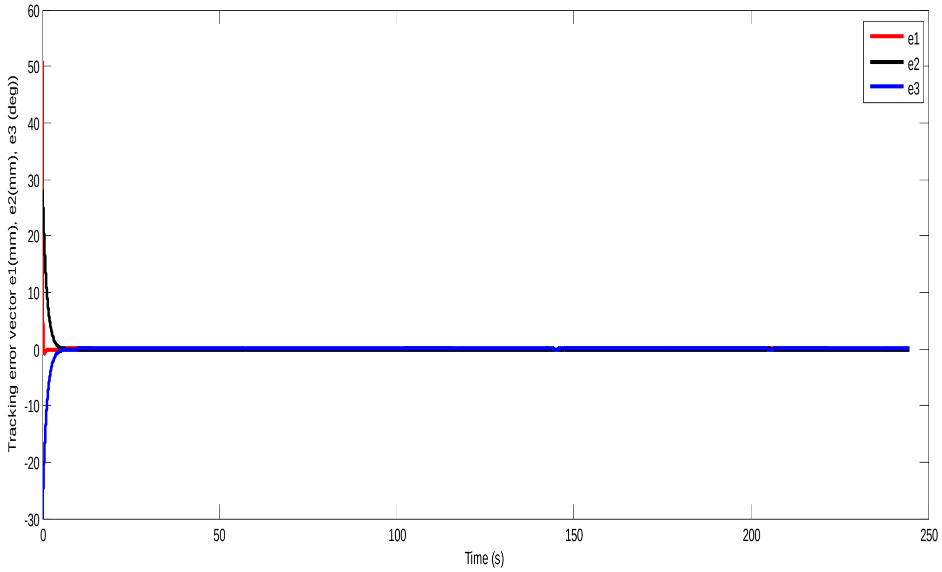

Fig. 7. Tracking errors for full time 245 seconds

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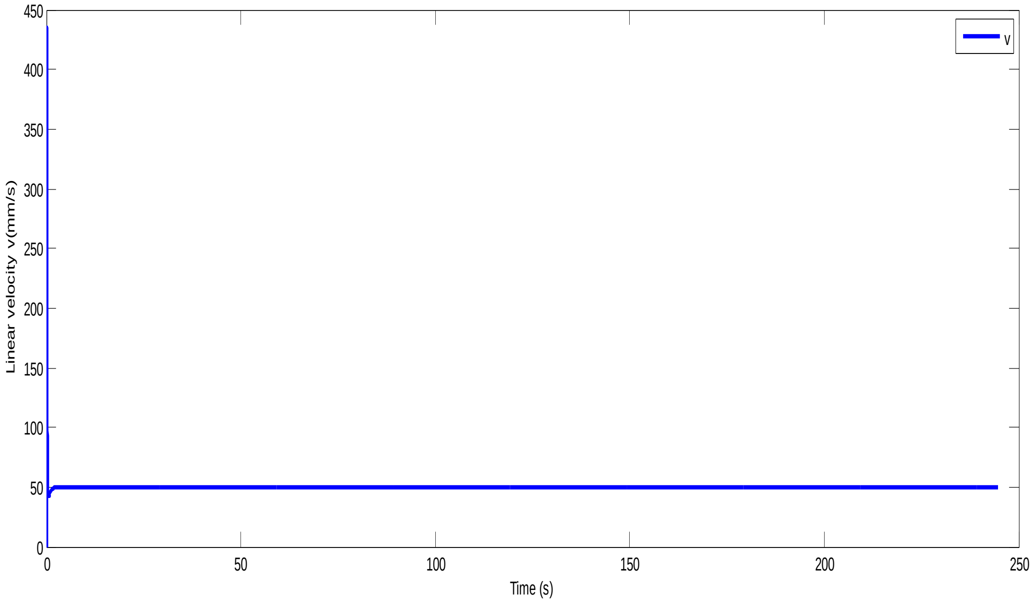

Fig. 8 . Linear velocity of the TWMR for full time 245 seconds

V. CONCLUSION

This paper proposes a new control scheme that makes the combination of a kinematic controller (KC) and an integral sliding mode dynamic controller (ISMC) for a two wheeled mobile robot (TWMR) to track a desired trajectory with a constant velocity. The system stability is guaranteed by Lyapunov theory. The simulation results show that TWMR has the good trajectory tracking and the proposed controller can be applicable in the practical applications. The future studies are continuing studying the developed controller that considering the uncertainty parameters of TWMR such as inertia moment of the body in working process.

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