ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 6, Issue 5, September 2017

Velocity profiles in an enclosure with buoyant driven flows

Peninnah Ngina [1], Dr. Eustace Mwenda [2], Prof. David Theuri [3]
[1] Meru University of Science and Technology, Kenya
[2]Meru University of Science and Technology, Kenya
[3] Jomo Kenyatta University of Agriculture and Technology, Kenya

Abstract*— A buoyant driven air flow in an infinitely long enclosure with a uniformly heated wall was numerically investigated. A 2-dimensional, laminar, steady and incompressible flow was considered and the primary velocity profiles brought about by varying the aspect ratio (AR=$\frac{L}{W}$) of the enclosure and Grashoff number (Gr) were investigated. The concept of buoyant driven flows is applied in the electronic industry for cooling heat generating electronics, in agriculture sector for drying and preservation of produce and in architecture for thermal insulation of buildings. The finding in this study equips engineers with the necessary knowledge required to design thermal structures and devices that will enhance heat transfer so as to improve the performance and optimize the structures and devices. The continuity, momentum and the energy equations simulating the flow were solved by the finite volume method. Results showed that for any given AR and Gr value, the primary velocities of flow of hot air decreased smoothly along the horizontal length of the cavity until a critical velocity was reached then the flow attained a free stream velocity. Increase in Gr caused an increase in the rate at which the critical velocity was attained while increase in AR increases the rate at which free stream velocity is attained.

***Index Terms*— Aspect ratio, buoyancy forces, Grashoff number.**

I. INTRODUCTION

This study investigates a 2-dimensional buoyant driven flow of air in an infinitely long enclosure with a uniformly heated vertical wall. Buoyancy occurs as cold air comes into contact with the heated wall. The study numerically investigates primary velocity profiles brought about by varying the aspect ratio of the enclosure and Grashoff number. Aspect ratio is the ratio of the convective length to any horizontal length. The geometry considered in this study is such that the convective length is the height of the enclosure set along the x-axis while the horizontal length is the free stream along the y-axis. Grashoff number is a non-dimensional number which measures the relative strength of buoyancy force to the opposing viscous force. Flow of air flow in the enclosure is characterized by recirculation with the less dense mass of air near the heated wall rising and the more dense cold air away from the heated wall sinking. This repeated recirculation leads to the formation of convective currents that are responsible for distributing heat in the enclosure. The convective currents play a great role in determining the velocity profiles in the system. The rising of the hot air along the convective length of the enclosure constitutes the primary velocities. The flow of air is assumed to be laminar, steady and incompressible. Effects of varying aspect ratio on flow profiles and heat transfer in enclosures with buoyant driven flows have been widely studied numerically by many scholars. Elsherbiny *et al.* [1] numerically studied laminar natural convection of air in an inclined rectangular cavity with a localized heat source and the effect of aspect ratio affects on Nusselt number was established for AR less than unity. Falahat [2] studied the effect of aspect ratio on laminar natural convection of water in a partially heated enclosure. Aspect ratio and Rayleigh number were varied and their effect on Nusselt number was established. Salman *et al.*[3] on natural convection heat transfer in rectangular enclosure with sinusoidal boundary conditions analyzed a problem for different aspect ratio number and Rayleigh numbers. The effect of these values on Nusselt number was established. Kulacti [4] studied numerically the effect of aspect ratio on flow structure and heat transfer. The study investigated the effect of aspect ratio on the rate of heat transfer in the cavity. Nagueira *et al.* [5] studied numerically natural convection in rectangular cavities with different Aspect ratios. The effect of varying aspect ratio on flow behavior and heat transfer in the cavity was established. Mohammed and Sezzai [6] studied natural convection from discrete heat source at the bottom of a horizontal enclosure. Variation of Nusselt number as a function of aspect ratio and Rayleigh number was reported.

.

II. MATHEMATICAL FORMULATION

The physical model with boundary conditions considered for this study is as depicted in figure 1. A 2-dimensional, laminar, steady and incompressible fluid flow is considered. The flow of air in the system is buoyant driven with gravitational acceleration acting in the x-direction only. The wall at the left hand side is heated at temperature T h such that $T_h > T_c$, the temperature of air in the free stream.

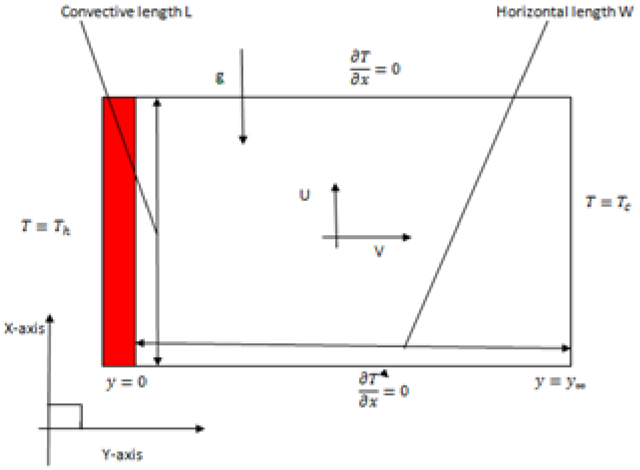

Fig 1: physical model with boundary conditions

The continuity, momentum and energy equations governing the flow are given as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\vartheta\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+g\beta\left(T_h-T_c\right) \qquad (2)$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\vartheta\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \qquad (3)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \qquad (4)$$

Equations (1),(2) , (3) and (4) can be non-dimensionalised by using the scaling variables given as

$$x^*=\frac{x}{L},\ y^*=\frac{y}{L},u^*=\frac{u}{u_0},v^*=\frac{v}{v_0},\ p^*=\frac{p}{\rho u_0^2},\ \theta=\frac{T_h-T_c}{\Delta T^*},\ T^*=\frac{T}{\theta}$$

and non-dimensional numbers

$$Gr=\frac{L^3\beta g\Delta T}{\vartheta^2},\ \mathrm{Re}=\frac{UL}{\vartheta},\ \Pr=\frac{\vartheta}{\alpha}$$

Non-dimensionalised equations together with the boundary conditions are necessary in determining flow properties such as velocity profiles investigated in this study. The non-dimensionalised form of the momentum equation (2) describing the primary velocities along the u-direction can be written as

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+Gr\theta \qquad (5)$$

III. METHOD OF SOLUTION

Equation (5) is solved numerically by the finite volume method. In this method of discretization, the solution domain is sub-divided into a number of non- overlapping control volumes. Focus is the made on one of them as the reference control volume over which integrals are to be evaluated. The advantage of the finite volume method over other numerical methods is that the principle of conservation is satisfied over each control volume and thus in the whole calculation domain. In addition the discretised equations obtained by this method have the same physical interpretation as the corresponding partial differential equations.

The integral form of equation (5) over the reference control volume can be written as

$$\int u\frac{\partial u}{\partial x}dxdy+\int v\frac{\partial u}{\partial y}dydx=-\int\frac{\partial p}{\partial x}dxdy+\frac{1}{\text{Re}}\int\left(\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)\right)dxdy+\int Gr\theta dxdy \quad (6)$$

Evaluating the integrals in a backward staggered grid equation (6) takes the form

$$a_p u_{i,J}=a_E u_{i+1,J}+a_W u_{i-1,J}+a_N u_{i,J+1}+a_S u_{i,J-1}+\frac{p_{I-1,J}-p_{I,J}}{\partial_{xn}}\Delta v_u+\overline{s_n\Delta x} \qquad (7)$$

Since the u-velocity was staggered from E to W, the tri-diagonal matrix for equation (7) can be written as

$$\begin{bmatrix} -\alpha_1 & D_1 & -\beta_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\alpha_2 & D_2 & -\beta_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\alpha_3 & D_3 & -\beta_3 & 0 & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & -\alpha_{n-2} & D_{n-2} & -\beta_{n-2} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\alpha_{n-1} & D_{n-1} & -\beta_{n-1} \end{bmatrix}\begin{bmatrix} u_{1,J} \\ u_{2,J} \\ u_{3,J} \\ \vdots \\ \vdots \\ u_{n-1,J} \end{bmatrix}=\begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \\ \vdots \\ \vdots \\ \gamma_{n-1} \end{bmatrix}\begin{bmatrix} u_{1,J+1} \\ u_{2,J+1} \\ u_{3,J+1} \\ \vdots \\ \vdots \\ u_{n-1,J+1} \end{bmatrix}+\begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \vdots \\ \vdots \\ \theta_{n-1} \end{bmatrix}\begin{bmatrix} u_{1,J-1} \\ u_{2,J-1} \\ u_{3,J-1} \\ \vdots \\ \vdots \\ u_{n-1,J-1} \end{bmatrix}$$

$$+\begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ \vdots \\ \vdots \\ c_{n-1} \end{bmatrix}+\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \vdots \\ \vdots \\ \omega_{n-1} \end{bmatrix}$$

With the help of a computer coding, the tri-diagonal matrix obtained is solved by applying the SIMPLEX algorithm and results on the velocity profiles brought about by varying aspect ratio and Grashoff number are presented and analyzed.

IV. RESULTS

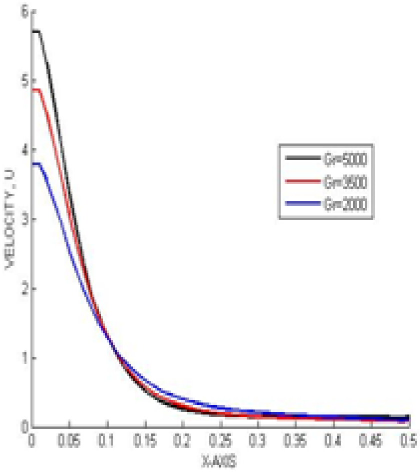

Fig (i) velocity profiles for AR=0.5 and Gr varying

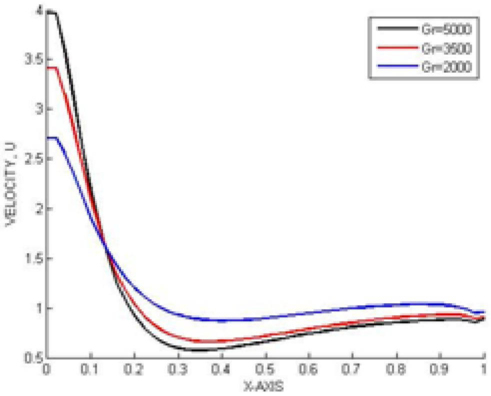

Fig (ii) velocity profiles for AR=1.0 and Gr varying

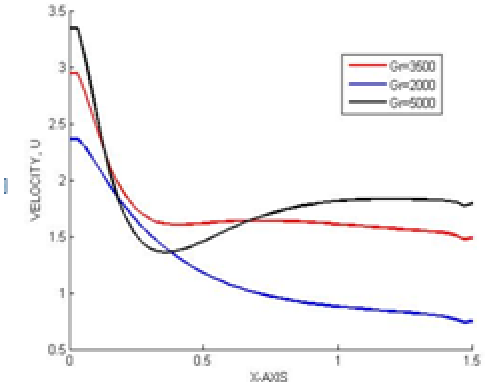

Fig (iii) velocity profiles for AR=1.5 and Gr varying

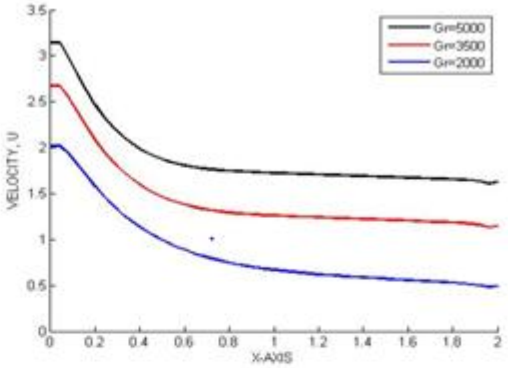

Fig (iv) velocity profiles for AR=2.0 and Gr varying

Figures (i), (ii), (iii) and (iv) show the results on the primary velocity profiles as aspect ratio of an enclosure and Grashoff number is varied. From the results it was established that buoyancy occurred and there was rising of hot air to the upper part of the cavity. This was due to density gradients of the less dense hot air in the lower part of the cavity and the more dense cold air in the upper part of the cavity. The velocity of the flow of air was high near the heated wall and low at the free stream. As hot air rises, its concentration at the upper part of the cavity increases. This leads to decrease in the temperature difference between mass of air in the lower and the upper part of the cavity, resulting to decrease of buoyancy forces along the convective length thus decrease in the velocity profiles. For a given AR value, the velocity decreases smoothly along the horizontal length of the cavity until a critical velocity was reached then the flow attained a free stream velocity for all Gr values. Critical velocity is the velocity of fluid flow up to when the flow attains a free stream velocity above which the flow becomes turbulent .The velocity of the rising of hot air near the heated wall is higher for higher Gr values and the rate at which the critical velocity is attained increases with increase in Gr values for all AR values. Gr is a non-dimensional number which measures the relative strength of buoyancy forces to the opposing viscous forces. When Gr is increased, buoyancy increases and the velocity of flow of air near the heated wall increases leading to substantial increase in the amount of hot air in the upper part of the cavity. This increases the rate at which the temperature difference of the mass of air in the lower and upper cavity decreases along the horizontal length thus the critical velocity is easily attained. Results also show that for AR values less than unity increase in AR causes increase in the critical velocity value. For AR values greater than one, increase in AR increases the rate at which free stream velocity is attained. Critical velocity depends on the coefficient viscosity of a fluid, density of the fluid and cross sectional length of a cavity. Since aspect ratio also depends on the cross-sectional length, increase in aspect ratio causes increase in the value of the critical velocity. However when the cross-sectional length is increased until it exceeds the length of a cavity flow likely becomes laminar and hence dependent on viscosity. In addition increase in AR leads to increase in the volume of air to be heated and the flow likely becomes laminar

A. *Abbreviations and Acronyms*

AR (aspect ratio), Gr (Grashoff number), Re (Reynolds number), α (thermal diffusivity)
ϑ (kinematic viscosity), β (volume expansion coefficient), g (gravitational acceleration)

B. *Other Recommendations*

Similar study could be carried out on a cavity of different geometry and different medium.

REFERENCES

[1] Elsherbiny, S. M., &Ragab, E. H. (2013). Laminar natural convection in inclined rectangular cavities with a localized heat source. Alexandria Engineering Journal, 52(3), 249-257. Universal Journal of Mechanical Engineering, 2(1), 28-33.

[2] Falahat, A. (2014). Effect of Aspect Ratio on Laminar Natural Convection in Partially Heated enclosure.

[3] Salman, A. M., Ahmed, I. S., &Saib, A. H. (2012). Natural convection heat transfer in rectangular enclosure with sinusoidal boundary condition. Journal of Engineering and Development, 16(1), 1813-7822.

[4] Kulacti, F. A. (1984). Corrective Heat Transfer in a Rectangular Porous Cawitw—Effect of Aspect Ratio on Flow Structure and Heat Transfer. Transfer, 1000, s2.

[5] Nagueira, R. M., Martins, M. A., &Ampessan, F. (2012). Natural convection in rectangular cavities with different aspect ratios. ThermEng, 10(01-02), 44-9.

[6] Sezzai, I &Mohammad, A. A. (2000). Natural convection from a discrete heat source on the bottom of a horizontal enclosure. International Journal of Heat and Mass Transfer, 43(13), 2257-2266.

AUTHOR BIOGRAPHY

Peninnah Ngina, affiliated to Meru University of Science and Technology, holds a bachelor of education degree in Mathematics and Chemistry from Kenyatta University, Kenya. She is currently pursuing a Masters of Science degree in applied mathematics from Meru University of Science and Technology, Kenya. She has lots of interests in investigating flow of fluid in enclosures with natural heat convection.

She has a taught mathematics and chemistry in many secondary schools. She is currently a trainer in Mitunguu Technical Trainining Institute where she majors in teaching engineering mathematics and applied sciences. She has authored several revision books in mathematics and chemistry for secondary schools.

Peninnah is a trained examiner of mathematics with KNEC-(Kenya national examinations council) - Kenya’s most respected and the official examination body in high school education. She is also a teacher trainer with the CEMASTEA; an institution charged with improvement of mathematics and sciences (biology, chemistry and physics) in Kenyan secondary schools.