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A Sub linearly Convergent Algorithm for Generalized Linear Complementarity Problem over a Polyhedral Cone

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***Abstract*—For the generalized linear complementarity problem (GLCP) over a polyhedral cone, we propose a new projection-type algorithm for solving GLCP without the backtracking line search to find a suitable step size, its global convergence is proved under mild conditions. Furthermore, the local error bound for GLCP without any assumption is also given, based on this, we prove that the method has sub linearly convergence rate.**

***Index Terms*—GLCP, projection-type algorithm, global convergence, sub linearly convergence rate.**

I. INTRODUCTION

Let $f(x) = Mx + p, g(x) = Nx + q$, where $M, N \in R^{m\times n}$, $p, q \in R^m$. The generalized linear complementarity problem, abbreviated as GLCP, is to find a vector $x^* \in R^n$ such that

$$f(x^*) \in K, \quad g(x^*) \in K^0, \quad f(x^*)^{\bullet} g(x^*) = 0, \qquad (1.1)$$

where K is a polyhedral cone in R^m, that is, there exists $A \in R^{s\times m}, B \in R^{t\times m}$, such that $K = \{v \in R^m \mid Av \geq 0, Bv = 0\}$. It is easy to verify that its polar cone K° assumes the following from ([1, 2, 3]) $K^{\circ} = \{u \in R^m \mid u = A^{\bullet}\lambda_1 + B^{\bullet}\lambda_2, \lambda_1 \in R^s_+, \lambda_2 \in R^t\}$. Throughout this paper, the solution set of the GLCP is denoted by X^*, which is always assumed to be nonempty.

The GLCP is a special case of the generalized nonlinear complementarity over a polyhedral cone (GCP) which was firstly considered in [1, 2, 3]. The GCP plays a significant role in economics, operation research, and nonlinear analysis, etc, and has been received much attention of researchers ([1, 2, 4, 5]).

In recent years, many effective methods have been proposed for solving GLCP. The basic idea of some methods is to reformulate the problem as an unconstrained or simply constrained optimization problem ([1, 2, 6, 7, 8]), the condition which the nonsingularity of Jacobian at a solution guarantees that these method have global convergence. Some projection-type algorithm also be proposed by Sun et al. ([9, 10]), and show that these methods are global convergence under different conditions. At the same time, as an important tool for a mathematical problem, the global error bound estimation for GLCP was deeply discussed in [11, 12, 13, 14]. This motivates us to consider the new method and error bound for the GLCP under mild conditions, which are different from the algorithms and error bound listed above.

The rest of this paper is organized as follows. In Section 2, some equivalent reformulations of the GLCP are established. In Section 3, we propose a new projection-type algorithm (PTA) without the backtracking line search to find a suitable step size, and show that the new PTA is global convergence under mild condition. Furthermore, we present a local error bound for GLCP without any assumption, based on this, the sublinearly convergence rate analysis of the proposed algorithm is established. Compared with the existing solution methods in [6, 7, 8, 9, 10], the conditions guaranteed for convergence are weaker. Finally, some concluding remarks are drawn in Section 4.

Some notations used in this paper are in order. R^n be a real Euclidean space with standard inner product, whose inner product is denoted by $\langle \cdot, \cdot \rangle$, the norm $\|\cdot\|$ denotes the Euclidean 2-norm, we denote the transpose of a matrix M by $M^{\bullet}$.

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II. EQUIVALENT REFORMULATIONS OF THE PROBLEM GLCP

In this section, we will convert the GLCP into a variational inequality problem, and state some well known properties of the projection operator.

Firstly, we give the equivalent reformulation of the GLCP.

Theorem 2.1 $K^0 = \overline{K}$, where $\overline{K} := \{u \in R^n \mid u^T v \geq 0, \forall v \in K\}$.

Proof For any $u \in K^0$, there exist $\lambda_1 \in R^s_+, \lambda_2 \in R^t$ such that $u = A^T \lambda_1 + B^T \lambda_2$. For any $v \in K$, $u^T v = (A^T \lambda_1 + B^T \lambda_2)^T v \geq 0$, therefore $u \in \overline{K}$. On the other hand, for any $u \in \overline{K}$ and $v \in K$, we have $u^T v \geq 0$, it is easy to know that there exist $\lambda_1 \in R^s_+, \lambda_2 \in R^t$ such that $u = A^T \lambda_1 + B^T \lambda_2$. Thus $u \in K^0$.

From Theorem 2.1, the GLCP can be equivalently converted into the following problem: find $x^* \in R^n$, such that

$$f(x^*) \in K, g(x^*) \in \overline{K}, f(x^*)^T g(x^*) = 0, \quad (2.1)$$

in the sense that $x^* \in R^n$ is a solution of the GLCP if and only if x^* is a solution of (2.1).

Theorem 2.2 x^* is a solution of (1.1) if and only if x^* is a solution of the following problem

$$g(x^*)^T ((f(x) - f(x^*)) \geq 0, \quad \forall f(x) \in K. \quad (2.2)$$

Proof Suppose that x^* is a solution of (2.2). Since vector $0 \in K$, by substituting $f(x) = 0$ into (2.2}), one has $g(x^*)^T f(x^*) \leq 0$. On the other hand, since $f(x^*) \in K$, then $2f(x^*) \in K$. By substituting $f(x) = 2f(x^*)$ into (2.2), we have $g(x^*)^T f(x^*) \geq 0$. Thus, $g(x^*)^T f(x^*) = 0$. For any $f(x) \in K$, one has

$$g(x^*)^T f(x) = g(x^*)^T [f(x) - f(x^*)] \geq 0,$$

i.e., $g(x^*) \in \overline{K}$. Thus, x^* is a solution of (2.1), i.e., x^* is a solution of (1.1).

On the contrary, suppose that x^* is a solution of (1.1), then x^* solves (2.1), for any $f(x) \in K$, one has $g(x^*)^T f(x) \geq 0$ by $g(x^*) \in \overline{K}$, combining this with $g(x^*)^T f(x^*) = 0$, we get $g(x^*)^T [f(x) - f(x^*)] \geq 0$. So x^* is a solution of (2.2).

By a simple manipulation, we can establish the following equivalent formulation of (1.1): find $x^* \in R^n$ such that

$$(x - x^*)^T (M^T N x^* + M^T q) \geq 0, \quad \forall x \in \Omega, \quad (2.3)$$

where $\Omega = \{x \in R^n \mid A(Mx + p) \geq 0, B(Mx + p) = 0\}$. Obviously, the solution set of (2.3) is nonempty under the nonempty assumption of the solution of (1.1), is denoted by Ω^*. (2.3) is also an equivalent reformulation of the problem (1.1).

Secondly, we recall the definition of projection operator and some related properties ([15, 16]). For a nonempty closed convex set $K \subset R^n$ and vector $x \in R^n$, the orthogonal projection of x onto K, i.e., $\arg\min\{\| y - x \| \mid y \in K\}$, is denoted by $P_K(x)$.

Proposition 2.1 Let K be a closed convex subset of R^n. For any $x, y \in R^n$ and $z \in K$, the following statements hold.

$$(i) \ \langle P_K(x) - x, z - P_K(x) \rangle \geq 0;$$

$$(ii) \, | P_K(x) - P_K(y) \|^2 \leq \| x - y \|^2 - \| (P_K(x) - x) - (P_K(y) - y) \|^2.$$

For (2.3) and $x \in R^n$, define the projection residue $r(x, \beta) := x - P_\Omega(x - \beta F(x)),$ where $\beta > 0$ is a constant, $F(x) = M^T N x - M^T p$. The projection residue is intimately related to the solution of (2.3) as shown by the following well-known result, which is due to Noor ([3]).

Proposition 2.2 x^* is a solution of (2.3) if and only if $r(x^*, \beta) = 0$ with some $\beta > 0$.

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III. ALGORITHM AND GLOBAL CONVERGENCE

In this section, based on the equivalent reformulations described in Section 2, we shall propose a new projection-type algorithm to solve GLCP without the backtracking line search to find a suitable step size, and prove global convergence of the new PTA in detail. Furthermore, the sub linearly convergence rate of the method is also established. We first give the following assumptions ([17]) which is crucial to our method analysis.

Assumption 3.1 Suppose that the mapping F is pseudo monotone, which F is defined in (2.3).

Remark 3.1 From (2.3), if F is pseudo monotone, then we have $\langle F(x), x-x^*\rangle \geq 0, \forall x\in\Omega, x^*\in\Omega^*$. On the other hand, if the mapping F is monotone, then the mapping F is pseudo monotone, we have Assumption 3.1 holds. But the converse is not true in general. For example, $F(x)=2-x\ (x\in R)$, the mapping F is pseudo monotone in interval $[0,1]$, i.e., for any $x, y\in[0,1]$, if $\langle F(y), x-y\rangle\geq 0$, then $\langle F(x), x-y\rangle \geq 0$, but the mapping F is not monotone.

Now, we formally state our algorithm.

Algorithm 3.1.

Step0. Select any $\sigma,\gamma,\beta\in(0,1), x^{-1}=x^0\geq 0, k:=0.$

Step1. For $x^k\in\Omega$, compute $z^k\in\Omega$ such that $z^k=P_\Omega(x-\beta F(x))$. If $\|r(x^k,\beta)\|=0$, stop. Otherwise, go to Step 2.

Step2. Compute $y^k\in\Omega$ such that $y^k=(1-\eta_k)x^k+\eta_k z^k$, where $\eta_k=r^{m_k}$ with m_k being the smallest non-negative integer m satisfying

$$\langle F(x^k), r(x^k,\beta)\rangle \geq (\sigma+\gamma^m|\lambda|_{max})\|r(x^k,\beta)\|^2, \tag{3.1}$$

Where $|\lambda|_{max}$ is the absolute value of eigenvalue of maximum for the matrix $M^{\bullet}N$. Let

$$x^{k+1}:=P_{H_k^1\bigcap H_k^2\bigcap\Omega}(x^k),$$

Where $H_k^1:=\{x\in\Omega\,|\,\langle x-y^k, F(y^k)\rangle\leq 0\}$, $H_k^2:=\{x\in\Omega\,|\,\langle x-x^k, x^{k-1}-x^k\rangle\leq 0\}$. Go to Step 1 with $k:=k+1$.

Remark 3.2. Firstly, we discuss the feasibility of step size rule (3.1). If Algorithm 3.1 terminates with $\|r(x^k,\beta)\|=0$, then x^k is a solution of (2.3). Otherwise, from Proposition 2.1 (i), we have that

$$\begin{aligned}
&\langle \beta F(x^k), r(x^k,\beta)\rangle - \|r(x^k,\beta)\|^2\\
&=\langle \beta F(x^k), x^k-P_\Omega(x^k-\beta Fx^k)\rangle\\
&\quad -\langle x^k-P_\Omega(x^k-\beta Fx^k), x^k-P_\Omega(x^k-\beta Fx^k)\rangle\\
&=\langle P_\Omega(x^k-\beta Fx^k)-(x^k-\beta F(x^k), x^k-P_\Omega(x^k-\beta Fx^k)\rangle\geq 0.
\end{aligned} \tag{3.2}$$

i.e., $\langle F(x^k), r(x^k,\beta)\rangle \geq \beta^{-1}\|r(x^k,\beta)\|^2.$ (3.3)

From (3.3), for large enough number m, we have that (3.1) holds.

Secondly, from the following lemma, we know that if the solution set (1.1) is nonempty, then $H_k^1\bigcap H_k^2\bigcap\Omega$ is a nonempty set, which implies the feasibility of Algorithm 3.1.

Lemma 3.1 Suppose that Assumption 3.1 holds, if the solution set of (1.1) is nonempty, then $X^*\subseteq H_k^1\bigcap H_k^2\bigcap\Omega$ for all k, and $H_k^1\bigcap H_k^2\bigcap\Omega$ is a nonempty set.

Proof For any $x^*\in\Omega^*$. From (2.3) and $y^k\in\Omega$, it follows that $\langle F(x^*), y^k-x^*\rangle\geq 0.$ (3.4)

Using Assumption 3.1, one has $\langle F(y^k), y^k-x^*\rangle \geq 0.$ Thus, $x^*\in H_k^1\bigcap\Omega.$

Next, we will prove that $x^*\in H_k^2$ for any $k\geq 0$. The proof will be given by induction.

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Obviously, $x^* \in H_0^2 = R^n$. Suppose $x^* \in H_k^2$, then $\Omega^* \subseteq H_k^1 \bigcap H_k^2 \bigcap \Omega$. Combining Proposition 2.1 (i) with $x^{k+1} = P_{H_k^1 \bigcap H_k^2 \bigcap \Omega}(x^k)$, it holds that $\langle x^* - x^{k+1}, x^k - x^{k+1} \rangle \le 0$, and that $x^* \in H_{k+1}^2$. This shows that $\Omega^* \subseteq H_k^2$ for all k and the desired result follows.

If the solution set of (1.1) is empty, then Ω^* is nonempty. Thus, we have $H_k^1 \bigcap H_k^2 \bigcap \Omega$ is nonempty.
In this following, we discuss the convergence and convergence rate of Algorithm 3.1. For this purpose, we first prove that the sequence η_k generated by Algorithm 3.1 has a positive bound from below.

Lemma 3.2 The sequence $\{\eta_k\}$ generated by Algorithm 3.1 has a positive bound from below.

Proof By the line search procedure for updating η_k, if $\eta_k < \gamma$, then

$$\langle F(x^k), r(x^k, \beta) \rangle < (\sigma + |\lambda|_{max} \eta_k / \gamma) \| r(x^k, \beta) \|^2. \tag{3.5}$$

Combining this with (3.3), one has

$$\beta^{-1} \| r(x^k, \beta) \|^2 < (\sigma + |\lambda|_{max} \eta_k / \gamma) \| r(x^k, \beta) \|^2. \tag{3.6}$$

By (3.6), we obtain $\eta_k > \gamma(\beta^{-1} - \sigma) / |\lambda|_{max}$. Let $\eta := \min\{1, \gamma(\beta^{-1} - \sigma)/|\lambda|_{max}\}$, for all k, one has $\eta_k > \eta > 0$.

If Algorithm 3.1 terminates at Step 1, then x^k is a solution of (2.3). Otherwise, it generates an infinite sequence, and $\| r(x^k, \beta) \| > 0$. Combining this with the definition of H_k^1 in Algorithm 3.1, one has $x^k \notin H_k^1$. In fact,

$$\begin{aligned} \langle x^k - y^k, F(y^k) \rangle &= \langle x^k - [(1-\eta_k)x^k + \eta_k z^k], M^{\bullet} N((1-\eta_k)x^k + \eta_k z^k) - M^{\bullet} p \rangle \\ &= \langle \eta_k [x^k - z^k], [M^{\bullet} N x^k - M^{\bullet} p] - \eta_k M^{\bullet} N [x^k - z^k] \rangle \\ &= \langle \eta_k r(x^k, \beta), [M^{\bullet} N x^k - M^{\bullet} p] - \eta_k M^{\bullet} N r(x^k, \beta) \rangle \\ &= \eta_k \langle r(x^k, \beta), F(x^k) \rangle - \eta_k^2 \langle r(x^k, \beta), M^{\bullet} N r(x^k, \beta) \rangle \\ &\ge \eta_k (\sigma + \eta_k |\lambda|_{max}) \| r(x^k, \beta) \|^2 - \eta_k^2 |\lambda|_{max} \| r(x^k, \beta) \|^2 \\ &\ge \eta_k \sigma \| r(x^k, \beta) \|^2 > 0. \end{aligned} \tag{3.7}$$

By (3.7), we obtain

$$\begin{aligned} \eta_k \sigma \| r(x^k, \beta) \|^2 &\le \langle x^k - y^k, F(y^k) \rangle \\ &= \langle x^k - [(1-\eta_k)x^k + \eta_k z^k], F(y^k) \rangle \\ &= \langle \eta_k (x^k - z^k), F(y^k) \rangle \\ &= \langle \eta_k r(x^k, \beta), F(y^k) \rangle \\ &= \eta_k \| r(x^k, \beta) \| \; \| F(y^k) \|. \end{aligned}$$

i.e.,

$$\| F(y^k) \| \ge \sigma \| r(x^k, \beta) \|. \tag{3.8}$$

Now, we give the global convergence result of algorithm 3.1.

Theorem 3.1 Suppose that Algorithm 3.1 generates an infinite sequence $\{x^k\}$, and the solution set of (1.1) is nonempty. Then, the sequence $\{x^k\}$ is bounded and globally converges to a solution of (2.3).

Proof For any $x^* \in \Omega^*$. Since $x^* \in \Omega_k$ and $x^{k+1} = P_{\Omega_k}(x^k)$, where $\Omega_k = H_k^1 \bigcap H_k^2 \bigcap \Omega$. By Proposition 2.1 (ii), one has

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$$\begin{aligned} \| x^{k+1} - x^* \|^2 &= \| P_{\Omega_k}(x^k) - P_{\Omega_k}(x^*) \|^2 \\ &\le \| x^k - x^* \|^2 - \| (P_{\Omega_k}(x^k) - x^k) - (P_{\Omega_k}(x^*) - x^*) \|^2 \\ &= \| x^k - x^* \|^2 - \| P_{\Omega_k}(x^k) - x^k \|^2 \\ &= \| x^k - x^* \|^2 - \| x^{k+1} - x^k \|^2, \end{aligned} \tag{3.9}$$

From (3.9), we obtain that the sequence $\{\| x^k - x^* \|\}$ is non-increasing and bounded. Thus, it converges, and one has $\lim_{k\to\infty} \| x^{k+1} - x^k \|^2 = 0.$ (3.10)

On the other hand, since

$$P_{H_k^1(x^k)} = x^k - \frac{\langle F(y^k), x^k - y^k \rangle}{\| F(y^k) \|^2} F(y^k). \tag{3.11}$$

Combining (3.11) with $x^{k+1} \in H_k^1$, and by $x^k \notin H_k^1$, we obtain

$$\begin{aligned} \| x^k - x^{k+1} \| &\ge \| x^k - P_{H_k^1}(x^k) \| \\ &= \left\| \frac{\langle F(y^k), x^k - y^k \rangle}{\| F(y^k) \|^2} F(y^k) \right\| \\ &= \frac{\| \langle F(y^k), x^k - y^k \rangle \|}{\| F(y^k) \|} \\ &= \frac{\| \langle F((1-\eta_k)x^k + \eta_k z^k), x^k - [(1-\eta_k)x^k + \eta_k z^k] \rangle \|}{\| F(y^k) \|} \\ &= \frac{\| \langle M^{\bullet} N((1-\eta_k)x^k + \eta_k z^k) - M^{\bullet} p, \eta_k [x^k - z^k] \rangle \|}{\| F(y^k) \|} \\ &= \frac{\| \langle [M^{\bullet} N x^k - M^{\bullet} p] - \eta_k M^{\bullet} N [x^k - z^k], \eta_k [x^k - z^k] \rangle \|}{\| F(y^k) \|} \\ &= \frac{\| \langle F(x^k), \eta_k [x^k - z^k] \rangle - \langle \eta_k M^{\bullet} N [x^k - z^k], \eta_k [x^k - z^k] \rangle \|}{\| F(y^k) \|} \\ &= \frac{\| \eta_k \langle F(x^k), r(x^k, \beta) \rangle - \eta_k^2 \langle M^{\bullet} N r(x^k, \beta), r(x^k, \beta) \rangle \|}{\| F(y^k) \|} \\ &\ge \frac{\eta_k \| \langle F(x^k), r(x^k, \beta) \rangle \| - \eta_k^2 |\lambda|_{max} \| r(x^k, \beta) \|^2}{\| F(y^k) \|} \\ &\ge \frac{\eta_k (\sigma + \eta_k |\lambda|_{max}) \| r(x^k, \beta) \|^2 - \eta_k^2 |\lambda|_{max} \| r(x^k, \beta) \|^2}{\| F(y^k) \|} \\ &= \frac{\sigma \eta_k \| r(x^k, \beta) \|^2}{\| F(y^k) \|} \\ &\ge \frac{\sigma \eta \| r(x^k, \beta) \|^2}{\| F(y^k) \|}, \end{aligned} \tag{3.12}$$

where the third inequality is obtained by using (3.1), the last equality is obtained by Lemma 3.2. Combining (3.10) with (3.12) ,we have

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$$\lim_{k\to\infty}\frac{\sigma\eta\| r(x^k,\beta)\|^2}{\| F((1-\eta_k)x^k+\eta_k z^k)\|}=0. \quad (3.13)$$

Since $\{x^k\}$ is bounded, so $\{z^k\}$ is also bounded, and thus $F((1-\eta_k)x^k+\eta_k z^k)$ is bounded. Combining (3.13), It follow that $\lim_{k\to\infty}\| r(x^k,\beta)\|^2=0.$ (3.14)

For any convergent subsequence $\{x^{k_j}\}$ of $\{x^k\}$, denote its limit by $\hat{x},$ combining this with (3.14), we obtain

$$\| r(\hat{x},\beta)\|^2=\lim_{j\to\infty}\| r(x^{k_j},\beta)\|^2=0, \quad (3.15)$$

and thus $\hat{x}$ is a solution of (2.3).

We can take $x^*=\hat{x}$ in the preceding arguments , in particular, in (3.9). Thus the sequence $\{\| x^k-\hat{x}\|\}$ converges. Since $\hat{x}$ is limit point of the subsequence $\{x^{k_j}\}$, it easily follows that $\| x^k-\hat{x}\|$ converges to zero, i.e., that $\{x^k\}$ converges to $\hat{x}\in\Omega^*$. Thus, the global convergence can be obtained.

Next, we discuss the convergence rate of algorithm 3.1. To this end, we present the following lemma.

Lemma 3.3 For given a constant $c_0>0$, for any $x\in R^n$ with $\| x\|\le c_0$. Then, there exists a constant $\tau>0$ such that $dist(x,\Omega^*)\le\tau\| r(x,\beta)\|,$ (3.16)

where $dist(x,\Omega^*)$ denotes the distance from point x to the solution set Ω^* of (2.3).

Proof Assume that the theorem is false. Then there exist positive sequence $\{\tau_m\}$ and sequence $\{x^m\}\in R^n$ with $\| x^m\|\le c_0$ such that $\tau_m\to\infty$ as $m\to\infty$ and $dist(x^m,\Omega^*)>\tau_m\| r(x^m,\beta)\|.$ (3.17)

Hence, $\| r(x^m,\beta)\|/dist(x^m,\Omega^*)\to 0$ as $m\to\infty$. (3.18)

Since $\| x^m\|\le c_0$ and $r(x^m,\beta)$ is continuous, by (3.18), we have $r(x^m,\beta)\to 0$ as $m\to\infty$. Since $\| x^m\|\le c_0$ again, there exists a subsequence $\{x^{m_i}\}$ of $\{x^m\}$ such that $\lim_{m_i\to\infty}x^{m_i}=\bar{x}$ with $r(\bar{x},\beta)=0$. Hence $\bar{x}\in\Omega^*$. By (3.18), we further obtain

$$\lim_{m_i\to\infty}\| r(x^{m_i},\beta)\|/\| x^{m_i}-\bar{x}\|\le\lim_{m_i\to\infty}\| r(x^{m_i},\beta)\|/dist(x^{m_i},\Omega^*)=0. \quad (3.19)$$

On the other hand, from $x^{m_i}\to\bar{x}$ and $r(x^{m_i},\beta)\to 0$, we know that, for all sufficiently large m_i,

$$\begin{aligned}\frac{\| r(x^{m_i},\beta)\|}{\| x^{m_i}-\bar{x}\|}&=\frac{\| r(x^{m_i},\beta)-r(\bar{x},\beta)\|}{\| x^{m_i}-\bar{x}\|}\\
&=\frac{\|[x^{m_i}-P_\Omega(x^{m_i}-\beta F(x^{m_i}))]-[\bar{x}-P_\Omega(\bar{x}-\beta F(\bar{x}))]\|}{\| x^{m_i}-\bar{x}\|}\\
&=\frac{\|[x^{m_i}-\bar{x}]+[P_\Omega(x^{m_i}-\beta F(x^{m_i}))-P_\Omega(\bar{x}-\beta F(\bar{x}))]\|}{\| x^{m_i}-\bar{x}\|}\\
&\ge\frac{\| x^{m_i}-\bar{x}\|-\| P_\Omega(x^{m_i}-\beta F(x^{m_i}))-P_\Omega(\bar{x}-\beta F(\bar{x}))\|}{\| x^{m_i}-\bar{x}\|}\\
&\ge\frac{\| x^{m_i}-\bar{x}\|-\|(x^{m_i}-\beta F(x^{m_i}))-(\bar{x}-\beta F(\bar{x}))\|}{\| x^{m_i}-\bar{x}\|}\\
&=\frac{\| x^{m_i}-\bar{x}\|-\|(I-\beta M^\bullet N)(x^{m_i}-\bar{x})\|}{\| x^{m_i}-\bar{x}\|}\\
&\ge 1-\|(I-\beta M^\bullet N)\|>0,\end{aligned}$$

this contradicts (3.19). Thus, (3.16) holds.

Theorem 3.2 Suppose that Assumption 3.1 holds, for the sequence $\{x^k\}$ generated by Algorithm 3.1, there exists a constant $c_1 > 0$ such that, for all k sufficiently large,

$$dist(x^k, \Omega^*) \le c_1 \big/ \sqrt{k+1}.$$

Proof From Theorem 3.1, we know that $\{y^k\}$ is a bounded sequence, so there exists a positive constant c_2 such that, for any k $\| F(y^k) \| \le c_2$. Combining this with (3.8}), one has

$$\frac{\| r(x^k, \beta) \|}{c_2} \le \frac{\| r(x^k, \beta) \|}{\| F(y^k) \|} \le \frac{1}{\sigma}. \tag{3.20}$$

Using (3.9) with $x^* = \hat{x} \in \Omega^*$, which is the closest to x^k, a direct computation yields that

$$\begin{aligned} dist^2(x^{k+1}, \Omega^*) &\le \| x^{k+1} - \hat{x} \|^2 \\ &\le \| x^k - \hat{x} \|^2 - \| x^{k+1} - x^k \|^2 \\ &\le \| x^k - \hat{x} \|^2 - \left(\frac{\sigma\eta \| r(x^k, \beta) \|^2}{\| F(y^k \|} \right)^2 \\ &\le dist^2(x^k, \Omega^*) - \left(\frac{\sigma\eta}{\tau^2 c_2} \right)^2 dist^4(x^k, \Omega^*), \end{aligned}$$

where the third inequality is by (3.12), the fourth inequality is obtained by (3.20) and (3.16). Since the sequence $dist^2(x^k, \Omega^*)$ satisfies the conditions of Lemma 6 in Chapter 2 of Ref.[18], there exists a positive constant c_1 such that

$$dist(x^k, \Omega^*) \le c_1 \big/ \sqrt{k+1},$$

for all k sufficiently large.

IV. CONCLUSION

In this paper, we propose a new projection-type algorithm for solving GLCP without the backtracking line search to find a suitable step size, and its global convergence is established in detail. Furthermore, the global sub linear convergence rate of the method also is shown. According to its limitations, the parameters of Algorithm 3.1 is adjusted dynamically to further enhance the efficiency of the corresponding method, which is worthy of research.

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