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A first approximation of the generalized planetary equations based upon Riemannian geometry and the golden metric tensor

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***Abstract:** In this paper we introduced super general geodesic equation and golden metric tensors. We derived linear velocity vector and linear acceleration vector using golden metric tensor in spherical polar coordinates. Coefficients of affine connection were evaluated based upon golden metric tensor. Based on the evaluated linear velocity vector and acceleration vector, we are in position to obtain super general geodesic planetary equation in terms of $r,\theta,$ to obtain Riemannian acceleration due to gravity in terms of $r,\theta,$ known as gravitational scalar potential that played an important role in dealing with planetary phenomenon. Also in this paper derived generalized planetary equations based upon Riemannian geometry and the golden metric tensor. The planetary equation obtain in this paper contained Newton's planetary equation, Einstein's planetary equation and the added or contribution term to the existing planetary equations known as post Newton's planetary equation and post Einstein's planetary equation. Also in this paper we offered solutions to both Newton and post Newton's planetary equation to obtain planetary parameters such as eccentricity. Generalized amplitude was obtained for two sets of equations, that is Newton and post Newton planetary equations.*

Keywords: **Golden metric tensor, Geodesic equation, coefficient of affine connection, Newton's planetary equation, Einstein's planetary equation.**

I. INTRODUCTION

The properties of geodesics differ from those of straight lines. For example, on a plane, parallel lines never meet, but this is not so far geodesics on the surface of the earth. For example, lines of longitude are parallel at the equator, but intersect at the poles. Analogously, the world lines of test particles in free fall are space time geodesics, the straightest possible lines in space time. But still there are crucial differences between them and the truly straight lines that can be traced out in the gravity-free space time of special relativity [1]. Einstein's equations are the centre piece of general relativity. They provide a precise formulation of the relationship between space time geometry and the properties of matter, using the language of mathematics. More concretely, they are formulated using the concepts of Riemannian geometry, in which the geometric properties of a space (or a space time) are described by a quantity called a metric. The metric encodes the information needed to compute the fundamental geometric notions of distance and angle in a curved space (or space time) [2]. The metric function and its rate of change from point to point can be defining a geometrical quantity called the Riemann curvature tensor, which describes exactly how the space (or space time) is curved at each point. In general relativity, the metric and the Riemann curvature tensor are quantities define at each point in space time. It is based on the above argument explanation. Howusu introduced, by postulation, a second natural and satisfactorily generalization or extension of the Schwarzschild's metric tensor from the gravitational fields of all static homogeneous spherical distribution of mass to the gravitational fields of all spherical distributions of mass-named as the golden metric tensor for all gravitational fields in nature [3]

II. GOLDEN METRIC TENSOR

In this paper we introduced golden metric tensor for all gravitational fields in nature as follows [4]. The covariant form of all golden metric tensor for all gravitational fields in nature as

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$$g_{11} = \left\{1 + \frac{2}{c^2} f(r, \theta, \phi, x^0)\right\}^{-1} \quad (2.1)$$

$$g_{22} = r^2 \left\{1 + \frac{2}{c^2} f(r, \theta, \phi, x^0)\right\}^{-1} \quad (2.2)$$

$$g_{33} = r^2 \sin^2\theta \left\{1 + \frac{2}{c^2} f(r, \theta, \phi, x^0)\right\}^{-1} \quad (2.3)$$

$$g_{00} = -\left\{1 + \frac{2}{c^2} f(r, \theta, \phi, x^0)\right\} \quad (2.4)$$

$$g_{uv} = 0, \text{otherwise} \quad (2.5)$$

Where f is the gravitational scalar potential of the space time, the golden metric tensor and also contains the following physical effects:

- Gravitational space contraction
- Gravitational time dilation
- Gravitational polar angle contraction
- Gravitational azimuthal angle contraction

While the contra variant form of golden metric tensor given as:

$$g^{11} = \left\{1 + \frac{2}{c^2} f(r, \theta, \phi, x^0)\right\} \quad (2.6)$$

$$g^{22} = \frac{1}{r^2} \left\{1 + \frac{2}{c^2} f(r, \theta, \phi, x^0)\right\} \quad (2.7)$$

$$g^{33} = \frac{1}{r^2 \sin^2\theta} \left\{1 + \frac{2}{c^2} f(r, \theta, \phi, x^0)\right\} \quad (2.8)$$

$$g^{00} = -\left\{1 + \frac{2}{c^2} f(r, \theta, \phi, x^0)\right\}^{-1} \quad (2.9)$$

$$g^{uv} = 0, \text{otherwise} \quad (2.10)$$

III. FORMULATION OF THE GENERAL DYNAMICAL LAWS OF GRAVITATION

It is well known that all of Newton's dynamical laws of gravitation are founded on the experimental physical facts available in his day. The instantaneous active mass passive mass and the inertial mass of a particle of non-zero rest mass are given by:

$$m_A = m_P = m_I = m_0 \quad (3.1)$$

In all proper inertial reference frames and proper times. This statement may be called Newton's principle of mass. According to the classic scientific method introduced by G. Galileo (the father of mechanics) and Newton the natural laws of mechanics are determined by experimental physical facts. Therefore today's experimental revisions of the definitions of inertial, passive and active masses of a particle on non-zero rest mass induce a corresponding revision of Newton's dynamical laws of gravitation which are now formulated (Howusu, 1991).

Super General Planetary Equation

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$$(m_I)_H = (m_p)_H = \left[1 - \frac{u^2}{c^2}\left(1 + \frac{2}{c^2}f\right)^{-1}\right]^{-\frac{1}{2}} \cdot \left(1 + \frac{2}{c^2}f\right)^{\frac{1}{2}} m_0 \qquad (3.2)$$

as given by equation (3.1) above

$\underline{g}_H$ = Riemannian acceleration due to gravity

$\underline{u}_H$ = Riemannian velocity vector

But force is defined as the rate of change of momentum

$$\frac{d}{d\tau}\left\{\left[1 - \frac{u^2}{c^2}\left(1 + \frac{2}{c^2}\right)^{-1}\right]^{-\frac{1}{2}} \left(1 + \frac{2}{c^2}f\right)^{\frac{1}{2}} m_0 \underline{u}_H\right\} = \left[1 - \frac{u^2}{c^2}\left(1 + \frac{2}{c^2}f\right)^{-1}\right]^{-\frac{1}{2}} \cdot \left(1 + \frac{2}{c^2}f\right)^{\frac{1}{2}} m_0 \underline{g}_H \quad (3.3)$$

Where (is Riemannian factor.

Equivalently,

$$\frac{d}{d\tau}\{(m_I)_H \cdot \underline{u}_H\} = (m_p)_H \underline{g}_H \qquad (3.4)$$

$$(m_I)_H \, \underline{a}_H + \left\{\frac{d}{d\tau}[(m_I)_H]\right\}\underline{u}_H = (m_p)_H \underline{g}_H \qquad (3.5)$$

$$\underline{a}_H + \frac{1}{(m_I)_H}\left\{\frac{d}{d\tau}[(m_I)_H]\right\}\underline{u}_H = \underline{g}_H \qquad (3.6)$$

$\underline{a}_H \equiv$ general acceleration vector.

Equation (3.6) above referred to as super general geodesics equation of motion.

Velocity Tensor

$$\dot{x}^\mu = \{\dot{x}^1, \dot{x}^2, \dot{x}^3, \dot{x}^0\} \qquad (3.7)$$

$x^1 = r, \;\; x^2 = \theta, \;\; x^3 = \phi, \;\; x^0 = ict$

$$\dot{x}^\mu = \{\dot{r}, \dot{\theta}, \dot{\phi}, ct\} \qquad (3.8)$$

Velocity vector

$$\underline{u}_H = \left(\sqrt{g_{11}}\dot{x}^1, \sqrt{g_{22}}\dot{x}^2, \sqrt{g_{33}}\dot{x}^3, \sqrt{g_{00}}\dot{x}^0\right) \qquad (3.9)$$

Based upon golden metric tensor

$$u_r = \sqrt{g_{11}}\dot{r} = \left(1 + \frac{2}{c^2}f\right)^{-\frac{1}{2}} \dot{r} \qquad (3.10)$$

$$u_\theta = \sqrt{g_{22}}\dot{\theta} = r\left(1 + \frac{2}{c^2}f\right)^{-\frac{1}{2}} \dot{\theta} \qquad (3.11)$$

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$$u_\varphi = \sqrt{g_{33}}\dot{\phi} = r\sin\theta\left(1+\frac{2}{c^2}f\right)^{-\frac{1}{2}}\dot{\phi} \quad (3.12)$$

$$u_0 = \sqrt{g_{00}}\dot{x}^0 = \left(1+\frac{2}{c^2}f\right)^{-\frac{1}{2}} ic\dot{t} \quad (3.13)$$

Where we have make use of Golden metric tensor in equations (2.1)-(2.4)

IV. THEORETICAL ANALYSIS

Acceleration tensors define by geodesics equation as:

$$a^\mu = \ddot{x}^\mu + \Gamma^\mu_{\alpha\beta}\dot{x}^\alpha\dot{x}^\beta \quad (4.1)$$

Where is defined as coefficient of affine connection gives as:

$$\Gamma^\mu_{\alpha\beta} = \frac{1}{2}g^{\mu\varepsilon}\left(g_{\alpha\varepsilon,\beta} + g_{\varepsilon\beta,\alpha} - g_{\alpha\beta,\varepsilon}\right) \quad (4.2)$$

Putting

$$a^1 = \ddot{r} + \Gamma^1_{11}(\dot{r})^2 + \Gamma^1_{22}(\dot{\theta})^2 + \Gamma^1_{33}(\dot{\phi})^2 \quad (4.3)$$

Putting

$$a^2 = \ddot{\theta}^2 + 2\Gamma^2_{12}(\dot{r}\dot{\theta}) + \Gamma^2_{33}(\dot{\phi})^2 \quad (4.4)$$

Putting

$$a^3 = \ddot{\phi}^2 + 2\Gamma^3_{13}(\dot{r}\dot{\phi}) + 2\Gamma^3_{23}(\dot{\theta}\dot{\phi}) \quad (4.5)$$

Putting

$$a^0 = c\ddot{t} + 2c\Gamma^0_{01}(\dot{t}\dot{r}) \quad (4.6)$$

Using

$x^0 = ct$

By employing equation (4.2) above, gives as [9]

$$\Gamma^1_{00} = \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)f_{,1} \quad (4.7)$$

$$\Gamma^1_{01} = \Gamma^1_{10} = -\frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,0} \quad (4.8)$$

$$\Gamma^1_{11} = -\frac{1}{c^2}\left(1+\frac{1}{c^2}f\right)^{-1} f_{,1} \quad (4.9)$$

$$\Gamma^1_{12} = \Gamma^1_{21} = -\frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,2} \quad (4.10)$$

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$$\Gamma^1_{13} = \Gamma^1_{31} = -\frac{1}{c^2}\left(1 + \frac{2}{c^2}f\right)^{-1} f_{,3} \tag{4.11}$$

$$\Gamma^1_{22} = -\frac{r^2}{c^2}\left(1 + \frac{2}{c^2}f\right)^{-1} f_{,1} - r \tag{4.12}$$

$$\Gamma^1_{33} = \frac{r^2\sin^2\theta}{c^2}\left(1 + \frac{2}{c^2}f\right)^{-1} f_{,1} - r\sin^2\theta \tag{4.13}$$

and

$$\Gamma^2_{00} = \frac{1}{c^2r^2}\left(1 + \frac{2}{c^2}f\right) f_{,2} \tag{4.14}$$

$$\Gamma^2_{02} = \Gamma^2_{20} = \frac{1}{c^2}\left(1 + \frac{2}{c^2}f\right)^{-1} f_{,0} \tag{4.15}$$

$$\Gamma^2_{11} = \frac{1}{c^2r^2}\left(1 + \frac{1}{c^2}f\right)^{-1} f_{,2} \tag{4.16}$$

$$\Gamma^2_{12} = \Gamma^2_{21} = \frac{1}{r} - \frac{1}{c^2}\left(1 + \frac{2}{c^2}f\right)^{-1} f_{,2} \tag{4.17}$$

$$\Gamma^2_{22} = -\frac{1}{c^2}\left(1 + \frac{2}{c^2}f\right)^{-1} f_{,2} \tag{4.18}$$

$$\Gamma^2_{23} = \Gamma^2_{32} = -\frac{2}{c^2}\left(1 + \frac{2}{c^2}f\right)^{-1} f_{,3} \tag{4.19}$$

$$\Gamma^2_{33} = \frac{\sin^2\theta}{c^2}\left(1 + \frac{2}{c^2}f\right)^{-1} f_{,2} - \sin\theta\cos\theta \tag{4.20}$$

and

$$\Gamma^3_{00} = \frac{1}{c^2r^2\sin^2\theta}\left(1 + \frac{2}{c^2}f\right) f_{,3} \tag{4.21}$$

$$\Gamma^3_{03} = \Gamma^3_{30} = \frac{1}{c^2}\left(1 + \frac{2}{c^2}f\right)^{-1} f_{,3} \tag{4.22}$$

$$\Gamma^3_{11} = \frac{1}{c^2r^2\sin^2\theta}\left(1 + \frac{2}{c^2}f\right) f_{,3} \tag{4.23}$$

$$\Gamma^3_{13} = \Gamma^3_{31} = \frac{1}{r} - \frac{1}{c^2}\left(1 + \frac{2}{c^2}f\right)^{-1} f_{,1} \tag{4.24}$$

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$$\Gamma^3_{22} = \frac{1}{c^2 \sin^2\theta}\left(1 + \frac{2}{c^2}f\right)^{-1} f_{,3} \tag{4.25}$$

$$\Gamma^3_{23} = \Gamma^3_{32} = \cot\theta - \frac{1}{c^2}\left(1 + \frac{2}{c^2}f\right)^{-1} f_{,2} \tag{4.26}$$

$$\Gamma^3_{33} = -\frac{1}{c^2}\left(1 + \frac{2}{c^2}f\right)^{-1} f_{,3} \tag{4.27}$$

and

$$\Gamma^0_{00} = \frac{1}{c^2}\left(1 + \frac{2}{c^2}f\right)^{-1} f_{,0} \tag{4.28}$$

$$\Gamma^0_{01} = \Gamma^0_{10} = \frac{1}{c^2}\left(1 + \frac{2}{c^2}f\right)^{-1} f_{,1} \tag{4.29}$$

$$\Gamma^0_{02} = \Gamma^0_{20} = \frac{1}{c^2}\left(1 + \frac{2}{c^2}f\right)^{-1} f_{,2} \tag{4.30}$$

$$\Gamma^0_{03} = \Gamma^0_{30} = \frac{1}{c^2}\left(1 + \frac{2}{c^2}f\right)^{-1} f_{,3} \tag{4.31}$$

$$\Gamma^0_{01} = -\frac{1}{c^2}\left(1 + \frac{2}{c^2}f\right)^{-3} f_{,0} \tag{4.32}$$

$$\Gamma^0_{22} = -\frac{r^2}{c^2}\left(1 + \frac{2}{c^2}f\right)^{-3} f_{,0} \tag{4.33}$$

$$\Gamma^0_{33} = -\frac{r^2 \sin^2\theta}{c^2}\left(1 + \frac{2}{c^2}f\right)^{-3} f_{,0} \tag{4.34}$$

$$\Gamma^\mu_{\alpha\beta} = 0; \quad \text{otherwise} \tag{4.35}$$

Hence the acceleration vector

$$\underline{a}_H\left(\sqrt{g_{11}}a^1, \sqrt{g_{22}}a^2, \sqrt{g_{33}}a^3, \sqrt{g_{00}}a^0\right) \tag{4.36}$$

By putting the results of coefficients of affine connection, covariant form of golden metric tensors into equation (4.15), we obtained acceleration vector equations as:

$$a_r = \left(1 + \frac{2}{c^2}\right)^{-\frac{1}{2}}\left[\ddot{r} - \frac{1}{c^2}\left(1 + \frac{2}{c^2}f\right)^{-1} f_{,1}(\dot{r})^2 - r\left(1 + \frac{2}{c^2}f\right)(\theta)^2 - r\sin^2\theta\left(1 + \frac{2}{c^2}f\right)(\dot{\phi})^2\right] \tag{4.37}$$

$$a_\theta = r\left(1 + \frac{2}{c^2}f\right)^{-\frac{1}{2}}\left[\ddot{\theta} + \frac{2}{r}(\dot{r}\dot{\theta}) - \sin\theta\cos\theta(\dot{\phi})^2\right] \tag{4.38}$$

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$$a_\phi = r\sin\theta\left(1+\frac{2}{c^2}f\right)^{-\frac{1}{2}}\left[\ddot{\phi}+\frac{2}{r}(\dot{r}\dot{\theta})+2\cot\theta\,(\dot{\theta}\dot{\phi})\right] \qquad (4.39)$$

$$a_0 = \left(1+\frac{2}{c^2}f\right)i\left[c\ddot{t}+\frac{2}{c}\left(1+\frac{2}{c^2}\right)^{-1}f_{,1}\right] \qquad (4.40)$$

Hence equations (4.37), (4.38), (4.39) an, are linear acceleration vector equations based upon golden metric tensor.

V. RIEMANNIAN ACCELERATION DUE TO GRAVITY

Riemannian acceleration due to gravity in terms of $r, \theta,$ is given by [10]

$$a_H + \frac{1}{(m_I)_H}\left\{\frac{d}{d\tau}[(m_I)_H]\right\}u_H = g_H \qquad (5.1)$$

Where a_H = Riemannian acceleratio [6]

u_H = Riemannian velocity vector

g_H = Riemannian acceleration due to gravity

$(m_I)_H$ = Riemannian inertial mass

Using

$$u_r = \sqrt{g_{11}}\dot{r} = \left(1+\frac{2}{c^2}f\right)^{-\frac{1}{2}}\dot{r}$$

and

$$a_r = a^1\left(1+\frac{2}{c^2}f\right)^{-\frac{1}{2}} \qquad (5.2)$$

From equation (4.37) that is

$$a_r = \left(1+\frac{2}{c^2}\right)^{-\frac{1}{2}}\left[\ddot{r}-\frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}f_{,1}(\dot{r})^2 - r\left(1+\frac{2}{c^2}f\right)(\dot{\theta})^2 - r\sin^2\theta\left(1+\frac{2}{c^2}f\right)(\dot{\phi})^2\right]$$

Hence Riemannian radial acceleration due to gravity is given by

$$a_r + \frac{1}{(m_I)_r}\left\{\frac{d}{d\tau}[(m_I)_r]\right\}u_r = g_r \qquad (5.3)$$

Putting (4.37) into equation (5.3) we have

$$\left(1+\frac{2}{c^2}\right)^{-\frac{1}{2}}\left[\ddot{r}-\frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}f_{,1}(\dot{r})^2 - r\left(1+\frac{2}{c^2}f\right)(\dot{\theta})^2 - r\sin^2\theta\left(1+\frac{2}{c^2}f\right)(\dot{\phi})^2\right]$$

$$+\frac{1}{(m_I)_r}\left\{\frac{d}{d\tau}[(m_I)_r]\right\}\left(1+\frac{2}{c^2}\right)^{-\frac{1}{2}}\dot{r} = g_r \qquad (5.4)$$

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Riemannian acceleration due to gravity is given by

$$a_\theta + \frac{1}{(m_I)_\theta}\left\{\frac{d}{d\tau}[(m_I)_\theta]\right\}u_\theta = g_\theta \qquad (5.5)$$

Putting equation (4.38) into equation (5.5) we get

$$\left(1+\frac{2}{c^2}f\right)^{-\frac{1}{2}} r\left[\ddot{\theta}+\frac{2}{r}(\dot{r}\dot{\theta}) - \sin\theta\cos\theta\left(\dot{\phi}\right)^2\right] + \frac{1}{(m_I)_\theta}\left\{\frac{d}{d\tau}[(m_I)_\theta]\right\}\left(1+\frac{2}{c^2}f\right)^{-\frac{1}{2}}\dot{\theta} = g_\theta \quad (5.6)$$

Riemannian acceleration due to gravity is given by

$$a_\phi + \frac{1}{(m_I)_\phi}\left\{\frac{d}{d\tau}[(m_I)_\phi]\right\}u_\phi = g_\phi \qquad (5.7)$$

Putting equation (4.39) into equation (5.7) we get

$$\left(1+\frac{2}{c^2}f\right)^{-\frac{1}{2}} r\sin\theta\left[\ddot{\phi}+\frac{2}{r}(\dot{r}\dot{\theta}) + 2\cot\theta\left(\dot{\theta}\dot{\phi}\right)\right] + \frac{1}{(m_I)_\phi}\left\{\frac{d}{d\tau}[(m_I)_\phi]\right\} r\sin\theta\left(1+\frac{2}{c^2}f\right)^{-\frac{1}{2}}\dot{\phi} = g_\phi \quad (5.8)$$

Riemannian acceleration due to gravity is given by

$$a_0 + \frac{1}{(m_I)_0}\left\{\frac{d}{d\tau}[(m_I)_0]\right\}u_0 = g_0 \qquad (5.9)$$

Putting equation (4.40) into equation (5.9) we have

$$\left(1+\frac{2}{c^2}f\right) i\left[c\ddot{t}+\frac{2}{c}\left(1+\frac{2}{c^2}\right)^{-1} f_{,1}\right] + \frac{1}{(m_I)_0}\left\{\frac{d}{d\tau}[(m_I)_0]\right\}\left(1+\frac{2}{c^2}f\right)^{-\frac{1}{2}} ic t = g_0 \qquad (5.10)$$

Hence equation (5.4), (5.6), (5.8) and (5.10) are super general planetary equations for the components of r, θ, ς

Using equation (3.2) above we get

$$(m_I)_H = (m_I)_P = \left[1-\frac{u^2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}\right]^{\frac{1}{2}}\left(1+\frac{2}{c^2}f\right)^{\frac{1}{2}} m_0$$

$$(m_I)_H = \left[1-\frac{u^2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}\right]^{\frac{1}{2}}\left(1+\frac{2}{c^2}f\right)^{\frac{1}{2}} m_0 \qquad (5.11)$$

Putting equations (5.11) into the equation (5.4), (5.6), (5.8) and (5.10) we get,

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$$\left(1+\frac{2}{c^2}\right)^{-\frac{1}{2}}\left[\ddot{r}-\frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,1}\,(\dot{r})^2 - r\left(1+\frac{2}{c^2}f\right)\left(\dot{\theta}\right)^2 - r\left(1+\frac{2}{c^2}f\right)\sin^2\theta\left(1+\frac{2}{c^2}f\right)\left(\dot{\phi}\right)^2\right]$$

$$+\left(1+\frac{2}{c^2}\right)^{-\frac{1}{2}}\left[\frac{1}{\left[1-\frac{u^2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}\right]^{\frac{1}{2}}}\left\{\frac{d}{d\tau}\left[1-\frac{u^2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}\right]^{\frac{1}{2}}\left(1+\frac{2}{c^2}\right)^{-\frac{1}{2}}\right\}\dot{r}\right] \equiv g_r \quad (5.12)$$

Equation (5.12) above give rise to super general radial gravitational intensity (or acceleration due to gravity) based upon the golden metric tensor.
Hence equation (5.6) becomes

$$\left(1+\frac{2}{c^2}\right)^{-\frac{1}{2}} r\left[\ddot{\theta}+\frac{2}{r}\left(\dot{r}\dot{\theta}\right)-\sin\theta\cos\theta\left(\dot{\phi}\right)\right]$$

$$+\left(1+\frac{2}{c^2}\right)^{-\frac{1}{2}}\left[\frac{1}{\left[1-\frac{u^2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}\right]^{\frac{1}{2}}}\left\{\frac{d}{d\tau}\left[1-\frac{u^2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}\right]^{\frac{1}{2}}\left(1+\frac{2}{c^2}\right)^{-\frac{1}{2}}\right\}r\dot{\theta}\right] \equiv g_\theta \, (5.13)$$

Equation (5.12) above give rise to general gravitational intensity (or acceleration due to gravity) based upon the golden metric tensor.
Equation (5.8) becomes

$$\left(1+\frac{2}{c^2}\right)^{-\frac{1}{2}} r\sin\theta\left[\ddot{\phi}+\frac{2}{r}\left(\dot{r}\dot{\phi}\right)+2\cot\theta\left(\dot{\theta}\dot{\phi}\right)\right]$$

$$+\left(1+\frac{2}{c^2}\right)^{-\frac{1}{2}}\left[\frac{1}{\left[1-\frac{u^2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}\right]^{\frac{1}{2}}}\left\{\frac{d}{d\tau}\left[1-\frac{u^2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}\right]^{\frac{1}{2}}\left(1+\frac{2}{c^2}\right)^{-\frac{1}{2}}\right\}r\sin\theta\,\dot{\phi}\right] \equiv g_\phi \, (5.14)$$

Equation (5.16) above give rise to gravitational intensity (or acceleration due to gravity) based upon the golden metric tensor.
Equation (5.10) reduced to

$$\left(1+\frac{2}{c^2}f\right)i\left[c\ddot{t}+\frac{2}{c}\left(1+\frac{2}{c^2}\right)^{-1} f_{,1}\right]$$

$$+\left(1+\frac{2}{c^2}\right)^{-\frac{1}{2}}\left[\frac{1}{\left[1-\frac{u^2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}\right]^{\frac{1}{2}}}\left\{\frac{d}{d\tau}\left[1-\frac{u^2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}\right]^{\frac{1}{2}}\left(1+\frac{2}{c^2}\right)^{-\frac{1}{2}}\right\}ic\dot{t}\right] \equiv g_0 \qquad (5.15)$$

Equation (5.15) above give rise to gravitational intensity (or acceleration due to gravity) based upon the golden metric tensor. Hence equation (5.12), (5.13), (5.14) and (5.15) are referred to as general gravitational intensity (or acceleration due to gravity) based upon the golden metric tensor in spherical polar coordinates.

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VI. GENERAL GRAVITATIONAL INTENSITY OR ACCELERATION DUE TO GRAVITY BASED UPON THE GOLDEN METRIC TENSOR IN SPHERICAL POLAR COORDINATES

$$g_r = -\,(\dot{x}^0)^2\Gamma^1_{00} \qquad (6.1)$$

$$g_\theta = -\,(\dot{x}^0)^2\Gamma^2_{00} \qquad (6.2)$$

$$g_\phi = -\,(\dot{x}^0)^2\Gamma^3_{00} \qquad (6.3)$$

$$g_0 = -\,(\dot{x}^0)^2\Gamma^0_{00} \qquad (6.4)$$

where , , , are coefficient of affine connection evaluated using the formula

$$\Gamma^\mu_{\alpha\beta} = \frac{1}{2}g^{\mu\varepsilon}\left(g_{\alpha\varepsilon,\beta} + g_{\varepsilon\beta,\alpha} + g_{\alpha\beta,\varepsilon}\right) \qquad (6.5)$$

Equations (6.1), (6.2), (6.3) and (6.4) reduce to

$$g_r = -\,(\dot{x}^0)^2\frac{1}{c^2}\left(1+\frac{1}{c^2}f\right)f_{,1} \qquad (6.6)$$

$$g_\theta = -\,(\dot{x}^0)^2\frac{1}{c^2r^2}\left(1+\frac{1}{c^2}f\right)f_{,2} \qquad (6.7)$$

$$g_\phi = -\,(\dot{x}^0)^2\frac{1}{c^2r^2\sin^2\theta}\left(1+\frac{1}{c^2}f\right)f_{,3} \qquad (6.8)$$

$$g_0 = -\,(\dot{x}^0)^2\frac{1}{c^2}\left(1+\frac{1}{c^2}f\right)^{-1}f_{,0} \qquad (6.9)$$

Where
$\dot{x}^0 = c\dot{t}$

$$(\dot{x}^0)^2 = c^2\dot{t}^2 \qquad (6.10)$$

equating equation (5.15) with equation (6.6) we get

$$\left(1+\frac{2}{c^2}\right)^{-\frac{1}{2}}\left[\ddot{r} - \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}f_{,1}(\dot{r})^2 - r\left(1+\frac{2}{c^2}f\right)(\theta)^2 - r\left(1+\frac{2}{c^2}f\right)\sin^2\theta\left(1+\frac{2}{c^2}f\right)(\dot{\phi})^2\right]$$

$$+\left(1+\frac{2}{c^2}\right)^{-\frac{1}{2}}\left[1-\frac{u^2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}\right]^{\frac{1}{2}}\left\{\frac{d}{d\tau}\left[1-\frac{u^2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}\right]^{\frac{1}{2}}\left(1+\frac{2}{c^2}\right)^{-\frac{1}{2}}\right\}\dot{r}$$

$$= -\dot{t}^2\frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)f_{,1} \qquad (6.11)$$

Equating equation (5.16) with equation (6.7) we get

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$$\left(1+\frac{2}{c^2}\right)^{-\frac{1}{2}} r\left[\ddot\theta+\frac{2}{r}(\dot r\dot\theta)-\sin\theta\cos\theta\,(\dot\phi)\right]$$

$$+\left(1+\frac{2}{c^2}\right)^{-\frac{1}{2}}\left[\left[1-\frac{u^2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}\right]^{\frac{1}{2}}\left\{\frac{d}{d\tau}\left[1-\frac{u^2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}\right]^{\frac{1}{2}}\left(1+\frac{2}{c^2}\right)^{-\frac{1}{2}}\right\}\dot r\dot\theta\right]$$

$$=-t^2\frac{1}{r^2}\left(1+\frac{2}{c^2}f\right)f_{,2} \qquad (6.12)$$

equating equation (5.17) with equation (6.8) we get

$$\left(1+\frac{2}{c^2}\right)^{-\frac{1}{2}} r\sin\theta\left[\ddot\phi+\frac{2}{r}(\dot r\dot\phi)+2\cot\theta\,(\dot\theta\dot\phi)\right]$$

$$+\left(1+\frac{2}{c^2}\right)^{-\frac{1}{2}}\left[\left[1-\frac{u^2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}\right]^{\frac{1}{2}}\left\{\frac{d}{d\tau}\left[1-\frac{u^2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}\right]^{\frac{1}{2}}\left(1+\frac{2}{c^2}\right)^{-\frac{1}{2}}\right\}r\sin\theta\,\dot\phi\right]$$

$$=-t^2\frac{1}{r^2\sin^2\theta}\left(1+\frac{2}{c^2}f\right)f_{,3} \qquad (6.13)$$

equating equation (5.18) with equation (6.9) we get

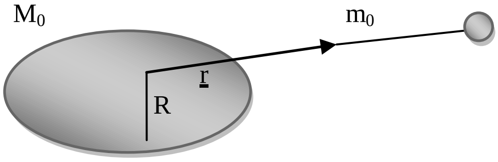
$$\left(1+\frac{2}{c^2}f\right)i\left[c\ddot t+\frac{2}{c}\left(1+\frac{2}{c^2}\right)^{-1}f_{,1}\right]$$

$$+\left(1+\frac{2}{c^2}\right)^{-\frac{1}{2}}\left[\left[1-\frac{u^2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}\right]^{\frac{1}{2}}\left\{\frac{d}{d\tau}\left[1-\frac{u^2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}\right]^{\frac{1}{2}}\left(1+\frac{2}{c^2}\right)^{-\frac{1}{2}}\right\}ic\dot t\right]$$

$$=-t^2\left(1+\frac{2}{c^2}f\right)^{-1}f_{,0} \qquad (6.14)$$

VII. APPLICATION TO PLANETARY THEORY

Consider a planet of rest mass moving in the gravitational field exterior to the sun which is assumed to be a homogeneous spherical body of radius R and rest mass , shown in the diagram below



In the first place let us assume that the gravitational field is that of Newton:

$$f(r,\theta,\phi,x^0)=-\frac{GM_0}{r}$$

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$$= -\frac{K}{r} \quad (7.1)$$

Where I

Where is the universal gravitational constant?

$$f_{,1} = \frac{\partial}{\partial r} f = \frac{K}{r^2} \quad (7.2)$$

$$f_{,2} = 0 \quad (7.3)$$

$$f_{,3} = 0 \quad (7.4)$$

$$f_{,0} = 0 \quad (7.5)$$

$$u^2 = \dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin^2\theta\dot{\phi}^2 \quad (7.6)$$

$$\frac{u^2}{c^2}\left(1 + \frac{2}{c^2}f\right)^{-1} = \frac{1}{c^2}\left(\dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin^2\theta\dot{\phi}^2\right) \times \left(1 - \frac{2K}{c^2 r}\right)^{-1} \quad (7.7)$$

$$\dot{t} = \left[1 - \frac{u^2}{c^2}\left(1 + \frac{2}{c^2}f\right)^{-1}\right]^{\frac{1}{2}}\left(1 + \frac{2}{c^2}f\right)^{-\frac{1}{2}} \quad (7.8)$$

Then super general geodesic equation of motion become: hence equation (6.11) becomes

$$\left(1 + \frac{K}{c^2 r}\right)\left[\ddot{r} - \left(\frac{K}{c^2 r^2} + \frac{2K^2}{c^4 r^3}\right)(\dot{r})^2 - r\left(1 - \frac{2K}{c^2 r}\right)(\dot{\theta})^2 - r\left(1 - \frac{2K}{c^2 r}\right)\sin^2\theta(\dot{\phi})^2\right]$$

$$+\left[\dot{t}\dot{r}\left\{\frac{d}{d\tau}\left[\frac{1}{\dot{t}}\right]\right\}\right]\frac{1}{c^2} = -\dot{t}^2\left(\frac{K}{r^2} - \frac{2K^2}{c^2 r^3}\right) \quad (7.9)$$

or

$$\left(1 + \frac{K}{c^2 r}\right)\left[\ddot{r} - \left(\frac{K}{c^2 r^2}\right)(\dot{r})^2 - r\left(1 - \frac{2K}{c^2 r}\right)(\dot{\theta})^2 - r\left(1 - \frac{2K}{c^2 r}\right)\sin^2\theta(\dot{\phi})^2\right]$$

$$-\frac{1}{c^2}\left[\dot{r}\dot{t}^{-1}\right] = -\dot{t}^2\left(\frac{K}{r^2} - \frac{2K^2}{c^2 r^3}\right) \quad (7.10)$$

Equation (6.12) becomes

$$\left(1 - \frac{2K}{c^2 r}\right)^{-\frac{1}{2}} r\left[\ddot{\theta} + \frac{2}{r}(\dot{r}\dot{\theta}) - \sin\theta\cos\theta\,(\dot{\phi})\right]$$

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$$+\left(1-\frac{2K}{c^2r}\right)^{-\frac{1}{2}}\left[t\left(1-\frac{2K}{c^2r}\right)^{\frac{1}{2}}\left\{\frac{d}{d\tau}\left[\frac{1}{t}\right]\right\}\dot{r}\dot{\theta}\right]=-t^2\frac{1}{r^2}\left(1-\frac{2K}{c^2r}\right)f_{,2} \qquad (7.11)$$

or

$$\left(1-\frac{2K}{c^2r}\right)r\left[\ddot{\theta}+\frac{2}{r}(\dot{r}\dot{\theta})-\sin\theta\cos\theta\,(\dot{\phi})\right]-\left(\frac{\dot{r}\dot{\theta}}{t}\right)\frac{1}{c^2}\equiv 0 \qquad (7.12)$$

Equation (6.13) becomes

$$\left(1-\frac{2K}{c^2r}\right)^{-\frac{1}{2}} r\sin\theta\left[\ddot{\phi}+\frac{2}{r}(\dot{r}\dot{\phi})+2\cot\theta\,(\dot{\theta}\dot{\phi})\right]$$

$$+\left(1-\frac{2K}{c^2r}\right)^{-\frac{1}{2}}\left[t\left(1-\frac{2K}{c^2r}\right)^{\frac{1}{2}}\left\{\frac{d}{d\tau}\left[\frac{1}{t}\right]r\sin\theta\,\dot{\phi}\right\}x\frac{1}{c^2}\right]=-t^2\frac{1}{r^2\sin^2\theta}\left(1-\frac{2K}{c^2r}\right)f_{,2} \qquad (7.13)$$

$$\left(1+\frac{2K}{c^2r}\right)r\sin\theta\left[\ddot{\phi}+\frac{2}{r}(\dot{r}\dot{\phi})+2\cot\theta\,(\dot{\theta}\dot{\phi})\right]-\frac{1}{c^2}\left[\frac{1}{t}\dot{\phi}\right]r\sin\theta=0 \qquad (7.14)$$

Note that planetary equation do not contain that is time parameter or in the case parameter, also the same equation that planetary equation contains no polar angle , but retained only azimuthal angle , radial portion of the equation also retained. Based on this argument equation (6.14) and equation (7.11) vanishes only equation (7.10) and equation (7.14) remain for further evaluations. Equation (7.10) reduce to

$$\left(1+\frac{K}{c^2r}\right)\left[\ddot{r}-\left(\frac{K}{c^2r^2}\right)(\dot{r})^2-r\left(1-\frac{2K}{c^2r}\right)(\dot{\theta})^2-r\left(1-\frac{2K}{c^2r}\right)\sin^2\theta(\dot{\phi})^2\right]$$

$$-\frac{1}{c^2}\left[\left(1+\frac{u^2}{2c^2}-\frac{K}{c^2r}\right)(\dot{r})\right]$$

$$=-\frac{K}{r^2}+\frac{u^2K}{c^2r^2} \qquad (7.15)$$

Equation (7.14) becomes

$$\left(1+\frac{K}{c^2r}\right)\left[r\sin\theta\,\ddot{\phi}+2\sin\theta\,(\dot{r}\dot{\phi})+2r\cot\theta\sin\theta\,(\dot{\theta}\dot{\phi})\right]$$

$$-\left[\frac{1}{c^2}\left(1+\frac{u^2}{2c^2}-\frac{K}{c^2r}\right)r\sin\theta\dot{\phi}\right]=0 \qquad (7.16)$$

For motion along the equatorial plane

$$\theta=\frac{a}{2} \quad \dot{\theta}=0 \quad \ddot{\theta}=0 \quad \sin\frac{\pi}{2}=1$$

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$$\ddot{r} = -\frac{K}{r^2} + \frac{K^2}{c^2 r^3} + \frac{2K}{c^2 r^2}(\dot{r})^2 + \frac{1}{c^2} + \frac{l^2}{r^3} - \frac{3Kl^2}{c^2 r^4}$$

$$\ddot{r} = -Ku^2 + \frac{K^2 u^2}{c^2} + \frac{2Ku^2}{c^2}(\dot{r})^2 + \frac{1}{c^2}(\dot{r}) + l^2 u^3 - \frac{3Kl^2 u^2}{c^2} \qquad (7.17)$$

Note that

$$u = \frac{1}{r}$$

For motion along the equatorial plane

$$\theta = \frac{a}{2} \quad \dot{\theta} = 0 \quad \ddot{\theta} = 0 \quad \sin\frac{\pi}{2} = 1$$

Equation (7.16) become

$$\ddot{\phi} + \frac{2}{r}(\dot{r}\dot{\phi}) = \frac{1}{c^2}\dot{\phi} \qquad (7.18)$$

Solution of equation (7.18) above gives

$$\ddot{\phi} + \frac{2\dot{r}}{r}\dot{\phi} \approx 0$$

First approximation

Where is a constant of the motion? Hence the radial equation becomes

VIII. SOLUTION OF RADIAL EQUATION OF MOTION

Solution of radial equation of motion, equation (1.17) above becomes

let $$\dot{\phi} = \frac{l}{r^2} \quad \Rightarrow \quad \dot{\phi}^2 = \frac{l^2}{r^4}$$

$$\ddot{r} = -\frac{K}{r^2} + \frac{K^2}{c^2 r^3} + \frac{2K}{c^2 r^2}(\dot{r})^2 + \frac{1}{c^2} + \frac{l^2}{r^3} - \frac{3Kl^2}{c^2 r^4} \qquad (8.1)$$

let $$u(\phi) = \frac{1}{r(\phi)}$$

$$\dot{r} = \frac{dr}{d\tau} = \frac{dr}{d\phi}\cdot\frac{d\phi}{d\tau} = \dot{\phi}\frac{dr}{d\phi} = \dot{\phi}\frac{dr}{du}\frac{du}{d\phi} = \dot{\phi}\left(-\frac{1}{u^2}\right)\frac{du}{d\phi}$$

$$\dot{r} = \frac{l}{r^2}\left(-\frac{1}{u^2}\right)\frac{du}{d\phi} = lu^2\left(-\frac{1}{u^2}\right)\frac{du}{d\phi} = -l\frac{du}{d\phi} \qquad (8.2)$$

Hence

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$$\ddot{r} = -lu^2 \frac{d}{d\phi}\left(l\frac{du}{d\phi}\right) = -l^2u^2\frac{du^2}{d\phi^2} \qquad (8.3)$$

Putting into equation (8.1) we get

$$-l^2u^2\frac{du^2}{d\phi^2} = -Ku^2 + \frac{K^2u^2}{c^2} + \frac{2Ku^2}{c^2}(\dot{r})^2 + \frac{1}{c^2}(\dot{r}) + l^2u^2 - \frac{3Kl^2u^2}{c^2} \qquad (8.4)$$

Finally obtain

$$\Rightarrow \frac{d^2u}{d\phi^2} + u = \frac{K}{l^2} + \frac{3Ku^2}{c^2} - \frac{K^2u}{c^2l^2} + \frac{2K}{c^2}\left(\frac{du}{d\phi}\right)^2 + \frac{1}{u^2c^2l}\left(\frac{du}{d\phi}\right) \qquad (8.5)$$

By neglecting the last two terms or where contribution for the mean time

$$\frac{d^2u}{d\phi^2} + u = \frac{K}{l^2} \Rightarrow \text{Newton's planetary equation} \qquad (8.6)$$

$$\frac{d^2u}{d\phi^2} + u = \frac{K}{l^2} + \frac{3Ku^2}{c^2} \Rightarrow \text{Einstein's palnetary equation} \qquad (8.7)$$

The new equation obtained in this work gives two sets of equation

$$\frac{d^2u}{d\phi^2} + \left(1 + \frac{k^2}{c^2l^2}\right)u = \frac{K}{l^2} \qquad (8.8)$$

$\Rightarrow$ which give rise to post Netwton's planetary equation and

$$\frac{d^2u}{d\phi^2} + \left(1 + \frac{k^2}{c^2l^2}\right)u = \frac{K}{l^2} + \frac{3Ku^2}{c^2} \qquad (8.9)$$

$\Rightarrow$ which give rise to post Einstient's planetary equation.

IX. SOLUTION OF NEWTON'S PLANETARY EQUATION

Now consider Newton's part of equation (8.5) given by

$$\frac{d^2u}{d\phi^2} + u = \frac{K}{l^2} \qquad (9.1)$$

Complementary solution of (9.1) is

$$u_c(\phi) = A\cos(\phi + \alpha) \text{ or } B\sin(\phi + \alpha) \qquad (9.2)$$

Particular solution

$$u_p(\phi) = \frac{K}{l^2} \qquad (9.3)$$

Thus general solution

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$$u_c(\phi) + u_p(\phi) \qquad (9.4)$$

or

$$u(\phi) = \frac{K}{l^2} + A\cos(\phi + \alpha) \qquad (9.5)$$

or

$$r(\phi) = \frac{1}{\frac{K}{l^2} + A\cos(\phi + \alpha)}$$

$$= \frac{1}{\frac{K}{l^2}\{1 + \varepsilon\cos(\phi + \alpha)\}}$$

$$= \frac{\frac{l^2}{K}}{[1 + \varepsilon\cos(\phi + \alpha)]} \qquad (9.6)$$

where

$$\varepsilon = \frac{Al^2}{K} \qquad (9.7)$$

being the eccentricity, equation (9.6) is Newton's solution of the planetary equation. Information contained in equation (9.6) physically corresponds to the polar equation of a conic section. The conic section is characterized by the parameter (eccentricity) as follows:

$\varepsilon > 1 \Rightarrow hyperbola, \varepsilon = 1 \Rightarrow parabola, \varepsilon = 0 \Rightarrow circle\ and\ \varepsilon < 1\ ellipse.$

According to this equation, a planet traces the same path throughout its orbit. [7].

To solve the relativistic planetary equation we use the method of successive approximation iterate which is the solution of Newton's equation.

$$r(\phi) = \frac{l^2}{K}\frac{1}{[1 + \varepsilon\cos(\phi + \alpha)]} \qquad (9.8)$$

$$u(\phi) = \frac{K}{l^2}[1 + \varepsilon\cos(\phi + \alpha)] \qquad (9.9)$$

substituting this expression for into R.H.S of equation (9.1) by considering Einstein's planetary only

we get

$$\frac{d^2u}{d\phi^2} + u - \frac{K}{l^2} = \left\{\frac{K}{l^2}[1 + \varepsilon\cos(\phi + \alpha)]\right\} \qquad (9.10)$$

This relativistic equation is solved by find the particular and complementary solutions to obtain

$$u(\phi) = \frac{K}{l^2}\left[1 + \varepsilon\cos\left(\phi + \alpha - \frac{3k^2}{c^2 l^2}\right)\right] \qquad (9.11)$$

Now let perihelion be attained first at , then the next perihelion will be attained at an angle such that

$$\left(1 - \frac{3k^2}{c^2 l^2}\right)\phi = 2\pi \qquad (9.12)$$

$(\text{phase angle } \alpha = 0)$

Or

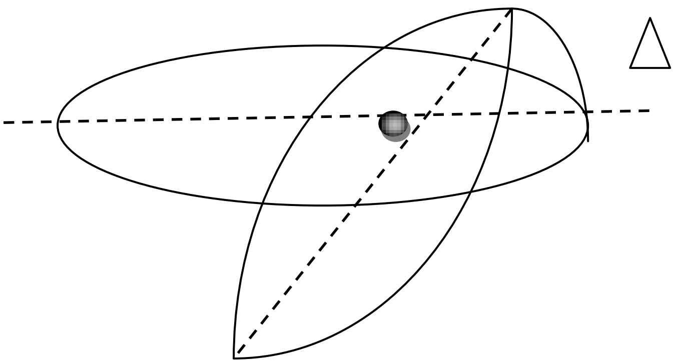
$$\phi = 2\pi\left(1 - \frac{3k^2}{c^2 l^2}\right)^{-1}$$

For small compared to 1, expanding gives

$$\phi = 2\pi\left(1 + \frac{3k^2}{c^2 l^2} + \cdots\right.$$

$$= 2\pi + \frac{6\pi k^2}{c^2 l^2} + \cdots \qquad (9.13)$$

Consequently, the perihelion of the orbit has advanced beyond that of the first orbit. Similarly, for the aphelion, and similarly for the whole orbit as shown in diagram below



Note that:
The displacement of the planetary orbit from revolution is called **PRECESION.**
Let be the angle through which the planetary orbit is displaced in each revolution, then

$$\Delta = \frac{6\pi k^2}{c^2 l^2} \ (\text{in rad per revolution}) \qquad (9.14)$$

The precession of planets in the solar system was discovered as far back as 1845 by a French man Leverrier.

X. SOLUTION OF POST NEWTON'S PLANETARY EQUATION (A FIRST APPROXIMATION)

Now consider post Newton's planetary equation or equation of the first approximation (8.5) given by

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$$\frac{d^2u}{d\phi^2} + \left(1 + \frac{k^2}{c^2 l^2}\right)u = \frac{K}{l^2} \qquad (10.1)$$

Complementary solution of (10.1) is

$$u_c(\phi) = A\cos\phi \text{ or } B\sin\phi$$

$$\frac{du_c(\phi)}{d\phi} = -A\sin\phi$$

$$\frac{d^2u_c(\phi)}{d\phi^2} = -A\cos\phi \qquad (10.2)$$

$$\frac{d^2u_c(\phi)}{d\phi^2} + \left(1 + \frac{k^2}{c^2 l^2}\right)u_c(\phi) = 0 \qquad (10.3)$$

Putting equation (10.2) into equation (10.3) we get

$$-A\cos\phi + \left(1 + \frac{k^2}{c^2 l^2}\right)A\cos\phi \qquad (10.4)$$

A is arbitrary constant.

Next is to obtain particular solution of equation (10.1)

Let $u_p(\phi) = 0 = D = \text{particular solution of equatio}$

$$\frac{d^2u}{d\phi^2} = 0$$

$$0 + \left(1 + \frac{k^2}{c^2 l^2}\right)D = \frac{K}{l^2}$$

Or

$$D = \frac{K}{l^2}\left(1 + \frac{k^2}{c^2 l^2}\right)^{-1} \qquad (10.5)$$

Complete solution is the sum of complementary solution and particular solution

$$u(\phi) = u_c(\phi) + u_p(\phi)$$

$$u(\phi) = A\cos\phi + D$$

$$A\cos\phi + = \frac{K}{l^2}\left(1 + \frac{k^2}{c^2 l^2}\right)^{-1} \qquad (10.6)$$

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$$r(\phi) = \frac{1}{u(\phi)}$$

$$= \frac{K}{l^2}\left(1 + \frac{k^2}{c^2 l^2}\right)^{-1} A\cos\phi$$

Or

$$r(\phi) = \frac{\left(1 + \frac{k^2}{c^2 l^2}\right)\frac{l^2}{K}}{1 + \frac{Al^2}{K}\left(1 + \frac{k^2}{c^2 l^2}\right)\cos\phi} \qquad (10.7)$$

With

$$A = \frac{l^2}{K}\left(1 + \frac{k^2}{c^2 l^2}\right) \qquad (10.8)$$

and

$$\varepsilon = \frac{Al^2}{K}\left(1 + \frac{k^2}{c^2 l^2}\right)$$

$$= \varepsilon_0\left(1 + \frac{k^2}{c^2 l^2}\right) \qquad (10.9)$$

Where

ε_0 = pure Newtonian eccentricity

$$\varepsilon_0 = \frac{Al^2}{K} \qquad (10.10)$$

But equation (10.7) reduce to

$$r(\phi) = \frac{A}{1 + \varepsilon\cos\phi} \qquad (10.11)$$

$$A = \frac{l^2}{K}\left(1 + \frac{k^2}{c^2 l^2}\right)$$

$$A = A_0\left(1 + \frac{k^2}{c^2 l^2}\right) \qquad (10.12)$$

With

$K = GM$

Where

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$$A_0 = \frac{l^2}{K} \qquad (10.13)$$

Is pure aphelion distance from sun angular momentum?

As a result of first approximation we have two different results of eccentricity and aphelion distance from the sun. Our new eccentricity gives rise to $\varepsilon = \varepsilon_0 ($ where is pure Newtonian eccentricity and our new aphelion distance from the sun give rise to $A = A_0 ($ where is pure aphelion distance from the sun. The two results obtained above differ from those obtained from Newton planetary equation by a factor of $($ where , G being the universal gravitational constant and M is the mass. The results obtained in this paper would be of paramount important in the astronomy and other related discipline. These change occurred as a result of using golden metric tensor.

XI. CONCLUSION AND RESULTS

In this paper we have succeeded in deriving equations [(3.10)-(3.13)] which referred to as velocity vectors equations based upon the golden metric tensors. We further obtained equations [(4.16)-(4.19)] which referred to as linear acceleration vector equations. Also in this paper concerted effort were made in order to obtained equation (5.15), (5.16), (5.17) and (5.18) are referred to as general gravitational intensity (or acceleration due to gravity) based upon the golden metric tensor in spherical polar coordinates. The results obtained in this paper are now available for both physics and mathematician alike to apply them in solving planetary problems based upon Riemannian geometry. More effort and time are being devoted in order to offer solution to equations (5.15-(5.18) obtained in this paper. Also in this paper we solved Newton's planetary equation and the results were obtained. We further solved post Newton's planetary equations (A first approximation) and the results were obtained. The results obtained from post Newton's planetary equations paved the way for mathematician and physics to search for more information in the planetary phenomena.

Appendix A

Expansion of , t^2 and t^{-1} to ord.

We get

$$t = \left[1 - \frac{u^2}{c^2}\left(1 + \frac{2}{c^2}f\right)^{-1}\right]^{\frac{1}{2}}\left(1 + \frac{2}{c^2}f\right)^{-\frac{1}{2}}$$

$$\approx \left[1 - \frac{u^2}{c^2}\left(1 + \frac{2}{c^2}f\right)^{-1} + \cdots\right]\left(1 - \frac{1}{c^2}f\ldots\right)$$

$$\approx 1 - \frac{1}{c^2}f - \frac{u^2}{2c^2} \qquad (1)$$

But

$$t^2 = \left[1 - \frac{u^2}{c^2}\left(1 + \frac{2}{c^2}f\right)^{-1}\right]\left(1 + \frac{2}{c^2}f\right)^{-1}$$

$$\approx \left[1 - \frac{u^2}{c^2}\left(1 - \frac{2}{c^2}f\ldots\right)\right]\left(1 - \frac{2}{c^2}f\ldots\right)$$

$$\approx \left[\left(1-\frac{u^2}{c^2}\dots\right)\left(1-\frac{2}{c^2}f\right)\right]$$

$$t^2 \approx \left[1-\frac{u^2}{c^2}-\frac{2}{c^2}f+\frac{2u^2}{c^4}f\dots\right]$$

$$\approx \left[1-\frac{u^2}{c^2}+\frac{2K}{c^2r}\dots\right] \qquad (2)$$

Next

$$\frac{1}{t}=(t)^{-1}$$

$$=\left[1-\frac{u^2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}\right]^{\frac{1}{2}}\left(1+\frac{2}{c^2}f\right)^{-\frac{1}{2}}$$

$$\approx \left[1+\frac{u^2}{2c^2}\left(1-\frac{2}{c^2}f\dots\right)\right]\left(1+\frac{2}{c^2}f\dots\right)$$

$$\approx \left[\left(1+\frac{u^2}{2c^2}\dots\right)\right]\left[\left(1+\frac{2}{c^2}f\dots\right)\right]$$

$$(t)^{-1}\approx 1+\frac{u^2}{2c^2}+\frac{1}{c^2}f+\cdots$$

$$\approx 1+\frac{u^2}{2c^2}+\frac{K}{c^2r} \qquad (3)$$

Appendix B

$$\left(1-\frac{2K}{c^2r}\right)^{-\frac{1}{2}}\approx\left(1+\frac{2K}{c^2r}\right)$$

$$\left(1-\frac{2K}{c^2r}\right)^{-\frac{1}{2}}\approx\left(1+\frac{2K}{c^2r}\right)$$

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