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Analysis of planar and spatial tensegrity structures with redundancies, by implementing a Comprehensive Equilibrium Equations Method with Force Densities

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Abstract— ***A general approach is presented to analyze tensegrity structures by examining their equilibrium. It belongs to the class of equilibrium equations methods with force densities. The redundancies are treated by employing Castigliano's second theorem, which gives the additional required equations. The partial derivatives, which appear in the additional equations, are numerically replaced by statically acceptable internal forces which are applied on the structure. For both statically determinate and indeterminate tensegrity structures, the properties of the resulting linear system of equations give an indication about structural stability. This method requires a relatively small number of computations, it is direct (there is no iteration procedure and calculation of auxiliary parameters) and is characterized by its simplicity. It is tested on both 2-D and 3-D tensegrity structures. Results obtained with the method compare favorably with those obtained by the Dynamic Relaxation Method or the Adaptive Force Density Method.***

Index Terms—**Equilibrium equations; force density; force density matrix; redundancy; Castigliano's second theorem; 2-D and 3-D tensegrity structures.**

I. INTRODUCTION

Cable networks [1-2] and tensegrity structures [3] are different from conventional structures, such as spatial steel frames or space steel trusses, in that they are lightweight structures with members which transmit only tension (cables and strings) or elements which transmit compression (bars before buckling). In this article we are studying only the behavior of tensegrity structures. These structures are usually defined as planar or spatial trusses with a discontinuous set of members under compression, inside a continuous network of members under tension. The word tensegrity is an artificial word and it combines the words "tension" and "integrity". This word was coined several decades ago. Professor B. Fuller, in the United States, was essentially involved in the invention of this technical word. In one of his last books, Fuller described the compression members as "islands of compression in a sea of tension" [3]. Using the same concept Emmerich [4] presented, in France, in 1963, his own tensegrity patent. Snelson, one of Fuller's students, describes this type of structures as "continuous tension and discontinuous compression structures" [5].

Pretension, applied by means of tension members, plays an essential role in the structural behavior of the tensegrities. For the design of such structures their stability is investigated under both static and dynamic loads. During the last decades many methods have been proposed for the analysis of the tensegrities. One of the most important methods is the Dynamic Relaxation Method (DRM) which was used in this research for checking the numerical results obtained with the numerical scheme proposed in the present work. DRM is one of the classical techniques. It belongs to the family of methods under the title of three-term recursive formulae. It is an iterative procedure which is based on the fact that a system undergoing damped vibration, excited by a constant force, ultimately comes to rest in the displaced position of static equilibrium, obtained under the action of the constant force. One of the numerous first papers, written by pioneers of this method, is that of Papadrakakis [6] which proposes an automatic procedure for the evaluation of the iteration parameters and it is mentioned here (without underestimating the importance of other works in this domain) as an example of an article which presents in a strict,

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clear and academic manner the way of implementing this numerical procedure. Recent research, performed in the domain of tensegrities, has given some new important techniques. However, a general review of the older and of the recent numerical schemes, developed in this area, is out of the scope of the present paper. Juan and Mirats Tur present an excellent review in their work [7] of the basic issues about the statics of tensegrity structures. Among the new methods, presented in literature during the last few years, is that of Zhang and Ohsaki [8] which is also considered in the present research for comparison purposes. Their method is an Adaptive Force Density Method (AFDM). It first finds a set of axial forces compatible with a given structure and then estimates the corresponding nodal coordinates under equilibrium conditions and constraints.

The technique implemented in the present work is a force density method which belongs to the class of the Equilibrium Equations Methods with Force Densities (EEMFD). With this method the system of equilibrium equations is created by considering the equilibrium of forces in all the joints. The redundancies are treated by employing Castigliano's second theorem which gives the additional equations required to have a solution [9]. The partial derivatives, which appear in the additional equations, are numerically replaced by statically acceptable internal forces acting along the members. Also, the properties of the matrix of the system of linear equations are exploited because they give a strong indication of the stability of the structure. The assumptions adopted in the present research are the following:

1) Joints are frictionless but their mass is considered in calculations unless otherwise specified.
2) The self-weight of a member (for structures within earth's gravity field) is not neglected unless otherwise specified. It is equally distributed at its ends.
3) Live loads and pretensions on members are transferred to the joints.
4) Displacements on the joints and deformations of the members of the structure are relatively small compared with the dimensions of the structure.
5) The axial force carried by a member is constant along its length.
6) The materials used for all members obey to Hooke's law for loadings below the yield stress.

The efficiency of this technique is based on its simplicity and the small number of calculations required. Thus, the objective is: (i) to present the general idea and formulation of this numerical scheme and (ii) to test the method in planar and spatial tensegrity structures with redundancies and of any type of complexity and to compare it with the DRM or the AFDM.

The outline of the rest of this paper is as follows: in section II we develop the formulation of a Comprehensive Equilibrium Equations Method with Force Densities (CEEMFD) and in section III we discuss the treatment of redundancies and we investigate the stability in this type of structures. In section IV some applications of the method are presented on planar and on spatial tensegrity structures.

II. A COMPREHENSIVE EQUILIBRIUM EQUATIONS METHOD WITH FORCE DENSITIES

A. *Equilibrium of an unconstrained node*

An important concept related with tensegrity structures is the "force density" [1-2]. For a member made of a material obeying to Hooke's Law, with ends at *i* and *j* and with a length $L_{i,j}$ and a longitudinal force $F_{i,j}$, the force density is defined as follows:

$$q_{i,j} = \frac{F_{i,j}}{L_{i,j}} = (EA)_{i,j} \frac{\varepsilon_{i,j}}{L_{i,j}} \qquad (1)$$

where $\varepsilon_{i,j}$ is the strain, *E* is the Young modulus [9] and *A* is the effective cross sectional area of the member. This definition will be useful in the analysis which is presented herewith. Force density has a negative sign in compression and a positive sign in tension.

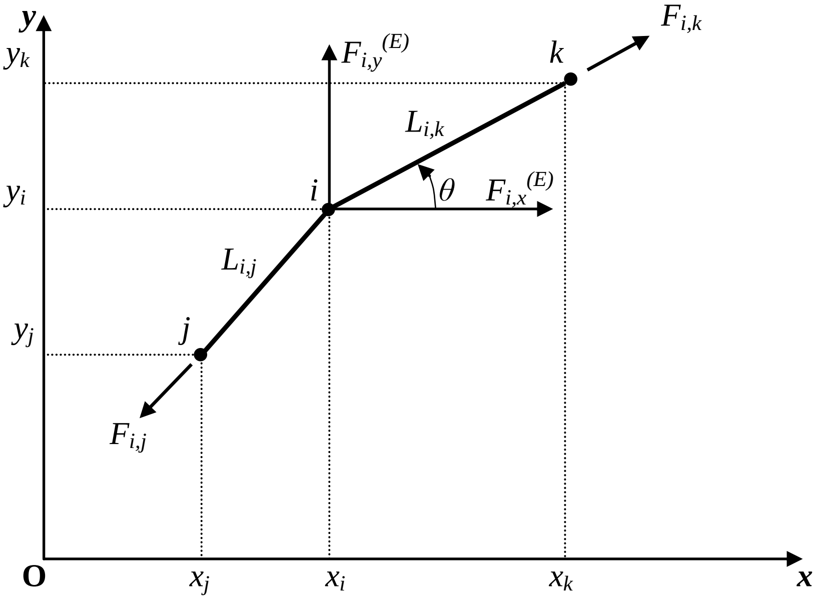

Fig. 1 Equilibrium of an unconstrained node of a 2-D tensegrity structure.

Figure 1 is used to exemplify the equilibrium of a typical unconstrained node *i*, on the *Oxy* plane, which is connected to joints *j* and *k*, through members which have lengths L_{ij} and L_{ik}. For the three-dimensional orthogonal Cartesian coordinate system *Oxyz*, where N_m joints are connected with joint *i*, through N_m members, the equilibrium equations, in the direction of the axes *Ox*, *Oy* and *Oz* are given by:

$$\sum_{j=1}^{N_m} \frac{x_i - x_j}{L_{i,j}} F_{i,j} = \sum_{j=1}^{N_m} (x_i - x_j) q_{i,j} = \sum_{j=1}^{N_m} q_{j,i} x_i - \sum_{j=1}^{N_m} q_{i,j} x_j = F_{i,x}^{(E)}, \quad (2)$$

$$\sum_{j=1}^{N_m} \frac{y_i - y_j}{L_{i,j}} F_{i,j} = \sum_{j=1}^{N_m} (y_i - y_j) q_{i,j} = \sum_{j=1}^{N_m} q_{j,i} y_i - \sum_{j=1}^{N_m} q_{i,j} y_j = F_{i,y}^{(E)}, \quad (3)$$

$$\sum_{j=1}^{N_m} \frac{z_i - z_j}{L_{i,j}} F_{i,j} = \sum_{j=1}^{N_m} (z_i - z_j) q_{i,j} = \sum_{j=1}^{N_m} q_{j,i} z_i - \sum_{j=1}^{N_m} q_{i,j} z_j = F_{i,z}^{(E)}, \quad (4)$$

Where i=1,…,N and N is the total number of nodes on the structure. In equations (2), (3) and (4) forces $F_{i,x}^{(E)}$, $F_{i,y}^{(E)}$ and $F_{i,z}^{(E)}$ are the external loads (Fig. 1) which include gravity loads, live loads, pretension and even reactions in the case where the node is a support. Gravity loads are assumed to act along the negative y-axis, for 2-D structures or along the negative z-axis for 3-D structures. External loads, which appear on the right hand side of equations (2), (3) and (4) have the following general expressions:

$$F_{i,x}^{(E)} = -\sum_{j=1}^{N_m} (x_i - x_j) t_{i,j} + B_{i,x} + R_{i,x}, \quad F_{i,y}^{(E)} = -\sum_{j=1}^{N_m} (y_i - y_j) t_{i,j} + B_{i,y} + R_{i,y},$$

and $$F_{i,z}^{(E)} = -G_i - \frac{1}{2} \sum_{j=1}^{N_m} w_{i,j} L_{i,j} - \sum_{j=1}^{N_m} (z_i - z_j) t_{i,j} + B_{i,z} + R_{i,z}, \quad (5)$$

Where G_i is the self-weight of joint *i* and $w_{i,j}$ is the weight per unit length of a member with nodes at *i* and *j*. Also, $t_{i,j}$ is the pretension per unit length applied on a member joining nodes *i* and *j*. Forces $B_{i,x}$, $B_{i,y}$ and $B_{i,z}$ are the concentrated live loads and $R_{i,x}$, $R_{i,y}$ and $R_{i,z}$ are the reaction forces (if they exist) on joint *i*. Both live loads and

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reaction forces are assumed to be acting along the positive directions of axes *Ox*, *Oy* and *Oz*.

B. The equilibrium equations of a statically determinate tensegrity and the force density matrix (FDM)

The equilibrium of the whole structure is considered by introducing the connectivity matrix ***P***. This matrix has elements $p_{a,c}$. Index a takes values from 1 to N_b (here N_b is the number of all members) and index c takes values from 1 to N. The elements $p_{a,c}$ of matrix ***P*** take the following values [1-2]:

$$p_{a,c} = \begin{cases} +1 & \text{if } c \text{ is the initial node of member } a \\ -1 & \text{if } c \text{ is the final node of member } a \\ 0 & \text{if } c \text{ does not belong to member } a. \end{cases} \tag{6}$$

It really doesn't matter which node of member a we consider first or last but once we consider a node c to be the initial node for the member, we also consider it as the initial node for any other member that is connected with this node.

Also, matrix ***S*** is introduced which has elements $s_{e,d}$. Index e takes values from 1 to N and index d takes values from 1 to N_s (where N_s is the number of the supports of the structure). Elements $s_{e,d}$ of matrix ***S*** are then defined as follows:

$$s_{e,d} = \begin{cases} -1 & \text{if node } e \text{ coincides with the support } d \\ 0 & \text{if node } e \text{ is a different node from support } d. \end{cases} \tag{7}$$

By introducing vectors $\boldsymbol{x_c}, \boldsymbol{y_c}$ and $\boldsymbol{z_c}$ which contain the x-coordinates, y-coordinates and z-coordinates, respectively, of all the N nodes of the structure, the set of linear equilibrium equations for all the joints of the structure can be expressed in block form as shown below:

$$A \cdot Q_f = \begin{bmatrix} P^T (P \cdot x_c)_{sq} & S & O & O \\ P^T (P \cdot y_c)_{sq} & O & S & O \\ P^T (P \cdot z_c)_{sq} & O & O & S \end{bmatrix} \cdot \begin{bmatrix} q \\ R_x \\ R_y \\ R_z \end{bmatrix} = \Psi^{(E)}, \tag{8}$$

Where vector $\boldsymbol{Q_f}$ contains all the force densities and reaction forces of the structure. For spatial tensegrities it has dimensions $(N_b+3N_s)\times 1$. Vector $\boldsymbol{q}$ contains all the unknown force densities $q_{i,j}$ which are inserted in $\boldsymbol{q}$ according to the ascending order of numbering of the elements of the structure and it has dimensions $N_b\times 1$. Vectors $\boldsymbol{R_x}$, $\boldsymbol{R_y}$ and $\boldsymbol{R_z}$ contain the reaction forces on the supports and each one of them has dimensions $N_s\times 1$. Vector $\boldsymbol{\Psi^{(E)}}$ contains all the known external loads acting in the x, y and z-directions, respectively, on all the joints of the structure (gravity loads, live loads, pretension etc.) and for 3-D tensegrities it has dimensions $3N\times 1$. Its components $\Psi_{i,x}^{(E)}$, $\Psi_{i,y}^{(E)}$ and $\Psi_{i,z}^{(E)}$ have the same expression as for $F_{i,x}^{(E)}$, $F_{i,y}^{(E)}$ and $F_{i,z}^{(E)}$ in equations (2), (3) and (4), respectively, **except** that they do not contain the reaction forces (Fig. 1). Matrix ***A*** is the global shape matrix of the tensegrity and is very sparse. This means a significantly smaller number of computations and memory space on a computer compared with the classical finite element method. For the creation of a computer code, definitions (6) and (7) and formulation (8) are easy to use.

In the case of statically determinate structures, the set of linear equilibrium equations (8) is the set of equations to solve to directly find the unknown values of the force densities $q_{i,j}$. Considering all the N joints of the structure and with the help of matrix ***P***, the set of equilibrium equations (2), (3) and (4), for the whole structure, takes the following block form :

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$$\begin{bmatrix} D & O & O \\ O & D & O \\ O & O & D \end{bmatrix} \cdot \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} = \begin{bmatrix} F_x^{(E)} \\ F_y^{(E)} \\ F_z^{(E)} \end{bmatrix} = \begin{bmatrix} (R_x)_{gl} + \Psi_x^{(E)} \\ (R_y)_{gl} + \Psi_y^{(E)} \\ (R_z)_{gl} + \Psi_z^{(E)} \end{bmatrix}, \tag{9}$$

Where $(R_x)_{gl}$, $(R_y)_{gl}$ and $(R_z)_{gl}$ are global vectors of dimensions $N\times 1$ which contain the unknown reaction forces. Also, matrix $\boldsymbol{D}$, which is known as the force density matrix (FDM) of the tensegrity, relates the nodal coordinates and the forces which are acting on the structure and is given by

$$\boldsymbol{D} = \boldsymbol{P}^T \boldsymbol{diag(q)P} . \tag{10}$$

The elements $d_{i,j}$ of matrix $\boldsymbol{D}$ are as follows [1-2]:

$$d_{i,j} = \begin{cases} -q_{i,j} & when\ i \neq j \\ \sum_{\substack{p=1 \\ (p\neq i)}}^{N_m} q_{p.j} & when\ i = j \\ 0 & if\ \ nodes\ i\ \ and\ \ j \\ & are\ not\ connected. \end{cases} \tag{11}$$

Matrix $\boldsymbol{D}$ is a Kirchoff matrix and it is a symmetric positive semi-definite matrix . It is also called discrete matrix or combinatorial Laplacian or admittance matrix and it contains elements with a positive or a negative sign [7]. It has dimensions $N\times N$.

C. General form of the equilibrium equations

A more general form of equations (9) is the following:

$$(I \otimes D) \cdot \begin{bmatrix} x_c - D^{-1}(R_x)_{gl} \\ y_c - D^{-1}(R_y)_{gl} \\ z_c - D^{-1}(R_z)_{gl} \end{bmatrix} = \begin{bmatrix} \Psi_x^{(E)} \\ \Psi_y^{(E)} \\ \Psi_z^{(E)} \end{bmatrix} = \Psi^{(E)} \tag{12}$$

where $\boldsymbol{I}$ is the 3×3 unit matrix. So, equations (8) and (12) give the following elegant and important, at the same time, relation:

$$(I \otimes D) \cdot \begin{bmatrix} x_c - D^{-1}(R_x)_{gl} \\ y_c - D^{-1}(R_y)_{gl} \\ z_c - D^{-1}(R_z)_{gl} \end{bmatrix} - A \cdot Q_f = 0 . \tag{13}$$

One may observe that equation (13) does not contain any values of the external loads except from the reaction forces on the supports.

III. REDUNDANCIES AND NUMERICAL REPRESENTATION OF CASTIGLIANO'S SECOND THEOREM

The values of the elements of matrix $\boldsymbol{A}$ in equation (8) depend directly on the values of the coordinates of the nodes. For planar problems it has dimensions $2N\times(N_b+2N_s)$ and for spatial structures it is of dimensions $3N\times(N_b+3N_s)$. For the statically determinate structures [9] and after inserting in the set of equations (8) the known values of reactions

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on the supports, matrix ***A*** is reduced to a square matrix $\boldsymbol{K_e}$ of dimensions $2N\times2N$, in two dimensions and $3N\times3N$ in three dimensions. Its properties, as a square matrix, give important information about the stability of the form of the structure under study. So, for statically determinate structures the set of linear equilibrium equations (8) has a solution if and only if it has a non-zero determinant, i.e. $\det|\boldsymbol{K_e}|\neq0$. Other equivalent conditions concerning square matrix $\boldsymbol{K_e}$, which secure the existence of a solution, are given in many references about numerical analysis (e.g. [10]). If matrix $\boldsymbol{K_e}$ has a determinant which is equal to zero then, most probably, the structure is unstable.

For the 2-D case the degree of redundancy is $N_r=N_b+N_{ur}-2N$ and for the 3-D problems it is $N_r=N_b+N_{ur}-3N$, where N_{ur} is the number of unknown reaction forces on the supports. We say that the system of linear equations has N_r parametric solutions. Thus, we need an additional number of N_r equations which, in this study, are provided by Castigliano's second theorem. According to this theorem, which is well presented in many classical books about solid mechanics (e.g. Fung [9]), all the forces in the bars or strings of the structure are expressed in terms of any N_r, in number, forces $Q^{(k)}$, which are considered as redundancies and are arbitrarily chosen among the N_b forces Q_i, acting as internal loads along the members of the structure. It is assumed that no local mechanisms are formed. Then, Castigliano's second theorem gives the additional N_r equations which are

$$\frac{\partial U}{\partial Q^{(k)}}=\sum_{i=1}^{N_b}\frac{Q_i}{(EA)_i}\cdot\frac{\partial Q_i}{\partial Q^{(k)}}L_i=0 \qquad k=1,\cdots,N_r\ , \tag{14}$$

Where U is the total potential energy of the structure. Internal forces Q_i are then expressed as

$$Q_i=Q_i^{(RL)}+\sum_{j=1}^{N_r}Q_i^{(j)}Q^{(j)}\ ,\ i=1,\dots,N \tag{15}$$

Where $Q^{(j)}$ are the unknown internal redundant forces. In (15) forces $Q_i^{(RL)}$ represent a set of forces acting along each member i and being in equilibrium with all other real forces acting on the same node. This set of forces is created by removing the redundancies and solving the resulting statically determinate structure (which is now called fundamental structure) under the action of its real loading. Then solution gives the values of $Q_i^{(RL)}$. Also, in (15) forces $Q_i^{(j)}$ represent a set of forces in static equilibrium which appears when no real loading is applied on the fundamental structure and when the redundancy members (one at a time) are replaced with a pair of unit forces, opposing each other and acting along the member's axis. One such a pair of forces is acting each time and for each pair the fundamental structure is solved. Forces $Q_i^{(j)}$ are then considered as quantities without the unit of force. They are used as the coefficients of $Q^{(j)}$ and their values constitute a group of statically acceptable internal forces, acting along the redundant members. Then the partial derivative of Q_i with respect to $Q^{(k)}$ in (14) is expressed as

$$\frac{\partial Q_i}{\partial Q^{(k)}}=Q_i^{(k)} \quad k=1,\cdots,N_r \qquad i=1,\cdots,N_b\ . \tag{16}$$

Using (15) and (16), equations (14) take the form

$$\sum_{i=1}^{N_b}\frac{Q_i^{(RL)}+\sum_{j=1}^{N_r}Q_i^{(j)}Q^{(j)}}{(EA)_i}\cdot Q_i^{(k)}L_i=0 \quad \text{or} \quad \delta_k^{(RL)}+\sum_{j=1}^{N_r}h_{kj}Q^{(j)}=0 \qquad k=1,\cdots,N_r \tag{17}$$

Where

$$\delta_k^{(RL)}=\sum_{i=1}^{N_b}\frac{Q_i^{(RL)}Q_i^{(k)}}{(EA)_i}L_i, \qquad h_{kj}=\sum_{i=1}^{N_b}\frac{Q_i^{(j)}Q_i^{(k)}}{(EA)_i}L_i\ . \tag{18}$$

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If there is a real relative displacement $\boldsymbol{\delta_k^{(s)}}$ between one end of the redundant member *k* and the joint at the other end, then equation (17) does not have a zero on the right hand side and it takes the form:

$$\delta_k^{(s)} = \delta_k^{(RL)} + \sum_{j=1}^{N_r} h_{kj} Q^{(j)} \qquad k = 1, \cdots, N_r \tag{19}$$

Equations (19) can be written in matrix form as follows:

$$\begin{bmatrix} \delta_1^{(s)} \\ \vdots \\ \delta_{N_r}^{(s)} \end{bmatrix} = \begin{bmatrix} \delta_1^{(RL)} \\ \vdots \\ \delta_{N_r}^{(s)} \end{bmatrix} + \begin{bmatrix} h_{1,i} & \cdots & h_{1,N_r} \\ \vdots & \vdots & \vdots \\ h_{1,N_r} & \cdots & h_{N_r,N_r} \end{bmatrix} \cdot \begin{bmatrix} Q^{(1)} \\ \vdots \\ Q^{(N_r)} \end{bmatrix}, \tag{20}$$

where $Q^{(j)}$ is also expressed in terms of force density $q^{(j)}$ as $Q^{(j)}=q^{(j)}L^{(j)}$. So, equilibrium equations (8) or (12) together with equations (20) make a system of $2N + N_r$ equations, in the case of planar problems or $3N + N_r$ equations, in the case of 3-D problems, which is solved to give the values of all the unknown forces on the members of the structure. Another way to find the unknown internal forces of the structure, is by solving separately the system of linear equations (20) for $Q^{(j)}$. Next by substituting the known values of $Q^{(j)}$ in (8) or (12) we may find the values of all other forces Q_i of the members of the tensegrtity structure. However, it's preferable to consider the complete system of $2N + N_r$ or $3N + N_r$ equilibrium equations in order to have the opportunity to investigate the stability of the whole structure.

The existence of solution, for the resulting system of linear equations $\boldsymbol{K_e \bullet \beta = \gamma}$, is an indication of the stability condition of the structure. If the solution exists then it is unique. One can be certain about the existence of the solution when square matrix $\boldsymbol{K_e}$ has one of the following properties [10]:

1. Matrix $\boldsymbol{K_e}$ has a rank ρ equal to the rank of the augmented matrix $(\boldsymbol{K_e / \gamma})$.
2. The determinant of the square matrix $\boldsymbol{K_e}$ is not equal to zero, i.e. $\det|\boldsymbol{K_e}| \neq 0$.
3. The inverse $\boldsymbol{K_e}^{-1}$ of the square matrix $\boldsymbol{K_e}$ exists and gives $\boldsymbol{K_e^{-1} K_e = I}$.
4. The number n_{ind} of linearly independent rows or columns of matrix $\boldsymbol{K_e}$ is equal to ρ.

If matrix $\boldsymbol{K_e}$ does not have one of the above *equivalent* properties then the solution does not exist and we say that the stability of our tensegrity structure is doubtful.

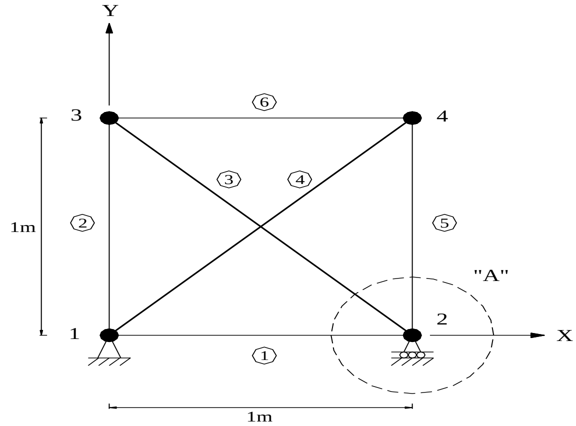

Fig. 2(a) The X-shape 2-D tensegrity truss.

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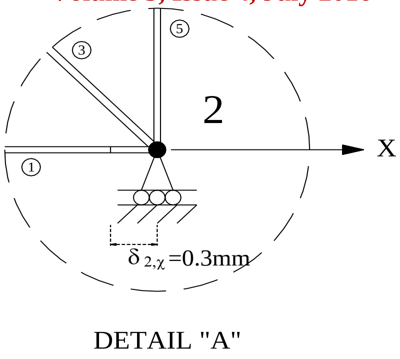

Fig. 2(b) Detail "A" of joint 2.

IV. NUMERICAL EXAMPLES

A. The X-shape 2-D tensegrity structure

One of the fundamental tensegrity configurations, used by Fuller [3] and Snelson [5] to create more complicated stable structures, is the X-shape 2-D tensegrity truss with N=4 nodes (Fig. 2(a)). Skelton [11] studied analytically this truss which has four steel cables (elements 1,2, 5 and 6) and two rods made of a special aluminum alloy (members 3 and 4). The technical characteristics of the cables and rods are presented in Table I. The weight of each joint is G=0.01kN. No other external loads are applied. Cable 1 in our example is shorter than the required length. During construction one end of the cable is connected to joint 1 and the other is extended by $\delta_{2,\chi}$= 3mm to meet joint 2, as it is explained in the exaggerated detail "A" (Fig. 2(b)), which, of course, is not to scale.

Table I: Technical characteristics of the members of the X-shape 2-D tensegrity.

Type of member	Outer diameter d_o (mm)	Inner diameter d_i (mm)	Cross sectional area A (mm^2)	Moment of inertia I_0 (m^4)	Specific weight γ (kN/m^3)	Young' s modulus E (GPa)	Yield stress σ_y (MPa)
Tension members (steel cables)	**8**	**---**	**50.24**	**---**	**78**	**210**	**360**
Compression members (aluminum rods)	**40**	**37**	**181.34**	**3.36×10^{-8}**	**31.85**	**89.17**	**260**

The complete set of linear equilibrium equations (8), for this problem, consists of 2N=8 equations. It has $N_b+N_{ur}-2N$=1 parametric solutions. We say that the structure has a redundancy equal to 1. Rod 1 is chosen as the redundant member of the structure (Fig. 2(a)). Using expression (19) and considering that $Q^{(1)}=L_{1,2}q_{1,2}$ we obtain the additional linear equation $7.28967\times10^{-4}q_{1,2}-3.0034122\times10^{-3}=0$ which gives $q_{1,2}$=4.1201 kN/m. Thus, the value of the force on this element is $F_{1,2}$=4.1201 kN. Also, this equation is added to equations (12) to give a complete system of linear equations. Expanded matrix $\boldsymbol{K_e}$ of the final system of equations is well-conditioned. The results for all forces on the members and the reactions on the supports are tabulated in Table II together with those obtained with the DRM. The CEEMFD and the DRM give results which are comparable. However, the CEEMFD is proved to be faster than the DRM (Table II) because it is a direct method (no iterations are necessary). Also, Table III presents the values of the vertical displacement $\delta_{top,y}$ of joint 3, obtained by the method, the DRM and the analytic approach proposed by Skelton [11] for the same problem. The results for $\delta_{top,y}$, with all three methods, are the same. It is also verified that the structure passes axial yield and Euler buckling criteria.

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Table II: Values of the forces on the members for the X-shape 2-D tensegrity structure.

Force density $q_{i,j}$	Value of $q_{i,j}$ with CEEMFD (kN/m)	Value of $q_{i,j}$ with DRM (kN/m)	Length $L_{i,j}$ of a member (m)	Value of force $F_{i,j}$ with CEEMFD (kN)	Value of force $F_{i,j}$ with DRM (kN)	Reaction force on a support	Value of reaction with CEEMFD (kN)	Value of reaction with DRM (kN)	CPU time with CEEMFD (sec)	CPU time with DRM (sec)
$q_{1,2}$	**4.1201**	**4.1196**	**1.0000**	**4.1201**	**4.1196**	$R_{1,x}$	**0**	**0**	**0.02**	**2.05**
$q_{1,3}$	**4.1021**	**4.1031**	**1.0000**	**4.1021**	**4.1031**	$R_{2,x}$	**0**	**0**		
$q_{1,4}$	**-4.1201**	**-4.1196**	**1.4142**	**-5.8267**	**-5.8260**	$R_{1,y}$	**0.0360**	**0.0345**		
$q_{2,3}$	**-4.1201**	**-4.1196**	**1.4142**	**-5.8267**	**-5.8260**	$R_{2,y}$	**0.0360**	**0.0345**		
$q_{2,4}$	**4.1021**	**4.1031**	**1.0000**	**4.1021**	**4.1031**					
$q_{3,4}$	**4.1201**	**4.1196**	**1.0000**	**4.1201**	**4.1196**					

Table III: Comparison between the results of the CEEMFD and those of the DRM and Skelton's approach.

	CEEMFD	*DRM*	*Skelton' s analytic approach* [11]
Vertical displacement of joint 3 (in mm)	0.39	0.39	0.39

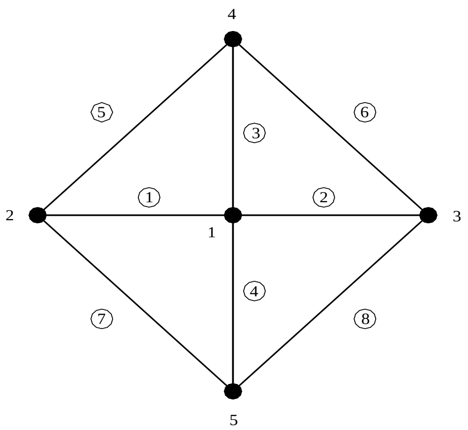

Fig. 3 A weightless two-dimensional tensegrity truss.

B. *A weightless 2-D tensegrity structure*

Zang and Ohsaki [8] use the Adaptive Force Density Method (AFDM) to solve a weightless two-dimensional tensegrity structure which is without any support and out of any gravitational field (Fig. 3). This structure has four cables (elements 1, 2, 3 and 4) and four struts (members 5, 6, 7 and 8). The steel cables and the aluminum rods have the same stiffness which in the current work is chosen to be $(EA)_r=(EA)_c=5426$ kN, without changing the generality of the problem. Also, the aluminum rods have a hollow cross section with a moment of inertia, about any diameter, equal to $I_r=2.8949\times10^{-9}$ m^4. The only load on the structure is the pretension on cable 1 which is equal to 1 kN (Fig. 3). In order to use the CEEMFD we introduce, temporarily, two auxiliary supports at joints 2 and 5 on which there are no reactions (it is as if they do not exist). These supports will only help to define the degree of redundancy. The complete set of linear equilibrium equations (12), for this problem, consists of $2N=10$ equations. It has $N_b+N_{ur}-2N=1$ parametric solutions. Thus the structure has a redundancy equal to 1. Cable 1 in the structure of Fig. 3 is chosen as the redundant member. In using the CEEMFD the additional linear equation, which is required in order to find the unknowns, is $-6.8284+6.8284q_{1,2}=0$, from which one obtains $q_{1,2}=1$ kN/m. The form of matrix $\boldsymbol{K_e}$ indicates that the structure is stable (i.e. det$|\boldsymbol{K_e}|\neq0$). The same problem is also solved with the DRM and the AFDM [8]. The results are presented in Table IV. One can see that the CEEMFD and the AFDM [8] give results which are comparable and more accurate than those obtained with the DRM. It is also verified that as per the materials employed for the structure, it passes axial yield and Euler buckling criteria.

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Table IV: Values of the forces on the members for the weightless 2-D tensegrity structure.

Force density $q_{i,j}$	Value of $q_{i,j}$ with CEEMFD (kN/m)	Value of $q_{i,j}$ with AFDM [8] (kN/m)	Value of $q_{i,j}$ with DRM (kN/m)	Length $L_{i,j}$ of a member (m)	Value of $F_{i,j}$ with method CEEMFD (kN)	Value of $F_{i,j}$ with method AFDM [8] (kN)	Value of force $F_{i,j}$ with DRM (kN)
$q_{1,2}$	**1.00000000**	**1.00000000**	**1.00000072**	1.00000000	**1.00000000**	**1.00000000**	**1.00000072**
$q_{1,3}$	**1.00000000**	**1.00000000**	**1.00000072**	1.00000000	**1.00000000**	**1.00000000**	**1.00000072**
$q_{1,4}$	**1.00000000**	**1.00000000**	**1.00000072**	1.00000000	**1.00000000**	**1.00000000**	**1.00000072**
$q_{1,5}$	**1.00000000**	**1.00000000**	**1.00000072**	1.00000000	**1.00000000**	**1.00000000**	**1.00000072**
$q_{2,4}$	**-0.50000000**	**-0.50000000**	**-0.50000036**	1.41421356	**-0.70710678**	**-0.70710678**	**-0.70710729**
$q_{2,5}$	**-0.50000000**	**-0.50000000**	**-0.50000036**	1.41421356	**-0.70710678**	**-0.70710678**	**-0.70710729**
$q_{3,4}$	**-0.50000000**	**-0.50000000**	**-0.50000036**	1.41421356	**-0.70710678**	**-0.70710678**	**-0.70710729**
$q_{3,5}$	**-0.50000000**	**-0.50000000**	**-0.50000036**	1.41421356	**-0.70710678**	**-0.70710678**	**-0.70710729**

C. A cantilever 2-D tensegrity beam

In this subsection a problem of a cantilever planar tensegrity beam is investigated (Fig. 4). This structure is supported at the two leftmost nodes 1 and 2 and loaded with a unit vertical force acting at the top-right node 10 (Fig. 4). This type of structure was well investigated by Masic et al. [12] who used a nonlinear large displacement model to find its static response and to make a design with optimal mass-to-stiffness ratio. In the present work no optimization was made. The CEEMFD was simply implemented on this problem, by considering the same geometry proposed by Masic et al. [12] and by arbitrarily choosing the material properties for the rods and the cables. So, this structure has thirteen steel cables (elements 1, 2, 3, 4, 5, 8, 9, 12, 13, 16, 17, 20 and 21) and eight aluminum struts (members 6, 7, 10, 11, 14, 15, 18 and 19). The technical characteristics of the cables and rods are given in Table V. According to the method, along each one of the cables 1, 2, 3 and 4 a pair of opposite axial forces, of value 1 kN, is acting each time. Each one of these forces is pushing an end node. Also, a gap exists between the right-end on each one of these members and its nearest joint (Fig. 4). This gap is the same for all the four members and is equal to $54.2640/(EA)_c$ or 10 mm (4 in. approximately). The truss is a statically indeterminate structure with a degree of redundancy $N_r=N_b+N_{ur}-2N=4$.

Table V: Technical characteristics of the members of the cantilever 2-D tensegrity beam.

Type of member	Outer diameter d_o (mm)	Inner diameter d_i (mm)	Cross sec. area A (mm^2)	Moment of inertia I_0 (m^4)	Specific weight γ (kN/m^3)	Young' s modulus E (GPa)	Yield stress σ_y (MPa)
Tension members (steel cables)	**5.74**	---	**25.84**	---	**78**	**210**	**480**
Compression members (alum. rods)	**77**	**75**	**238.76**	**1.72×10^{-7}**	**26.50**	**70**	**260**

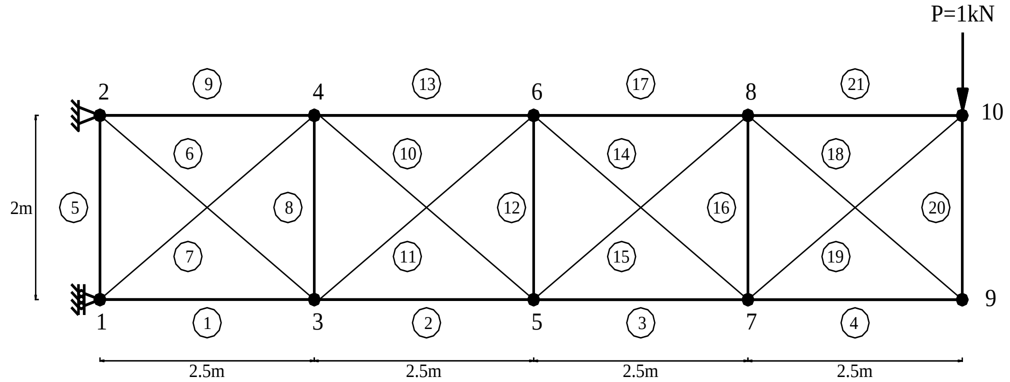

Fig. 4 A cantilever planar tensegrity beam.

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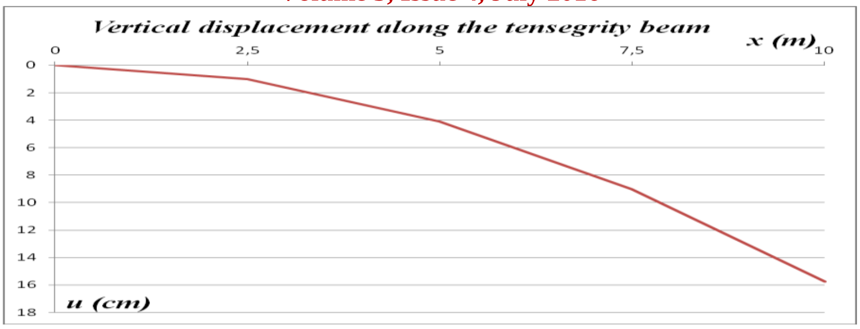

Fig. 5 Vertical displacements along the upper side of the tensegrity beam.

Table VI: Values of the forces on the members for the cantilever 2-D tensegrity beam.

Force density $q_{i,j}$	Value of $q_{i,j}$ with CEEMFD (kN/m) CPU time =0.07 sec	Value of $q_{i,j}$ with DRM (kN/m) CPU time =8.25 sec	Length $L_{i,j}$ of a member (m)	Value of force $F_{i,j}$ with CEEMFD (kN)	Value of force $F_{i,j}$ with DRM (kN)	Value of stress $\sigma_{i,j}$ with CEEMFD (MPa)
$q_{1,2}$	**0.240633**	**0.240613**	2.5000	**0.601583**	**0.601533**	**23.2811**
$q_{2,5}$	**0.651541**	**0.651521**	2.5000	**1.628852**	**1.628802**	**63.0361**
$q_{5,7}$	**1.196278**	**1.196278**	2.5000	**2.990696**	**2.990696**	**115.7390**
$q_{7,9}$	**1.966444**	**1.966444**	2.5000	**4.916110**	**4.916110**	**190.2519**
$q_{1,3}$	**2.461283**	**2.461415**	2.0000	**4.922566**	**4.922830**	**190.5018**
$q_{2,3}$	**-1.861511**	**-1.861511**	3.2016	**-5.959814**	**-5.959814**	**-24.9615**
$q_{1,4}$	**-2.454204**	**-2.454336**	3.2016	**-7.857380**	**-7.857803**	**-32.9091**
$q_{2,4}$	**4.147074**	**4.147074**	2.0000	**8.294147**	**8.294147**	**320.9809**
$q_{3,4}$	**4.075133**	**4.075096**	2.5000	**10.187833**	**10.187740**	**394.2660**
$q_{4,5}$	**-1.705920**	**-1.705788**	3.2016	**-5.461675**	**-5.461252**	**-22.8752**
$q_{2,6}$	**-2.272414**	**-2.272414**	3.2016	**-7.275360**	**-7.275360**	**-30.4714**
$q_{5,6}$	**3.969719**	**3.969475**	2.0000	**7.939438**	**7.938949**	**307.2538**
$q_{4,6}$	**3.326840**	**3.326539**	2.5000	**8.317101**	**8.316348**	**321.8692**
$q_{6,7}$	**-1.710358**	**-1.710114**	3.2016	**-5.475883**	**-5.475101**	**-22.9347**
$q_{5,8}$	**-2.250652**	**-2.250539**	3.2016	**-7.205687**	**-7.205327**	**-30.1796**
$q_{7,8}$	**4.204022**	**4.203534**	2.0000	**8.408044**	**8.407068**	**325.3887**
$q_{6,8}$	**2.764778**	**2.764233**	2.5000	**6.911946**	**6.910582**	**267.4902**
$q_{8,9}$	**-1.966421**	**-1.966045**	3.2016	**-6.295692**	**-6.294490**	**-26.3683**
$q_{7,10}$	**-2.480514**	**-2.480270**	3.2016	**-7.941615**	**-7.940834**	**-33.2619**
$q_{9,10}$	**1.973494**	**1.973118**	2.0000	**3.946987**	**3.946236**	**152.7472**
$q_{8,10}$	**2.480544**	**2.479735**	2.5000	**6.201359**	**6.199338**	**239.9907**
Reaction (kN)						
$R_H^{(1)}$				**5.534000**	**5.534380**	
$R_H^{(3)}$				**-5.534000**	**-5.533907**	
$R_v^{(3)}$				**1.213600**	**1.213864**	

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Tensions in cables 1, 2, 3 and 4 are chosen as redundancies. Matrix $\boldsymbol{K_e}$ is well-conditioned. The results obtained with CEEMFD are presented in Table VI together with those of the DRM which was also implemented to this problem. The two methods give results which are comparable but, again, the CEEMFD is proved to be faster than the DRM. It is also verified that the structure passes axial yield and Euler buckling criteria. Also, Table VII gives the values of the vertical displacement $\delta_y^{(k)}$ at joints 4, 6, 8 and 10 obtained by the method and the DRM. The two sets of results are again comparable. One may visualize the deformed shape of this structure with graph in Fig. 5, which presents the displacements along its upper side.

Table VII: Displacements with the CEEMFD and the DRM along the tensegrity beam.

Joint *i*	**1**	**4**	**6**	**8**	**10**
Vertical displacement of joint *i* with CEEMFD	0.0000	1.0091 cm or 0.40 in.	4.1011 cm or 1.61 in.	9.0438 cm or 3.56 in.	15.7700 cm or 6.21 in.
Vertical displacement of joint *i* with DRM	0.0000	1.0091 cm or 0.40 in.	4.1012 cm or 1.61 in.	9.0467 cm or 3.56 in.	15.7779 cm or 6.21 in.

D. A one-stage 3-D tensegity structure

One of the classical tensegrity structures is the one-stage 3-D tensegrity structure (Fig. 6) which contains a very basic 2-D configuration: the X-shape 2-D tensegrity truss. This 3-D structure can be made much more complicated by adding more bars and cables and more stages.

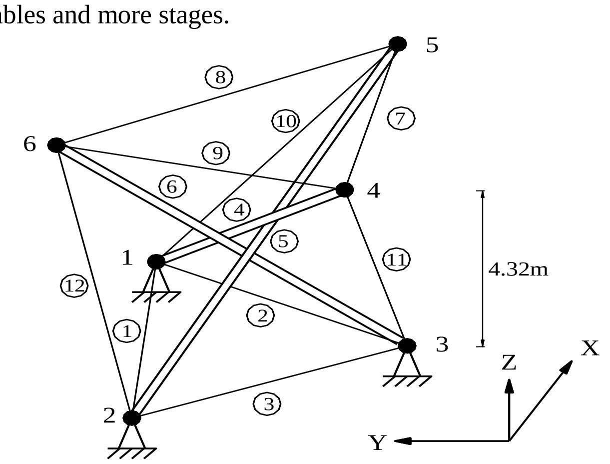

Fig. 6 A one-stage 3-D tensegrity structure.

In the present work the X-shape planar truss is formed by the bars 5 and 6 and cables 3, 8 and 12, as shown in Fig. 6. This stable planar structure is connected with bar 4 with the aid of cables 1, 2, 7, 9, 10 and 11, to create a stable three-dimensional structure. All elements are made of the same precious aluminum alloy which was used to fabricate the aluminum members of the structure in the first example. Linear elastic behavior is considered for loads below the yield stress. The technical characteristics of all members are shown in Table VIII. Also, the weight of each joint is G=0.01kN. Cables 1, 2, 3, 10, 11 and 12 have a constant pretension equal to 0.01kN/m along their length. The coordinates of all the 6 nodes of the structure are shown on Table IX. In applying the method the set of equilibrium equations (8) is obtained which has $3N$=18 independent linear equations with unknowns the force densities on the members and the reactions on the supports (Fig. 6). There are N_b+N_{ur}=12+3×3=21 unknowns indicating a degree of redundancy equal to 3. However, since tensions in cables 1, 2 and 3 have no effect on the values of force densities of the other members, one has $\boldsymbol{q_{1,2}}$= $\boldsymbol{q_{1,3}}$= $\boldsymbol{q_{2,3}}$=0.01kN/m. Then the number of the unknowns is reduced to 18. Thus, the system of 18 linear equations is sufficient to give the solution to our problem.

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Matrix $\boldsymbol{K_e}$ of the linear system of equations has all the properties given by statements 1. to 4. of Section III, verifying that our structure is stable as expected.

Table VIII: Technical characteristics of the one-stage 3-D tensegrity structure.

Type of member	Outer diameter d_o (mm)	Inner diameter d_i (mm)	Cross sectional area A (mm^2)	Moment of inertia I_0 (m^4)	Specific weight γ (kN/m^3)	Young' s modulus E (GPa)	Yield stress σ_y (MPa)
Tension members (aluminium cables)	**20**	**---**	**314.16**	**---**	**31.85**	**89.17**	**260**
Compression elements 4 & 6 (aluminum rods)	**63.9**	**61.7**	**214.87**	**1.07×10^{-7}**	**31.85**	**89.17**	**260**
Compression element 5 (aluminum rod)	**58.4**	**54.9**	**311.45**	**1.25×10^{-7}**	**31.85**	**89.17**	**260**

Table IX: Values of the coordinates of the joints of the one-stage 3-D tensegrity structure.

Node i	$\mathbf{x_i}$ (m)	$\mathbf{y_i}$ (m)	$\mathbf{z_i}$ (m)
1	3.0	-1.73205	0.0
2	0.0	3.46410	0.0
3	-3.0	-1.73205	0.0
4	-4.618802	0.0	4.32049
5	-0.479058	-0.82975	4.32049
6	2.309401	4.0	4.32049

The values of the reaction forces on the supports and of all the forces and the corresponding force densities on the members of the structure are presented in Table X. Again, the results obtained with the CEEMFD compare favorably with those of the DRM. Also, it is verified that the structure passes axial yield and Euler buckling criteria.

Table X: Values of the forces on the members of the one-stage 3-D structure.

Force density on a member.	Value of $q_{i,j}$ with CEEMFD (kN/m).	Value of $q_{i,j}$ with DRM (kN/m).	Length $L_{i,j}$ of a member (m).	Value of $F_{i,j}$ with CEEMFD (kN).	Value of $F_{i,j}$ with DRM (kN).	Reaction force on a support.	Value of reaction force with CEEMFD (kN).	Value of reaction force with DRM (kN).
$q_{1,2}$	**0.010000**	**0.010000**	6.000000	**0.060000**	**0.060000**	$R_{1,x}$	**0.321956**	**0.322034**
$q_{1,3}$	**0.010000**	**0.010000**	6.000000	**0.060000**	**0.060000**	$R_{2,x}$	**-0.207729**	**-0.207787**
$q_{1,4}$	**-0.093977**	**-0.093998**	8.928200	**-0.839048**	**-0.839231**	$R_{3,x}$	**0.529678**	**0.529822**
$q_{1,5}$	**0.077393**	**0.077414**	5.620020	**0.434951**	**0.435068**	$R_{1,y}$	**-0.031957**	**-0.031974**
$q_{2,3}$	**0.010000**	**0.010000**	6.000000	**0.060000**	**0.060000**	$R_{2,y}$	**0.423712**	**0.423814**
$q_{2,5}$	**-0.114623**	**-0.114644**	6.110100	**-0.700357**	**-0.700484**	$R_{3,y}$	**0.391755**	**0.391837**
$q_{2,6}$	**0.056172**	**0.056193**	4.928199	**0.276828**	**0.276931**	$R_{1,z}$	**0.157097**	**0.157097**
$q_{3,4}$	**0.054744**	**0.054765**	4.928199	**0.269789**	**0.269892**	$R_{2,z}$	**0.334522**	**0.334524**
$q_{3,6}$	**-0.096973**	**-0.096994**	8.928200	**-0.865798**	**-0.865984**	$R_{3,z}$	**0.264438**	**0.264437**
$q_{4,5}$	**0.125318**	**0.125340**	4.222079	**0.529100**	**0.529194**			
$q_{4,6}$	**0.013337**	**0.013342**	8.000000	**0.106699**	**0.106736**			
$q_{5,6}$	**0.096702**	**0.096720**	5.576919	**0.539297**	**0.539401**			

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E. A two-stage self-stressed 3-D tensegrity structure

Zhang and Ohsaki [8] have solved the problem of a two-stage self-stressed 3-D tensegrity structure (Fig. 7) by using the AFDM. In this example the structure, which is presented in Fig. 7, is considered to be weightless and is composed of 12 nodes and 30 members; i.e. N=12 and N_b=30.

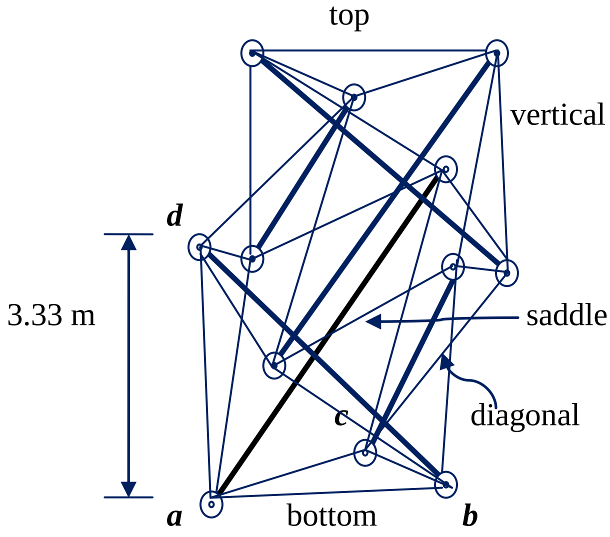

Fig. 7 A two-stage 3-D tensegrity structure (perspective view).

The structure contains 24 steel cables and 6 aluminum rods. The 24 cables are divided into four groups: (i) cables of the top and bottom bases, (ii) saddle cables, (iii) vertical cables and (iv) diagonal cables, as indicated clearly in Fig. 7. Its six rods are divided in two groups: (1) rods of the upper stage (2) rods of the lower stage. Thus, in total, we have six groups of elements. Since the initial force densities and independent nodal coordinates can be arbitrarily specified, one can have some control over the geometrical and mechanical properties of the structure. Thus, the steel cables and the aluminum rods are chosen to have a stiffness $(EA)_r=(EA)_c$=5426 kN. The aluminum rods have a hollow cross section with a moment of inertia, about any diameter, equal to $I_r=2.894\times10^{-9}$ m^4. The only loading on the structure is the initial set of force densities $\boldsymbol{q}^{(o)}$ for the six groups; i.e. for the two groups of rods it is $q_r^{(o)}= -1.5$ kN/m, for the saddle cables is $q_s^{(o)}= 2.0$ kN/m and for all the other cables is $q_c^{(o)}= 1.0$ kN/m.

By specifying the coordinates of nodes *a*, *b* and *c*, which are shown in Fig. 7, to make the bottom base located on the *xy*-plane, and node *d* in the lower stage, we can have the configuration of the tensegrity structure as shown in Fig. 7. The coordinates of these nodes are shown in Table XI. Then the CEEMFD gives the set of final values of force densities, for the six groups of elements, which is listed in Table XII. These values are compared with those obtained with the AFDM [8], after 158 iterations. The two sets of solutions compare favorably but in CEEMFD the solution is obtained directly. It is verified that the structure passes axial yield and Euler buckling criteria as in previous examples.

Table XI: Specified nodal coordinates in the two-stage 3-D tensegrity structure.

Node i	x_i (m)	y_i (m)	z_i (m)
a	-2.6667	0.0000	0.0000
b	1.3333	-2.3094	0.0000
c	1.3334	2.3094	0.0000
d	-1.8867	1.6666	3.3333

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Table XII: Force densities in the members of the two-stage 3-D tensegrity structure.

Group →	Rods of the upper stage (1)	Rods of the lower stage (2)	Cables (top & bottom) (i)	Saddle cables (ii)	Vertical cables (iii)	Diagonal cables (iv)
Initial value (kN/m)	-1.5000	-1.5000	1.0000	2.0000	1.0000	1.0000
Final value (CEEMFD)	-1.8376	-1.8376	0.9282	1.9920	1.1735	0.9957
Final value (AFDM) [8]	-1.8376	-1.8376	0.9281	1.9918	1.1737	0.9958

V. CONCLUSION

In this paper the CEEMFD is applied to analyze tensegrity structures. For statically indeterminate structures, Castigliano's 2^{nd} theorem is implemented. The partial derivatives, which then appear are numerically replaced by statically acceptable internal forces on the members. The final complete system of equilibrium equations is solved to give the values of the force densities. Five problems of planar and spatial tensegrities were solved. The results compare favorably with those obtained by the DRM and the AFDM [8] but the CEEMFD was proved to be faster since it is a direct method. As for future work we consider the application of the method on bridges for pedestrians.

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