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Finite semigroups of binary relations defined by semi lattices of the class $\Sigma_6(X,6)$

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ABSTRACT: The paper deals with the structure of idempotent and maximal subgroups of finite semi groups of binary relations that are identified by the semi lattices of class $\Sigma_6(X,6)$. For finite set X we found the formula of counting the number of idempotent.

AMS Subject Classification: 20M05
***Key words:* semi lattice, semi group of binary relations.**

I. INTRODUCTION

Let $X-$ be any non-empty set. D is a non-empty set of subsets of the set X, closed in relation to the operation set-theoretic union of elements from D. f is a map of set X in the set D. For each such map f we will compare binary relation α_f on the set X, satisfying

$$\alpha_f=\bigcup_{x\in X}\left(\{x\}\times f(x)\right).$$

Set of all such kind α_f $(f:X\to D)$ will be denoted by $B_X(D)$. It is easy to prove that $B_X(D)$ is subsemigroup of semi group B_X, which is called finite semi groups of binary relations defined with $X-$semi lattice of union D.

Furthermore, let Y - be a certain set of subset X and $\alpha-$be e certain binary relation on the set X. Let us denote with αY, set of all such elements $x\in X$, for which there is an element $y\in Y$, which $x\alpha y$. Moreover,

$$V(D,\alpha)=\{Y\alpha \mid Y\in D\},\ V(X^*,\alpha)=\{Y\alpha \mid \varnothing\neq Y\subseteq X\}\ \ \ddot{D}'_T=\{Z'\in D' \mid Z'\subseteq T\},$$

if $\varnothing\neq D'\subseteq D$ и $T\in D$.

Definition 1. *Let D be a certain set $X-$semi lattice of set $T\in D$, $\alpha\in B_X(D)$ and $Y_T^\alpha=\{y\in X \mid y\alpha=T\}$. If*

$$V[\alpha]=\begin{cases}V(X^*,\alpha), \text{ if } \varnothing\notin D,\\ V(X^*,\alpha), \text{ if } \varnothing\in V(X^*,\alpha),\\ V(X^*,\alpha)\cup\{\varnothing\}, \text{ if } \varnothing\notin V(X^*,\alpha) \text{ and } \varnothing\in D,\end{cases}$$

then it is obvious that any binary relation α of subgroup $B_X(D)$, can always be represented as

$$\alpha=\bigcup_{T\in V[\alpha]}\left(Y_T^\alpha\times T\right). \quad \dots (1)$$

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In the future, such a representation of a binary relation α *we shall call quasinormal and call normal if* $Y_T^{\alpha} \neq \varnothing$ *for any* $T \in V[\alpha]$.

Definition 2. *Let* D *is a finite* $X-$*semilattice of union,* $\varnothing \neq D' \subseteq D$ *and* $N(D,D') = \{Z \in D \mid Z \subseteq Z' \text{ for any } Z' \in D'\}$. *It is obvious that* $N(D,D')$ *is the set of all lower bounds of non-empty subsets* D' *from* D *in the semilattice* D. *If* $N(D,D') \neq \varnothing$, *then* $\cup N(D,D') \in D$ *is the exact lower bound of the set* D' *in* D. *Let us denote this element with* $\Lambda(D,D')$, *i.e.* $\Lambda(D,D') = \cup N(D,D')$.

Note, if the element $\Lambda(D,D')$ in the semi lattice D exists, then we can write $\Lambda(D,D') \in D$, otherwise - $\Lambda(D,D') \notin D$.

Definition 3. *If* $t \in \breve{D}$ *u* $D_t = \{Z \in D \mid t \in Z\}$. *It is said that finite* $X-$*semi lattice of union* D *is* $XI-$*semi lattice of union, if it satisfies the following two conditions:*

a) $\Lambda(D,D_t) \in D$, *for any* $t \in \breve{D}$,

b) $Z = \bigcup_{t \in Z} \Lambda(D,D_t)$, *for any non-empty element* Z *of semi lattice* D.

Theorem 1. *Binary relation* $\varepsilon \in B_X(D)$ *is a right unit of given semigroup unit if and only if* $\varepsilon-$*is idempotent and* $D = V(D,\varepsilon)$.

Theorem 2. *If* D *is finite* $X-$*semilattice of union. Semigroup* $B_X(D)$ *has a right unit if and only if* D *is* $XI-$ *semilattice of union.*

Theorem 3. *Let* D, $\Sigma(D)$, $E_X^{(r)}(D')$ *and* I *respectively denote the finite* $X-$ *semilattice of union, set of all* $XI-$*subsemigroup of semilattice* D, *the set of all the right units of the semigroup* $B_X(D')$ $(D' \in \Sigma(D))$ *and the set of all idempotent of the semigroup* $B_X(D)$. *Then sets* $E_X^{(r)}(D')$ *and* I *the following statements are true:*

a) *if* $\varnothing \in D$ *u* $\Sigma_{\varnothing}(D) = \{D' \in \Sigma(D) \mid \varnothing \in D'\}$, *then*

1) $E_X^{(r)}(D') \cap E_X^{(r)}(D'') = \varnothing$ *for any elements* D' *and* D'' *sets* $\Sigma_{\varnothing}(D)$, *satisfying the condition* $D' \neq D''$;

2) $I = \bigcup_{D' \in \Sigma_{\varnothing}(D)} E_X^{(r)}(D')$;

3) *for a finite set X the following equality is true* $|I| = \sum_{D' \in \Sigma_{\varnothing}(D)} \left| E_X^{(r)}(D') \right|$.

b) *If* $\varnothing \notin D$, *then*

1) $E_X^{(r)}(D') \cap E_X^{(r)}(D'') = \varnothing$ *for any elements* D' *and* D'' *of set* $\Sigma(D)$, *satisfying the condition* $D' \neq D''$;

2) $I = \bigcup_{D' \in \Sigma(D)} E_X^{(r)}(D')$;

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3) *for the finite* X *the following equality is true* $|I| = \sum_{D'\in\Sigma(D)} \left| E_X^{(r)}(D') \right|$.

Theorem 4. *Let* $Q=\{T_0,T_1,...,T_m\}$ $(m\geq 1)$ *there is a semilattice which* $T_0\subset T_1\subset ...\subset T_m$. *Then binary relation* α *of semigroup* $B_X(Q)$, *having a quasi-normal representation of the form* $\alpha=\bigcup_{i=0}^{m}(Y_i^\alpha\times T_i)$, *that* $Q=V(Q,\alpha)$, *is a right unit of semigroup* $B_X(Q)$ *if and only if* $Y_0^\alpha\cup Y_1^\alpha\cup...\cup Y_p^\alpha\supseteq T_p$ *u* $Y_q^\alpha\cap T_q\neq\varnothing$ *for any* $p=0,1,...,m-1$ *and* $q=1,2,...,m$.

Theorem 5. *Let* $Q=\{T_0,T_1,T_2,T_3,...,T_m\}$ $(m\geq 3)$ *be such a semilattice and* $j-$ *such a fixed natural number that* $0\leq j\leq m-3$ *and*

$$T_0\subset T_1\subset ...\subset T_j\subset T_{j+1}\subset T_{j+3}\subset ...\subset T_m,$$
$$T_0\subset T_1\subset ...\subset T_j\subset T_{j+2}\subset T_{j+3}\subset ...\subset T_m,$$
$$T_{j+1}\setminus T_{j+2}\neq\varnothing, T_{j+2}\setminus T_{j+1}\neq\varnothing, T_{j+1}\cup T_{j+2}=T_{j+3}.$$

Binary relation α *of semigroup* $B_X(Q)$, *having a quasi-normal representation of the form* $\alpha=\bigcup_{i=0}^{m}(Y_i^\alpha\times T_i)$,*that* $Q=V(Q,\alpha)$, *is a right unit of semigroup* $B_X(Q)$ *if and only if*

$$Y_0^\alpha\cup Y_1^\alpha\cup...\cup Y_j^\alpha\supseteq T_{j+1}\cap T_{j+2},\ Y_0^\alpha\cup Y_1^\alpha\cup...\cup Y_j^\alpha\cup Y_{j+2}^\alpha\supseteq T_{j+2},$$
$$Y_0^\alpha\cup Y_1^\alpha\cup...\cup Y_p^\alpha\supseteq T_p \text{ u } Y_q^\alpha\cap T_q\neq\varnothing$$

For any $p=0,1,2,...,m-1$, $q=1,2,...,m$ $(p\neq j+2,\ q\neq j+3)$.

Let X and $\Sigma_6(X,6)$ respectively, are certain non-empty set, and such a class X -semilattice of union, each element of which is isomorphic of some X -semilattice of union $D=\{Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\}$, satisfying the following conditions:

$$Z_5\subset Z_1\subset\breve{D},\ Z_5\subset Z_4\subset Z_2\subset\breve{D},\ Z_5\subset Z_3\subset\breve{D},$$
$$Z_1\setminus Z_2\neq\varnothing,\ Z_2\setminus Z_1\neq\varnothing,\ Z_1\setminus Z_4\neq\varnothing,\ Z_4\setminus Z_1\neq\varnothing,$$
$$Z_2\setminus Z_3\neq\varnothing,\ Z_3\setminus Z_2\neq\varnothing,\ Z_4\setminus Z_3\neq\varnothing,\ Z_3\setminus Z_4\neq\varnothing.$$

Semi lattices satisfying conditions (1), is given on the figure Fig. 1.

Further we will assume that $C(D)=\{P_5,P_4,P_3,P_2,P_1,P_0\}$, is a set of X , and there is such a map of semilattices D on the set $C(D)$, that $\varphi(\breve{D})=P_0$ and

Z_3 $\breve{D}$

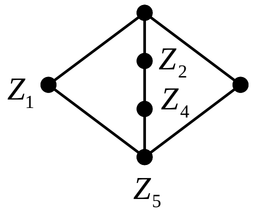

Fig. 1.

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$\varphi(Z_i) = P_i$, for any $i = 1,2,3,4,5$. Then the formal equality for a given semi lattices elements have the following form:

$$\begin{aligned}
\breve{D} &= P_0 \cup P_1 \cup P_2 \cup P_3 \cup P_4 \cup P_5,\\
Z_1 &= P_0 \cup P_1 \cup P_2 \cup P_3 \cup P_4 \cup P_5,\\
Z_2 &= P_0 \cup P_1 \cup P_3 \cup P_4 \cup P_5,\\
Z_3 &= P_0 \cup P_1 \cup P_2 \cup P_4 \cup P_5,\\
Z_4 &= P_0 \cup P_1 \cup P_3 \cup P_4 \cup P_5,\\
Z_5 &= P_0,
\end{aligned}$$

where $|P_1| \geq 1$, $|P_2| \geq 1$, $|P_3| \geq 1$, $|P_4| \geq 1$, $|P_0| \geq 0$, $|P_5| \geq 0$ and $|X| \geq 4$, i.e. elements P_1, P_2, P_3 and P_4 are basic sources and P_0 and P_5, $X -$semi lattice of union D (see. [2 or 3]).

Lemma 1. *Let $X -$is a finite set $|X| = n \geq 4$ and $|\Sigma_6(X,6)| = s$. Then* $s = \frac{1}{2}(7^n - 4\cdot 6^n + 6\cdot 5^n - 4\cdot 4^n + 3^n)$.

Proof. By definition of semi lattices of this class we have that number of basic sources $\delta = 4$. Moreover, any automorphisms of semilattices of class $\Sigma_6(X,6)$ are excluded bye the mapping of the form

$$\varphi_1 = \begin{pmatrix} Z_5 & Z_4 & Z_3 & Z_2 & Z_1 & \breve{D} \\ Z_5 & Z_4 & Z_3 & Z_2 & Z_1 & \breve{D} \end{pmatrix}, \quad \varphi_2 = \begin{pmatrix} Z_5 & Z_4 & Z_3 & Z_2 & Z_1 & \breve{D} \\ Z_5 & Z_4 & Z_1 & Z_2 & Z_3 & \breve{D} \end{pmatrix}.$$

Therefore their number is $q = 2$. now, considering the theorem 2.4. from [2], we will have:

$$s = \frac{1}{2}\sum_{P=4}^{6}\sum_{i=1}^{P+1}\frac{(-1)^{P+i+1}\cdot C_{m-3}^{P-3}\cdot(P!)\cdot i^n}{(i-1)!\cdot(P+1-i)!},$$

where $C_j^k = \frac{j!}{(k!)\cdot(j-k)!}$. After simplifying the last expression we obtain:

$$s = \frac{1}{2}(7^n - 4\cdot 6^n + 6\cdot 5^n - 4\cdot 4^n + 3^n).$$

The lemma is proved.

Example 1. Let $n = 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10$. Then, accordingly we will have:

$$s = 12,\ 300,\ 4560,\ 54750,\ 567252,\ 5366340,\ 47534520.$$

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$$\left|B_X\left(D\right)\right| = 1296,\ 7776,\ 46656,\ 279936,\ 1679616,\ 10077696,\ 60466176.$$

These numbers show, if for example, $\left|X\right| = 10$,then the number of elements in the class of semigroups in which each element is determined by a semilattice of class $\Sigma_6\left(X,6\right)$, equals to 47534520, and the number of elements in each semigroup belonging to this class equals to 60466176.

Our goal is to study finite semi groups of binary relations $B_X\left(D\right)$, determined by X -semilattice of union D of the class $\Sigma_6\left(X,6\right)$.

It is clear that, quazinormal representations of any element of α from the semigroup $B_X\left(D\right)$ has the following form:

$$\begin{aligned}\alpha = &\left(Y_5^\alpha \times Z_5\right) \cup \left(Y_4^\alpha \times Z_4\right) \cup \left(Y_3^\alpha \times Z_3\right) \cup \\ &\cup \left(Y_2^\alpha \times Z_2\right) \cup \left(Y_1^\alpha \times Z_1\right) \cup \left(Y_0^\alpha \times \breve{D}\right)\end{aligned} \quad \ldots(*)$$

where $Y_5^\alpha, Y_4^\alpha, Y_3^\alpha, Y_2^\alpha, Y_1^\alpha$ and Y_0^α are some disjoint subsets of the set X .

At first we will describe idempotent elements of the subgroup $B_X\left(D\right)$. In order to solve the problem, by Theorem 1.1.3 it is necessary first to identify $XI-$semilattice of union (see definition 1.1.3). in the semilattices D .

Lemma 2. *Let* $D = \left\{Z_5, Z_4, Z_3, Z_2, Z_1, \breve{D}\right\} \in \Sigma_6\left(X,6\right)$. *Then the subsets of the form*

a) $\left\{\breve{D}\right\}, \left\{Z_1\right\}, \left\{Z_2\right\}, \left\{Z_3\right\}, \left\{Z_4\right\}, \left\{Z_5\right\};$

b) $\left\{Z_1,\breve{D}\right\}, \left\{Z_2,\breve{D}\right\}, \left\{Z_3,\breve{D}\right\}, \left\{Z_4,\breve{D}\right\}, \left\{Z_5,\breve{D}\right\}, \left\{Z_5,Z_1\right\}, \left\{Z_5,Z_2\right\}, \left\{Z_5,Z_3\right\}, \left\{Z_5,Z_4\right\}, \left\{Z_4,Z_2\right\};$

c) $\left\{Z_5,Z_4,\breve{D}\right\}, \left\{Z_5,Z_3,\breve{D}\right\}, \left\{Z_5,Z_2,\breve{D}\right\}, \left\{Z_5,Z_1,\breve{D}\right\}, \left\{Z_4,Z_3,\breve{D}\right\},$
$\left\{Z_4,Z_2,\breve{D}\right\}, \left\{Z_4,Z_1,\breve{D}\right\}, \left\{Z_3,Z_2,\breve{D}\right\}, \left\{Z_3,Z_1,\breve{D}\right\}, \left\{Z_2,Z_1,\breve{D}\right\}, \left\{Z_5,Z_4,Z_2\right\};$

d) $\left\{Z_3,Z_2,Z_1,\breve{D}\right\}, \left\{Z_4,Z_3,Z_1,\breve{D}\right\}, \left\{Z_4,Z_2,Z_1,\breve{D}\right\}, \left\{Z_4,Z_3,Z_1,\breve{D}\right\}, \left\{Z_5,Z_2,Z_1,\breve{D}\right\},$
$\left\{Z_5,Z_3,Z_1,\breve{D}\right\}, \left\{Z_5,Z_3,Z_2,\breve{D}\right\}, \left\{Z_5,Z_4,Z_1,\breve{D}\right\}, \left\{Z_5,Z_4,Z_3,\breve{D}\right\}, \left\{Z_5,Z_4,Z_2,\breve{D}\right\};$

e) $\left\{Z_4,Z_3,Z_2,Z_1,\breve{D}\right\}, \left\{Z_5,Z_3,Z_2,Z_1,\breve{D}\right\}, \left\{Z_5,Z_4,Z_2,Z_1,\breve{D}\right\},$
$\left\{Z_5,Z_4,Z_3,Z_1,\breve{D}\right\}, \left\{Z_5,Z_4,Z_3,Z_2,\breve{D}\right\};$

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f) $\{Z_5, Z_4, Z_3, Z_2, Z_1, \breve{D}\}$

subsemilattice of the semilattice D *is excluded* .

Proof. Obviously, all the single-elements of subset of semilattice D are its subsemilattices.

The number of two-element of subsets of semilattices D equals $C_6^2 = 15$. They have the form:

$$\{Z_5, Z_4\}, \{Z_5, Z_3\}, \{Z_5, Z_2\}, \{Z_5, Z_1\}, \{Z_5, \breve{D}\}, \{Z_4, Z_3\}, \{Z_4, Z_2\}, \{Z_4, Z_1\},$$
$$\{Z_4, \breve{D}\}, \{Z_3, Z_2\}, \{Z_3, Z_1\}, \{Z_3, \breve{D}\}, \{Z_2, Z_1\}, \{Z_2, \breve{D}\}, \{Z_1, \breve{D}\}.$$

From these subsets of subsemilattices $X-$ semilattice of union D will be only those sets, which are of X - chain. The number of all three-element sets of X -set of unions D equals $C_6^3 = 20$.

$$\{Z_5, Z_4, Z_2\}, \{Z_5, Z_4, \breve{D}\}, \{Z_5, Z_3, \breve{D}\}, \{Z_5, Z_2, \breve{D}\}, \{Z_5, Z_1, \breve{D}\},$$
$$\{Z_4, Z_3, \breve{D}\}, \{Z_4, Z_2, \breve{D}\}, \{Z_4, Z_1, \breve{D}\}, \{Z_3, Z_2, \breve{D}\}, \{Z_3, Z_1, \breve{D}\}, \{Z_2, Z_1, \breve{D}\};$$
$$\{Z_5, Z_4, Z_3\}, \{Z_5, Z_4, Z_1\}, \{Z_5, Z_3, Z_2\}, \{Z_5, Z_3, Z_1\},$$
$$\{Z_5, Z_2, Z_1\}, \{Z_4, Z_3, Z_2\}, \{Z_4, Z_3, Z_1\}, \{Z_4, Z_2, Z_1\}, \{Z_3, Z_2, Z_1\};$$

It is easy to prove that from the given subsets the 9 sets are not subsemilattices of the semilattice D.

The number of all four elements subsets of semilattice D equals $C_6^4 = 15$. They have the form.

$$\{Z_3, Z_2, Z_1, \breve{D}\}, \{Z_4, Z_2, Z_1, \breve{D}\}, \{Z_4, Z_3, Z_1, \breve{D}\}, \{Z_4, Z_3, Z_2, \breve{D}\}, \{Z_5, Z_2, Z_1, \breve{D}\};$$
$$\{Z_5, Z_3, Z_1, \breve{D}\}, \{Z_5, Z_3, Z_2, \breve{D}\}, \{Z_5, Z_4, Z_1, \breve{D}\}, \{Z_5, Z_4, Z_2, \breve{D}\}, \{Z_5, Z_4, Z_3, \breve{D}\},$$
$$\{Z_4, Z_3, Z_2, Z_1\}, \{Z_5, Z_3, Z_2, Z_1\}, \{Z_5, Z_4, Z_2, Z_1\}, \{Z_5, Z_4, Z_3, Z_1\}, \{Z_5, Z_4, Z_3, Z_2\}.$$

It can be verified that from the given subsets the last 10 sets are not semilattices.

From the proved it immediately follows that the diagrams of type 2 all diagrams of own sublattices of semilattice D are excluded.

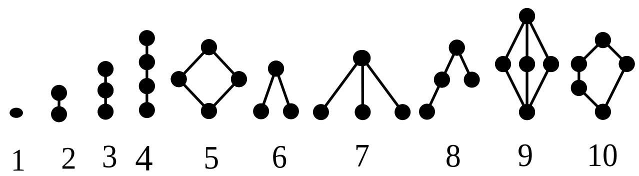

Fig.2

Lemma 3. Let $D = \{Z_5, Z_4, Z_3, Z_2, Z_1, \breve{D}\} \in \Sigma_6(X, 6)$. Then the subsets of form:

a) $\{\breve{D}\}, \{Z_1\}, \{Z_2\}, \{Z_3\}, \{Z_4\}, \{Z_5\}$,

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b) $\{Z_1,\breve{D}\},\{Z_2,\breve{D}\},\{Z_3,\breve{D}\},\{Z_4,\breve{D}\},\{Z_5,\breve{D}\},\{Z_5,Z_1\},\{Z_5,Z_2\},\{Z_5,Z_3\},\{Z_5,Z_4\},\{Z_4,Z_2\}$,

c) $\{Z_5,Z_4,Z_2\},\{Z_5,Z_4,\breve{D}\},\{Z_5,Z_3,\breve{D}\},\{Z_5,Z_2,\breve{D}\},\{Z_5,Z_1,\breve{D}\},\{Z_4,Z_2,\breve{D}\}$,

d) $\{Z_5,Z_4,Z_2,\breve{D}\}$,

e) $\{Z_5,Z_2,Z_1,\breve{D}\},\{Z_5,Z_3,Z_1,\breve{D}\},\{Z_5,Z_3,Z_2,\breve{D}\},\{Z_5,Z_4,Z_1,\breve{D}\},\{Z_5,Z_4,Z_3,\breve{D}\}$

all $XI-$subsemilattices of semilattice D are excluded.

Proof. The truth of given lemma follows directly from the theorem 6.2 from [1] and from the theorem 2 from [4]. Lemma is proved. From the proved lemma it directly follows that in Fig. 3. The given diagrams exclude all diagrams of X_I-subsemilattice of semi lattice D.

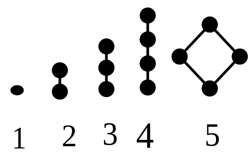

Fig.3

Theorem 1. *Let* $D=\{Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\}\in\Sigma_6(X,6)$ *and* $Z_5\neq\varnothing$. *Then binary relations* α *of semigroup* $B_X(D)$ *if and only if is idempotent element of this semi group, when it meets at least one of the conditions listed below:*

a) $\alpha=X\times T$, where $T\in D$;

в) $\alpha=(Y_T^\alpha\times T)\cup(Y_{T'}^\alpha\times T')$ for some $T,T'\in D,\ \ T\subset T'$ and $Y_T^\alpha,Y_{T'}^\alpha\notin\{\varnothing\}$, satisfying the conditions: $Y_T^\alpha\supseteq T$ и $Y_{T'}^\alpha\cap T'\neq\varnothing$;

c) $\alpha=(Y_T^\alpha\times T)\cup(Y_{T'}^\alpha\times T')\cup(Y_{T''}^\alpha\times T'')$ for some $T,T',T''\in D,\ \ T\subset T'\subset T''$ and $Y_T^\alpha,Y_{T'}^\alpha,Y_{T''}^\alpha\notin\{\varnothing\}$, satisfying the conditions: $Y_T^\alpha\supseteq T$, $Y_T^\alpha\cup Y_{T'}^\alpha\supseteq T'$, $Y_{T'}^\alpha\cap T'\neq\varnothing$ and $Y_{T''}^\alpha\cap T''\neq\varnothing$;

d) $\alpha=(Y_5^\alpha\times Z_5)\cup(Y_4^\alpha\times Z_4)\cup(Y_2^\alpha\times Z_2)\cup(Y_0^\alpha\times\breve{D})$ for some $Y_5^\alpha,Y_4^\alpha,Y_2^\alpha,Y_0^\alpha\notin\{\varnothing\}$, satisfying the conditions: $Y_5^\alpha\supseteq Z_5$, $Y_5^\alpha\cup Y_4^\alpha\supseteq Z_4$, $Y_5^\alpha\cup Y_4^\alpha\cup Y_2^\alpha\supseteq Z_2$, $Y_4^\alpha\cap Z_4\neq\varnothing$ и $Y_2^\alpha\cap Z_2\neq\varnothing$, $Y_0^\alpha\cap\breve{D}\neq\varnothing$.

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e) $\alpha=(Y_5^\alpha\times Z_5)\cup(Y_T^\alpha\times T)\cup(Y_{T'}^\alpha\times T')\cup(Y_0^\alpha\times\breve{D})$ for some $T,T'\in\{Z_4,Z_3,Z_2,Z_1\}$, $T\setminus T'\neq\varnothing$, $T'\setminus T\neq\varnothing$ и $Y_5^\alpha,Y_T^\alpha,Y_{T'}^\alpha\notin\{\varnothing\}$, satisfying the conditions: $Y_5^\alpha\supseteq Z_5$, $Y_5^\alpha\cup Y_T^\alpha\supseteq T$, $Y_5^\alpha\cup Y_{T'}^\alpha\supseteq T'$, $Y_T^\alpha\cap T\neq\varnothing$, $Y_{T'}^\alpha\cap T'\neq\varnothing$ и $Y_0^\alpha\cap\breve{D}\neq\varnothing$.

Proof. Let $D=\{Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\}\in\Sigma_6(X,6)$ and binary relation α is an idempotent element of the semi group $B_X(D)$. Then, the truth of the theorem follows directly from Lemma 2.3 from theorem 1.4 and 1.5.

The theorem is proved.

Theorem 2. *Let* $X-$ *finite set,* $D=\{Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\}\in\Sigma_6(X,6)$ *and* $Z_5\neq\varnothing$. *If by symbol* I *we denote the set of all idempotent elements of the semi group* $B_X(D)$, *then the number of elements* $|I|$ *in the set* I *can be calculated by the formula:*

$$\begin{aligned}
|I|=6&+\left(2^{|\breve{D}\setminus Z_5|}+2^{|\breve{D}\setminus Z_4|}+2^{|\breve{D}\setminus Z_3|}+2^{|\breve{D}\setminus Z_2|}+2^{|\breve{D}\setminus Z_1|}-5\right)\cdot 2^{|X\setminus\breve{D}|}+\left(2^{|Z_4\setminus Z_5|}-1\right)\cdot 2^{|X\setminus Z_4|}+\\
&+\left(2^{|Z_3\setminus Z_5|}-1\right)\cdot 2^{|X\setminus Z_3|}+\left(2^{|Z_2\setminus Z_5|}-1\right)\cdot 2^{|X\setminus Z_2|}+\left(2^{|Z_1\setminus Z_5|}-1\right)\cdot 2^{|X\setminus Z_1|}+\left(2^{|Z_2\setminus Z_4|}-1\right)\cdot 2^{|X\setminus Z_2|}+\\
&+\left(2^{|Z_4\setminus Z_5|}-1\right)\cdot\left(3^{|Z_2\setminus Z_4|}-2^{|Z_2\setminus Z_4|}\right)\cdot 3^{|X\setminus Z_2|}+\left(2^{|Z_4\setminus Z_5|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_4|}-2^{|\breve{D}\setminus Z_4|}\right)\cdot 3^{|X\setminus\breve{D}|}+\\
&+\left(2^{|Z_3\setminus Z_5|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_3|}-2^{|\breve{D}\setminus Z_3|}\right)\cdot 3^{|X\setminus\breve{D}|}+\left(2^{|Z_2\setminus Z_5|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_2|}-2^{|\breve{D}\setminus Z_2|}\right)\cdot 3^{|X\setminus\breve{D}|}+\\
&+\left(2^{|Z_1\setminus Z_5|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_1|}-2^{|\breve{D}\setminus Z_1|}\right)\cdot 3^{|X\setminus\breve{D}|}+\left(2^{|Z_2\setminus Z_4|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_2|}-2^{|\breve{D}\setminus Z_2|}\right)\cdot 3^{|X\setminus\breve{D}|}+\\
&+\left(2^{|Z_4\setminus Z_5|}-1\right)\cdot\left(3^{|Z_2\setminus Z_4|}-2^{|Z_2\setminus Z_4|}\right)\cdot\left(4^{|\breve{D}\setminus Z_2|}-3^{|\breve{D}\setminus Z_2|}\right)\cdot 4^{|X\setminus\breve{D}|}+\left(2^{|Z_2\setminus Z_1|}-1\right)\cdot\left(2^{|Z_1\setminus Z_2|}-1\right)\cdot 4^{|X\setminus\breve{D}|}+\\
&+\left(2^{|Z_3\setminus Z_1|}-1\right)\cdot\left(2^{|Z_1\setminus Z_3|}-1\right)\cdot 4^{|X\setminus\breve{D}|}+\left(2^{|Z_3\setminus Z_2|}-1\right)\cdot\left(2^{|Z_2\setminus Z_3|}-1\right)\cdot 4^{|X\setminus\breve{D}|}+\\
&+\left(2^{|Z_4\setminus Z_1|}-1\right)\cdot\left(2^{|Z_1\setminus Z_4|}-1\right)\cdot 4^{|X\setminus\breve{D}|}+\left(2^{|Z_4\setminus Z_3|}-1\right)\cdot\left(2^{|Z_3\setminus Z_4|}-1\right)\cdot 4^{|X\setminus\breve{D}|}.
\end{aligned}$$

Proof. The truth of the theorem follows directly from Theorem 6.5 from [1] and from Theorem 3.2 from [5].

REFERENCES

[1] Ya. I. Diasamidze. Complete Semi groups of Binary Relations. Journal of Mathematical Sciences, Plenum Publ. Cor., New York, Vol. 117, No. 4, 2003, 4271-4319.

[2] Ya. I. Diasamidze, Sh. I. Makharadze, G. J. Fartenadze O. T. Givradze. On finite $X-$ semilattices of unions. Journal of Mathematical Sciences, Plenum Publ. Cor., New York, 141, № 4, 2007, 1134-1181.

[3] Я. И. Диасамидзе, Ш. И. Махарадзе, Г. Ж. Партенадзе, О. Т. Гиврадзе. О конечных $X-$ полурешнтках объединений. Современная математика и ее приложения, Алгебра и геометрия, Тбилиси, 2005, т. 27, 46-94.

[4] Diasamidze Ya., Makharadze Sh., N. V. Rokva. On $XI-$ Semilattices of Unions. Bull. Georg. Acad. Sci., 175, № 4. 2008.

[5] Makharadze Sh. I., Diasamidze I. Ya., One class of complete semi groups of binary relations. J. of Math. Sciences, Plenum Publ. Cor., New York, 117, № 4, 2003, 4393-4424.