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Solving Fuzzy Differential Equations using Runge-kutta second order method for two stages contra-harmonic mean

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***Abstract*— In this paper, a study on numerical method for solving first order fuzzy differential equations using Runge-kutta second order method for two stages of Contra-harmonic mean is considered. Here the applicability and stability of the proposed method are illustrated by a numerical example with triangular fuzzy number.**

***Key wordss*— Contra-harmonic mean, Fuzzy Differential Equations, Runge-kutta second order method, Triangular Fuzzy Number.**

I. INTRODUCTION

The theory of fuzzy differential equation plays an important role in modeling of science and engineering problems because this theory represents a natural way to model dynamical systems under uncertainty. The applicability of the fuzzy differential equation leads to a several number of research works in the open literature. First order linear fuzzy differential equation is one of the simplest fuzzy differential equation, which appear in many applications. Some of the reviewed research papers are cited below for better understanding of the present paper. The concept of fuzzy derivative was first introduced by S.L.Chang and L.A.Zadeh in[6].D.Dubois and Prade [7] discussed differentiation with fuzzy features. M.L.puri, D.A.Ralescu [24] and R.Goetschel, W.Voxman[10] contributed towards the differential of fuzzy functions. The fuzzy differential equation and initial value problems were extensively studied by O.Kaleva[15,16] and by S.Seikkala[25].Recently many research papers are focused on numerical solution of fuzzy initial value problems. Numerical Solution of fuzzy differential equations has been introduced by M.Ma, M.Friedman, A.Kandel [19] through euler method and by S.Abbasbandy and T.Allahviranloo [1] by Taylor's method. Runge–Kutta methods have also been studied by authors [2,22].V.Nirmala, N.Saveetha, S.Chenthurpandiyan discussed on numerical Solution of fuzzy differential equations by Runge-Kutta method with higher order derivative approximations[21]. R.Gethsi sharmila and E.C.Henry Amirtharaj discussed on numerical solutions of first order fuzzy initial value problems by Non-linear trapezoidal formulae based on variety of Means[13].Runge-kutta second order method for two stages contra-harmonic mean was discussed by Osama Yusuf Ababneh,Rokiah Rozita[17].

Following the introduction, this paper is organized as follows: In section 2, some basic results of fuzzy numbers and definitions of fuzzy derivative are given. In section 3,the fuzzy initial value problem is discussed. Section 4 describes the structure of Runge-kutta second order method for two stages of contra-harmonic mean was proposed. In section 5, the proposed method is used for solving fuzzy differential equations and the numerical example is given to illustrate the applicability of the proposed method followed by the conclusion is given in the last section.

II. PRELIMINARIES

A. Definition (FUZZY NUMBER)

An arbitrary fuzzy number is represented by an ordered pair of functions $(\underline{u}(r),\overline{u}(r))$ for all $r\in[0,1]$ which satisfy the following conditions.

i) $\underline{u}(r)$ is a bounded left continuous non-decreasing function over $[0,1]$ with respect to any 'r'.

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ii) $\overline{u}(r)$ is a bounded right continuous non-decreasing function over $[0,1]$ with respect to any 'r'.

iii) $(\underline{u}(r) \leq \overline{u}(r))$ for all $r \in [0,1]$ then the r-level set is $[u]_r = \{x \setminus u(x) \geq r\};0 \leq r \leq 1$

Clearly, $[u]_0 = \{x \setminus u(x) \geq 0\}$ is compact, which is a closed bounded interval and we denote by $[u]_r = (\underline{u}(r), \overline{u}(r))$

B. *Definition (TRIANGULAR FUZZY NUMBER)*

A triangular fuzzy number 'u' is a fuzzy set in E that is characterized by an ordered triple $(u_l, u_c, u_r) \in R^3$ with $u_l \leq u_c \leq u_r$ such that $[u]_0 = [u_l; u_r]$ and $[u]_l = \{u_c\}$.

The membership function of the triangular fuzzy number 'u' is given by

$$u(x) = \begin{cases} \dfrac{x-u_l}{u_c-u_l} & ; \quad u_l \leq x \leq u_c \\ 1 & ; \quad x = u_c \\ \dfrac{u_r-x}{u_r-u_c} & ; \quad u_c \leq x \leq u_r \end{cases}$$

. we have :*(1)* $u > 0 \ if \ u_l > 0$

(2) $u \geq 0 \ if \ u_l \geq 0$

(3) $u < 0 \ if \ u_c < 0$ and

(4) $u \leq 0 \ if \ u_c \leq 0$.

C. α - Level Set

Let I be the real interval. A mapping $y: I \rightarrow E$ is called a fuzzy process and its α - *level Set* is denoted by $[y(t)]_\propto = [\underline{y}(t;\alpha), \overline{y}(t;\alpha)]$, $t \in I, 0 < \propto < 1$

D. *Seikkala Derivative*

The Seikkala derivative $y'(t)$ of a fuzzy process is defined by $[y'(t)]_\propto = [\underline{y}'(t;\alpha), \overline{y}'(t;\alpha)]$ $t \in I, 0 < \propto \leq 1$ provided that this equation defines a fuzzy number, as in [24]

E. *Lemma*

If the sequence of non-negative number $\{W_n\}_{n=0}^m$ satisfy $|Wn+1| \leq A|Wn| + B, 0 \leq n \leq N-1$ for the given positive constants *A* and *B*, then $|Wn| \leq A^n |W_0| + B\dfrac{A^n-1}{A-1}, 0 \leq n \leq N$

F. *Lemma*

If the sequence of non-negative numbers $\{W_n\}_{n=0}^m, \{V_n\}_{n=0}^N$ satisfy

$$|W_{n+1}| \leq |W_n| + A\, max\{|W_n|, |V_n|\} + B$$

$$|V_{n+1}| \leq |V_n| + A\, max\{|W_n|, |V_n|\} + B$$

for the given positive constants *A* and *B*, then $U_n = |W_n| + |V_n|, \ 0 \leq n \leq N$

we have, $U_n \leq \overline{A}^n U_0 + B\dfrac{\overline{A}^n-1}{A-1} \quad 0 \leq n \leq N$ where $\overline{A} = 1+2A$ and $\overline{B} = 2B$.

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G. Lemma

Let $F(t,u,v)$ and $G(t,u,v)$ belong to $C'(R_F)$ and the partial derivatives of F and G be bounded over R_F .Then for arbitrarily fixed r, $0 \le r \le 1$, $D(y(t_{n+1}), y^0(t_{n+1})) \le h^2 L(1+2C)$ where L is a bound of partial derivatives of F and G, and $C = Max\left\{\left|G\left[t_N, \underline{y}(t_N;r), \overline{y}(t_{N-1};r)\right]\right|, r \in [0,1]\right\} < \infty$

H. Theorem

Let $F(t,u,v)$ and $G(t,u,v)$ belong to $C'(R_F)$ and the partial derivatives of F and G be bounded over R_F .Then for arbitrarily fixed r, $0 \le r \le 1$ the numerical solutions of $\underline{y}(t_{n+1};r)$ and $\overline{y}(t_{n+1};r)$ converge to the exact solutions $\underline{Y}(t_{n+1};r)$ and $\overline{Y}(t_{n+1};r)$ uniformly in t.

I. Theorem

Let $F(t,u,v)$ and $G(t,u,v)$ belong to $C'(R_F)$ and the partial derivatives of F and G be bounded over R_F and $2Lh < 1$ .Then for arbitrarily fixed $0 \le r \le 1$,the iterative numerical solutions of $\underline{y}^{(j)}(t_n;r)$ and $\overline{y}^{(j)}(t_n;r)$ converge to the numerical solutions $\underline{y}(t_n;r)$ and $\overline{y}(t_n;r)$ in $t_0 \le t_n \le t_N$, when $j \to \infty$.

III. FUZZY INITIAL VALUE PROBLEM

A first-order fuzzy initial value problem is given by

$$\begin{cases} y'(t) = f(t, y(t)), t \in [t_0, T] \\ y(t_0) = y_0 \end{cases} \tag{3.1}$$

Where 'y' is a fuzzy function of 't', $f(t,y)$ is a fuzzy function of the crisp variable 't' and the fuzzy variable 'y', y' is the fuzzy derivative of y and $y(t_0) = y_0$ is a triangular or a triangular shaped fuzzy number.

We denote the fuzzy function 'y by $y = [\underline{y}, \overline{y}]$ .It means that the r-level set of $y(t)$ for $t \in [t_0, T]$ is $[y(t)]_r = [\underline{y}(t;r), \overline{y}(t;r)]$, $[y(t_0)]_r = [\underline{y}(t_0;\ r), \overline{y}(t_0;\ r)]$, $r \in (0,1]$,

we write $f(t,y) = [\underline{f}(t,y), \overline{f}(t,y)]$ and

$$\underline{f}(t,y) = F[t, \underline{y}, \overline{y}], \qquad \overline{f}(t,y) = G[t, \underline{y}, \overline{y}]$$

because of $y' = f(t,y)$ we have

$$\underline{f}(t, y(t); r) = F[t, \underline{y}(t;r), \overline{y}(t;r)], \tag{3.2}$$

$$\overline{f}(t, y(t); r) = G[t, \underline{y}(t;r), \overline{y}(t;r)] \tag{3.3}$$

by using the extension principle, we have the membership function

$$f(t, y(t))(s) = sup\{y(t)(\tau) \setminus s = f(t,\tau)\},\ s \in R \tag{3.4}$$

so the fuzzy number $f(t, y(t))$ follows that

$$[f(t, y(t))]_r = [\underline{f}(t, y(t); r), \overline{f}(t, y(t); r)],\ r \in (0,1] \tag{3.5}$$

where $\underline{f}(t, y(t); r) = min\{f(t,u) \mid u[y(t)]_r\}$ (3.6)

$$\overline{f}(t, y(t); r) = max\{f(t,u) \mid u[y(t)]_r\} \tag{3.7}$$

Definition 3.1 A function $f: R \to R_F$ is said to be fuzzy continuous function, if for an arbitrary fixed $t_0 \in R$ and $\varepsilon > 0$, $\delta > 0$ such that $|t - to| < \delta \Rightarrow D[f(t), f(t_0)] < \varepsilon$ exists.

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The fuzzy function considered are continuous in metric D and the continuity of $f(t, y(t); r)$ guarantees the existence of the definition of $f(t, y(t); r)$ for $t \in [t_0, T]$ and $r \in [0,1]$ [10]. Therefore, the functions G and F can be definite too.

IV. RUNGE-KUTTA SECOND ORDER METHOD FOR TWO STAGES CONTRA-HARMONIC MEAN

Runge-kutta second order method for two stages contra-harmonic mean is considered for approximating the solution of first order fuzzy initial value problem $y'(t) = f(t, y(t)) \;\; y(t_0) = y_0$.

The basis of all Runge-Kutta methods is to express the difference between the value of y at t_{n+1} and t_n as

$$y_{n+1} - y_n = \sum_{i=0}^{m} w_i k_i \tag{4.1}$$

where w_i's are constant for all 'i' and $k_i = hf(t_n + a_i h, y_n + \sum_{j=1}^{i-1} c_{ij} k_j)$ (4.2)

Increasing of the order of accuracy, the Runge-Kutta methods have been accomplished by increasing the number of taylor's series terms used and thus the number of functional evaluations required[5].The method proposed by Goeken.D and Johnson.O [9] introduces new terms involving higher order derivatives of 'f' in the Runge-Kutta terms(i >0) to obtain a higher order of accuracy without a corresponding increase in evaluations of 'f '. Runge-kutta second order method for two stages contra-harmonic mean was discussed by Osama Yusuf Ababneh, and Rokiah Rozita[17]

Consider $$y(t_{n+1}) = y(t_n) + h\left[\frac{k_1^2 + k_2^2}{k_1 + k_2}\right] \tag{4.3}$$

Where $$k_1 = hf(t_n, y(t_n)) \tag{4.4}$$
$$k_2 = hf(t_n + h, y(t_n) + a_1 k_1) \tag{4.5}$$

and the parameter a_1 is chosen to make y_{n+1} closer to $y(t_{n+1})$ .The value of parameters are $a_1 = 1$

V. SOLVING FUZZY DIFFERENTIAL EQUATIONS USING RUNGE-KUTTA SECOND ORDER METHOD FOR TWO STAGES CONTRA-HARMONIC MEAN

Let the exact solution $[Y(t)]_r = [\underline{Y}(t;r), \overline{Y}(t;r)]$,is approximated by some $[y(t)]_r = [\underline{y}(t;r), \overline{y}(t;r)]$.

The grid points at which the solutions is calculated are $h = \frac{T - t_0}{N}$, $t_i = t_0 + ih; 0 \le i \le N$

From 4.3 to 4.5 we define

$$\underline{y}(t_{n+1}, r) - \underline{y}(t_n, r) = h\left[\frac{\underline{k}_1^2(t_n, y(t_n, r)) + \underline{k}_2^2(t_n, y(t_n, r))}{\underline{k}_1(t_n, y(t_n, r)) + \underline{k}_2(t_n, y(t_n, r))}\right] \tag{5.1}$$

where

$$k_1 = hF[t_n, \underline{y}(t_n, r), \overline{y}(t_n, r)] \tag{5.2}$$

$$k_2 = hF[t_n + h, \;\; \underline{y}(t_n, r) + \underline{k}_1(t_n, y(t_n, r)), \;\; \overline{y}(t_n, r) + \overline{k}_1(t_n, y(t_n, r))] \tag{5.3}$$

and

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$$\overline{y}(t_{n+1},r)-\overline{y}(t_n,r)=h\left[\frac{\overline{k_1}^2(t_n,y(t_n,r))+\overline{k_2}^2(t_n,y(t_n,r))}{\overline{k_1}(t_n,y(t_n,r))+\overline{k_2}(t_n,y(t_n,r))}\right] \quad (5.4)$$

Where

$$k_1=hG[t_n,\underline{y}(t_n,r),\overline{y}(t_n,r)] \quad (5.5)$$

$$k_2=hG[t_n+h,\ \underline{y}(t_n,r)+\underline{k_1}(t_n,y(t_n,r)),\ \overline{y}(t_n,r)+\overline{k_1}(t_n,y(t_n,r))] \quad (5.6)$$

we define

$$F(t_n,y(t_n,r))=h\left[\frac{\underline{k_1}^2(t_n,y(t_n,r))+\underline{k_2}^2(t_n,y(t_n,r))}{\underline{k_1}(t_n,y(t_n,r))+\underline{k_2}(t_n,y(t_n,r))}\right] \quad (5.7)$$

$$G(t_n,y(t_n,r))=h\left[\frac{\overline{k_1}^2(t_n,y(t_n,r))+\overline{k_2}^2(t_n,y(t_n,r))}{\overline{k_1}(t_n,y(t_n,r))+\overline{k_2}(t_n,y(t_n,r))}\right] \quad (5.8)$$

Therefore we have

$$\underline{Y}(t_{n+1},r)=\underline{Y}(t_n,r)+F[t_n,Y(t_n,r)]]$$

$$\overline{Y}(t_{n+1},r)=\overline{Y}(t_n,r)+G[t_n,Y(t_n,r)] \quad (5.9)$$

and

$$\underline{y}(t_{n+1},r)=\underline{y}(t_n,r)+F[t_n,y(t_n,r)] \quad (5.10)$$

$$\overline{y}(t_{n+1},r)=\overline{y}(t_n,r)+G[t_n,y(t_n,r)]$$

clearly $\underline{y}(t;r)$ and $\overline{y}(t;r)$ converge to $\underline{Y}(t;r)$ and $\overline{Y}(t;r)$ whenever $h\to 0$

VI. NUMERICAL EXAMPLE

Consider a fuzzy initial value problem

$$\begin{cases} y'(t)=y(t), & t\geq 0 \\ y(0)=(0.75+0.25r,1.125-0.125r) \end{cases} \quad (6.1)$$

The exact solution is given by

$$Y(t,r)=[(0.75+0.25r)e^t,(1.125-0.125r)e^t]$$

at t=1we get

$$Y(1,r)=[(0.75+0.25r)e,(1.125-0.125r)e],\ 0\leq r\leq 1$$

The values of exact and approximate solution with h= 0.1 is given in Table : I .The exact and approximate solutions obtained by the proposed method is plotted in Fig:I and the error estimation of the exact and approximate solution is given in Fig II.

Table: I

r	Exact Solution t=1		Approximate Solution (h=0.1)		Error 1	Error 2
	$\underline{Y}(t;r)$	$\overline{Y}(t;r)$	$\underline{y}(t;r)$	$\overline{y}(t;r)$		
0.0	2.038711 ,	3.058067	2.039951 ,	3.059926	1.239558e-003	1.859337e-003
0.1	2.106668 ,	3.024089	2.107949 ,	3.025927	1.280876e-003	1.838677e-003
0.2	2.174625 ,	2.990110	2.175948 ,	2.991928	1.322195e-003	1.818018e-003
0.3	2.242583 ,	2.956131	2.243946 ,	2.957929	1.363513e-003	1.797359e-003

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0.4	2.310540 , 2.922153	2.311944 , 2.923930	1.404832e-003	1.776699e-003
0.5	2.378497 , 2.888174	2.379943 , 2.889930	1.446151e-003	1.756040e-003
0.6	2.446454 , 2.854196	2.447941 , 2.855931	1.487469e-003	1.735381e-003
0.7	2.514411 , 2.820217	2.515939 , 2.821932	1.528788e-003	1.714721e-003
0.8	2.582368 , 2.786239	2.583938 , 2.787933	1.570106e-003	1.694062e-003
0.9	2.650325 , 2.752260	2.651936 , 2.753934	1.611425e-003	1.673403e-003
1.0	2.718282 , 2.718282	2.719935 , 2.719935	1.652744e-003	1.652744e-003

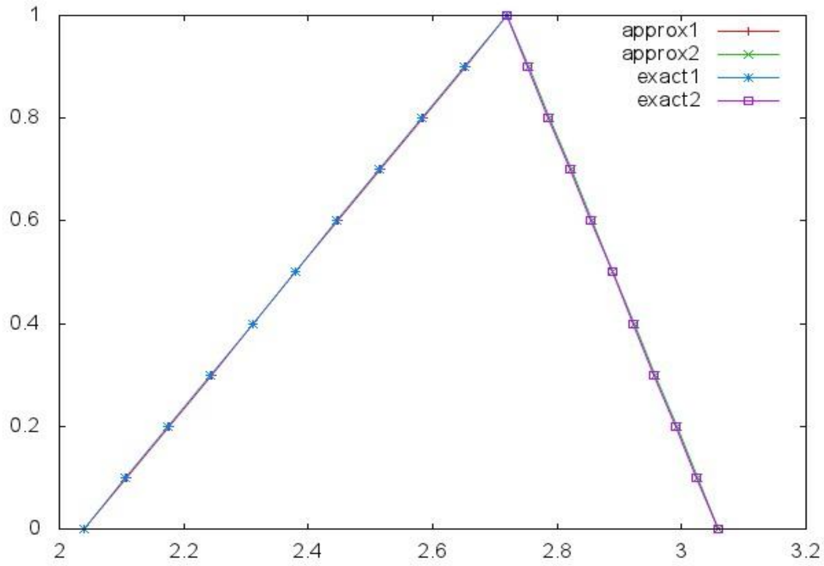

Fig – I (Exact & Approximate)

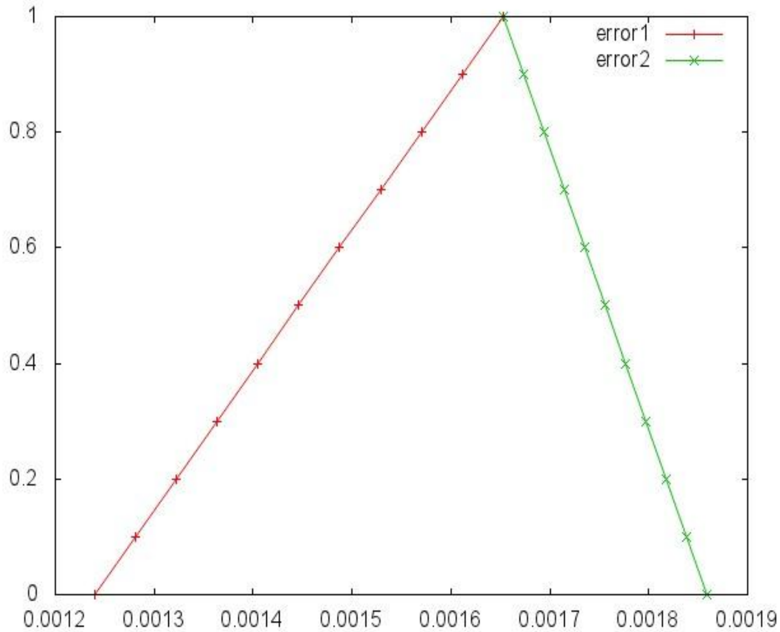

Fig – II (Error I & Error II)

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VII. CONCLUSION

In this paper the runge-kutta second order method for two stages contra-harmonic mean has been applied for finding the numerical solution of first order fuzzy differential equations using triangular fuzzy number. The efficiency and the accuracy of the proposed method have been illustrated by a suitable example. From the numerical example it has been observed that by minimizing the step size'h' the exact solution at 'h' and the approximate solution obtained by the proposed method almost coincides.

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