ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 5, Issue 1, January 2016

Solving multi objective inventory model of deteriorating items using intuitionistic fuzzy optimization technique

[1]A.Faritha Asma, [2]E.C.Henry Amirtharaj,
[1]Assistant professor, Department of Mathematics Government Arts College, Trichy-22.Tamilnadu, India
[2]Associate professor, Department of Mathematics, Bishop Heber College, Trichy-17.Tamilnadu, India

Abstract-In this paper a multi objective and multi item EOQ model with stock dependent demand for deteriorating items under the space and investment constraints is considered in fuzzy environment. Various inventory costs, the storage area and the amount of investment are taken as triangular fuzzy numbers. The aim of this paper is present a method in which a fuzzy inventory model is reduced to crisp using ranking function and then the crisp EOQ is solved by intuitionistic fuzzy programming technique. . Linear membership and non membership function is considered for fuzzy objectives. The method is illustrated with a numerical example.

Index Terms— **Fuzzy inventory, Deteriorating items, Triangular fuzzy number, Intuitionistic Fuzzy optimization, Fuzzy ware house capacity, Fuzzy maximum investment, Fuzzy ranking.**

I. INTRODUCTION

Inventory problems are common in manufacturing, maintenance service and business operations in general. Often uncertainties may be associated with demand, various relevant costs. Generally demand rate is considered to be constant, time dependent, ramp type and selling price dependent. However in present competitive market stock-dependent demand plays an important role in increase its demand. Deterioration is one of the factors in inventory system. Some items like food grains, vegetables, milk, eggs etc. deteriorating during their storage time and retailer suffers loss. In an inventory system available storage space, budget, number of orders etc. are always limited hence multi-item classical inventory models under these constraints have great importance.

In conventional inventory models, uncertainties are treated as randomness and are handled by probability theory. Furthermore, when addressing real world problems, frequently the parameters are imprecise numerical quantities. However in certain situations, uncertainties are due to fuzziness and in such cases the fuzzy set theory introduced by Zadeh [15] is applicable. There are several studies on fuzzy EOQ model. Lin et al. have developed a fuzzy model for production inventory problem. Katagiri and Ishii [5] have proposed an inventory problem with shortage cost as fuzzy quantity.

Now a days different modification and generalized form of fuzzy set theory have appeared intuitionistic fuzzy set (IFS).The concept of an IFS can be viewed as an alternative approach to define a fuzzy set in case where available information is not sufficient for the definition of an imprecise concept by means of a conventional fuzzy sets. In this paper, under limited storage area and investment, a multi-item multi-objective inventory model of deteriorating item with stock-dependent demand is formulated in crisp and fuzzy environment. We introduced a method in which a fuzzy inventory model is first reduced to crisp inventory model using ranking function suggested by Robust’s and the resulting one is solved by intuitionistic fuzzy programming technique of Atanassov [17] . The model is illustrated numerically and the results are obtained from different methods.

II. PRELIMINARIES

Definition (1):A fuzzy set is characterized by a membership function mapping elements of a domain, space or universe of discourse X to the unit interval [0,1].(i.e) $A=\{X,\mu_A(x):x\in X\}$,here $\mu_A:X\rightarrow[0,1]$ is a mapping called the degree of membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value of $x\in X$ in the fuzzy set A. These membership grades are often represented by real numbers ranking from [0,1].

ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 5, Issue 1, January 2016

Definition(2): Atanassov [17], Let X={$x_1, x_2, ...x_n$} be a finite universal set. An Atanassov's intuitionistic fuzzy set (IFS) in a given universal set X is an expression $\hat{A}$ is given by $\hat{A}=\{\langle x_i, \mu_{\hat{A}}(x_i), \nu_{\hat{A}}(x_i)\rangle : x_i \in X\}$

Where the functions $\mu_{\hat{A}} : X \to [0,1]$ i.e. $x_i \in X \to \mu_{\hat{A}}(x_i) :\in [0,1]$ and $\nu_{\hat{A}} : X \to [0,1]$ i.e. $x_i \in X \to \nu_{\hat{A}}(x_i) \in [0,1]$ define the degree of membership and the degree of non-membership respectively of an element $x_i \in X$ satisfy the condition: for every $x_i \in X$, $0 \le \mu_{\hat{A}}(x) + \nu_{\hat{A}}(x) \le 1$.

Definition(3): Let $\hat{A}$ and $\hat{B}$ be two Atanassov's IFSs in the set X. The intersection of $\hat{A}$ and $\hat{B}$ is defined as follows:

$$\hat{A} \cap \hat{B} = \left\{\left\langle x_i, \min\left(\mu_{\hat{A}}(x_i), \mu_{\hat{B}}(x_i)\right), \max\left(\nu_{\hat{A}}(x_i), \nu_{\hat{B}}(x_i)\right)\right\rangle \middle| x_i \in X\right\}.$$

Definition (4): (Triangular fuzzy number)

For a triangular fuzzy number A(x), it can be represented by A(a,b,c:1) with membership function $\mu_A(x)$ given by

$$\mu_A(x) = \begin{cases} \frac{(x-a)}{(b-a)}; a \le x \le b \\ 1 \quad ; x = b \\ \frac{(c-x)}{(c-b)}; b \le x \le c \\ 0 \quad ; otherwise \end{cases}$$

Definition (5): (α-cut of a fuzzy number)

The α-cut of a fuzzy number A(x) is defined as A(α)={x:μ(x)≥α, α∈[0,1]}

III. RANKING FUNCTION FOR FUZZY NUMBER

Assume that R:F($\mathcal{R}$) →$\mathcal{R}$ be linear ordered function that maps each fuzzy number in to the real number, in which F($\mathcal{R}$) denotes the whole fuzzy numbers. Accordingly for any two fuzzy numbers $\tilde{a}$ and $\tilde{b}$ we have

$$\tilde{a} \underset{R}{\ge} \tilde{b} \text{ iff } R(\tilde{a}) \ge R(\tilde{b})$$

$$\tilde{a} \underset{R}{>} \tilde{b} \text{ iff } R(\tilde{a}) > R(\tilde{b})$$

$$\tilde{a} \underset{R}{=} \tilde{b} \text{ iff } R(\tilde{a}) = R(\tilde{b})$$

We restrict our attention to linear ranking function, that is a ranking function R such that $R(k\tilde{a}+\tilde{b}) = kR(\tilde{a}) + R(\tilde{b})$ for any $\tilde{a}$ and $\tilde{b}$ in F(R) and any k ϵ R.

Robust's ranking function:

The ranking function proposed by Robust's for a triangular fuzzy number $\tilde{a}$ is defined by

$R(\tilde{a}) = \frac{1}{2}\int_0^1 (a_\alpha^L, a_\alpha^U) d\alpha$, where a_α^L, a_α^U is the lower and upper limit of the α-cut of a fuzzy number $\tilde{a}$

IV.ASSUMPTIONS AND NOTATIONS

(i) The scheduling period is constant and no lead time.
(ii)Replenishment rate is infinite.
(iii) Selling price is known and constant.
(iv) Demand rate is stock dependent.
(v)Shortages are not allowed.
(vi) Deteriorating rate is age specific failure rate.

ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 5, Issue 1, January 2016

T_i : Time period for each cycle for the i^th^ item.
R_i : Demand rate per unit time of i^th^ item. [$R_i = a_i + b_i q_i$]
θ_i : Deteriorating rate of i^th^ item.
$Q_i(t)$: Inventory level at time t of i^th^ item.
C_H : Total Holding cost.
C_{1i} : Holding cost per unit of i^th^ item.
C_{3i} : Setup cost for i^th^ item.
S_{di} : Total deteriorating units of i^th^ item.
P_i : Selling price per unit of i^th^ item.
Q_i : Initial stock level of i^th^ item.
N : Number of items.
TC (Q_i) : Sum of costs of the system
PF(Q_i) : Sum of profits of the system
WC(Q_i) : Sum of wastage costs of the system
(wavy bar (~) represents the fuzzification of the parameters)

V. MATHEMATICAL FORMULATION

A. Crisp model

As $Q_i(t)$ is the inventory level at time t of i^{th} item, then the differential equation describing the state of inventory is given by

$$\frac{d}{dt}Q_i(t)+\theta_i Q_i(t)=-(a_i+b_iQ_i(t)) \qquad 0\le t\le T_i$$

Solving the above differential equation using boundary condition $Q_i(t)= Q_i$ at t=0, we get

$$Q_i(t)=-\frac{a_i}{(\theta_i+b_i)}+\left[Q_i+\frac{a_i}{(\theta_i+b_i)}\right]e^{-(\theta_i+b_i)t} \qquad (1)$$

And using boundary condition $Q_i(t)= 0$ at $t=T_i$

$$\therefore\ T_i=\frac{1}{(\theta_i+b_i)}\log\left\{1+\frac{(\theta_i+b_i)Q_i}{a_i}\right\} \qquad (2)$$

The holding cost of i^{th} item in each cycle is

$$C_H = C_{1i}\, G_i\,(Q_i) \qquad (3)$$

Where $G_i(Q_i)=\int_0^{Q_i}\frac{q_i\,dq_i}{a_i+(\theta_i+b_i)q_i}$

$$=\frac{Q_i}{(\theta_i+b_i)}+\frac{a_i}{(\theta_i+b_i)^2}\log\left\{1+\frac{(\theta_i+b_i)Q_i}{a_i}\right\}$$

By neglecting the higher power terms, we get

$$G_i(Q_i)=\frac{Q_i^2}{2a_i}\left\{1-2\frac{(\theta_i+b_i)Q_i}{3a_i}\right\}$$

The total number of deteriorating units of the i^{th} item is

$$S_{di}(Q_i)=\theta_i G_i(Q_i)$$

The net revenue for the i^{th} item is

$$N(Q_i)=(P_i-C_i)Q_i-P_i.S_{di}(Q_i)$$
$$N(Q_i)=(P_i-C_i)Q_i-P_i\theta_i.G_i(Q_i) \qquad (4)$$

The profit of i^{th} item is

$$PF_i(Q_i)=N(Q_i)-C_{1i}G_i(Q_i)-C_{3i}, \qquad i=1,2,.....n.$$

$$PF_i(Q_i)=(P_i-C_i)Q_i-P_i\theta_i.G_i(Q_i)-C_{1i}G_i(Q_i)-C_{3i} \qquad i=1,2,.........n. \qquad (5)$$

$$PF_i(Q_i)=(P_i-C_i)Q_i-(C_{1i}+P_i\theta_i)G_i(Q_i)-C_{3i}, \ i=1,2,..............n.$$

ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 5, Issue 1, January 2016

The cost of i^{th} item is

$$TC_i(Q_i) = C_{1i}G_i(Q_i) + C_{3i} + C_iQ_i \qquad i=1,2,.............n$$

The wastage cost of i^{th} item is

$$WC_i(Q_i) = C_i\theta_iG_i(Q_i) \qquad i=1,2,.........n \qquad \textbf{(6)}$$

Hence the problem is

Maximize $PF(Q_i) = \sum_{i=1}^{n} PF_i(Q_i)$

Minimize $TC(Q_i) = \sum_{i=1}^{n} TC_i(Q_i)$

Minimize $WC(Q_i) = \sum_{i=1}^{n} WC_i(Q_i)$ **(7)**

Subject to: $\sum_{i=1}^{n} f_iQ_i \leq F$

$\sum_{i=1}^{n} C_iQ_i \leq B$

B. Fuzzy model

When the inventory parameters such as setup cost, holding cost, rate of deterioration, total storage area and maximum amount of investment are fuzzy, the said crisp model (7) is transformed to a fuzzy model and represented as

$$M\tilde{a}x\, PF = \sum_{i=1}^{n}\left[(P_i - C_i)Q_i - (\tilde{C}_{1i} + P_i\tilde{\theta}_i)G_i(Q_i) - \tilde{C}_{3i}\right]$$

$$M\tilde{i}n\, TC(Q_i) = \sum_{i=1}^{n} \tilde{C}_{1i}G_i(Q_i) + \tilde{C}_{3i} + C_iQ_i$$

$$M\tilde{i}n\, WC(Q_i) = \sum_{i=1}^{n} C_i\tilde{\theta}_iG_i(Q_i) \qquad \textbf{(8)}$$

Subject to: $\sum_{i=1}^{n} f_iQ_i \leq \tilde{F}$

$\sum_{i=1}^{n} C_iQ_i \leq \tilde{B}$

$Q_i \geq 0, \qquad i=1,2,..............n$

Now the above fuzzy multi objective inventory problem (FMIP) can be easily transformed into a classical form of a multi objective inventory problem (MOIP) by considering R as a linear ranking function. By implementing the R on the above model (8). We obtain the classical form of MOIP :

$$MaxPF' = \sum_{i=1}^{n}\left[(P_i - C_i)Q_i - (R(\tilde{C}_{1i}) + P_iR(\tilde{\theta}_i))G_i(Q_i) - R(\tilde{C}_{3i})\right]$$

ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 5, Issue 1, January 2016

$$MinTC^{'}=\sum_{i=1}^{n}\left[R(\tilde{C}_{1i})G_i(Q_i)+R(\tilde{C}_{3i})+C_iQ_i\right]$$

$$MinWC^{'}=\sum_{i=1}^{n}\left[C_iR(\tilde{\theta}_i)G_i(Q_i)\right]$$

$$\sum_{i=1}^{n}f_iQ_i\le R(\tilde{F})$$

$$\sum_{i=1}^{n}C_iQ_i\le R(\tilde{B}) \qquad \textbf{(9)}$$

VI. MATHEMATICAL ANALYSIS

Intuitionistic Fuzzy optimization technique

To solve the MOIP (9) we have used the following intuitionistic fuzzy programming technique.

Step 1: Solve the multi-objective programming problem as a single objective problem using only one objective at a time and ignoring the rest objectives subject to constraints of storage space. Let $X^{'}$ be the optimal solution for the i^{th} single objective problem.

Step 2: From the results of step 1, determine the corresponding values for every objective at each optimal solution derive. Using all the above optimal values of the objectives in step-1, construct a pay-off matrix (3x3) as follows:

$$\begin{array}{c} \\ X^1 \\ X^2 \\ X^3 \end{array}\begin{array}{ccc} PF(X) & TC(X) & WC(X) \\ \end{array}$$

$$\begin{matrix} X^1 \\ X^2 \\ X^3 \end{matrix}\begin{bmatrix} PF\left(X^1\right) & TC\left(X^1\right) & WC\left(X^1\right) \\ PF\left(X^2\right) & TC\left(X^2\right) & WC(X^2) \\ PF\left(X^3\right) & TC\left(X^3\right) & WC\left(X^3\right) \end{bmatrix}$$

Here, the Diagonal elements represent the optimal values of the corresponding objectives. From the pay-off matrix we find lower bounds

L_{PF} =Min $(PF(X^1),PF(X^2),PF(X^3))$, L_{TC} =Min $(TC(X^1),TC(X^2),TC(X^3))$, L_{WC} =Min $(WC(X^1),WC(X^2),WC(X^3))$.
And the upper bounds,
U_{PF} =Max $(PF(X^1),PF(X^2),PF(X^3))$, U_{TC} =Max $(TC(X^1),TC(X^2),TC(X^3))$, U_{WC} =Max$(WC(X^1),WC(X^2),WC(X^3))$.
Then the objective summations are estimated as

$L_{PF}\le PF(X)\le U_{PF}$ $L_{TC}\le TC(X)\le U_{TC}$ and $L_{WC}\le WC(X)\le U_{WC}$.

Step 3: From step 2, we may find for each objective the value L_K and U_K corresponding to the set of solutions. For the multi-objective problem (8), the membership and non-membership functions are $\mu_{PF}(X)$, $\mu_{TC}(X)$, $\mu_{WC}(X)$ and $\nu_{PF}(X)$, $\nu_{TC}(X)$, $\nu_{WC}(X)$ respectively. We have considered linear and non-membership functions which are defined below:

$$\mu_{PF}(X)=\begin{cases} 1 & ;PF(X)>U_{PF} \\ \dfrac{PF(X)-L_{PF}}{d_{PF}} & ;L_{PF}\le PF(X)\le U_{PF} \\ 0 & ;PF(X)<L_{PF} \end{cases}$$

ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 5, Issue 1, January 2016

$$\mu_{TC}(X)=\begin{cases}1 & ;TC(X)<L_{TC}\\ \dfrac{U_{TC}-TC(X)}{d_{TC}} & ;L_{TC}\le TC(X)\le U_{TC}\\ 0 & ;TC(X)>U_{TC}\end{cases}$$

$$\mu_{WC}(X)=\begin{cases}1 & ;WC(X)<L_{WC}\\ \dfrac{U_{WC}-WC(X)}{d_{WC}} & ;L_{WC}\le WC(X)\le U_{WC}\\ 0 & ;WC(X)>U_{WC}\end{cases}$$

(10) $$\nu_{PF}(X)=\begin{cases}1 & ;PF(X)<L_{PF}\\ \dfrac{U_{PF}-PF(X)}{d_{PF}} & ;L_{PF}\le PF(X)\le U_{PF}\\ 0 & ;PF(X)>U_{PF}\end{cases}$$

$$\nu_{TC}(X)=\begin{cases}1 & ;TC(X)>U_{TC}\\ \dfrac{TC(X)-L_{TC}}{d_{TC}} & ;L_{TC}\le TC(X)\le U_{TC}\\ 0 & ;TC(X)<L_{TC}\end{cases}$$

$$\nu_{WC}(X)=\begin{cases}1 & ;WC(X)>U_{WC}\\ \dfrac{WC(X)-L_{WC}}{d_{WC}} & ;L_{WC}\le WC(X)\le U_{WC}\\ 0 & ;WC(X)<L_{WC}\end{cases}$$

Step 4: After determining the membership and non-membership function defined in (10) for each objective functions following the problem (9) can be formulated an equivalent crisp model on the basis of definition 2 of this paper as

Max α , min β

$$\alpha\le\mu_{PF}(x)\,,\alpha\le\mu_{TC}(x),\alpha\le\mu_{WC}(x)$$

$$\beta\ge\nu_{PF}(x),\beta\ge\nu_{TC}(x),\beta\ge\nu_{WC}(x)$$

$$\sum_{i=1}^{n}f_iQ_i\le R(\tilde{F})$$

$$\sum_{i=1}^{n}C_iQ_i\le R(\tilde{B})$$

$$\alpha\ge\beta\ ;\ \alpha+\beta\le1;\alpha,\beta\ge0$$

$$0\le\alpha\le1,Q_i>0$$

Step 5: Now the above problem can be solved by a non-linear optimization technique and optimal solution of α (say α*) and β (say β*) are obtained.

VII. NUMERICAL EXAMPLE

For the model let us assume

For n=2,

ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 5, Issue 1, January 2016

P_1=10, C_1=7, a_1=110, b_1=0.5, θ_1=0.025, P_2= 10, C_2=6.75, a_2=100, b_2=0.5, θ_2=0.03, C_{11}=2, C_{12}=2.2, C_{31}=65, C_{32}=50, B=1800.

Taking $\tilde{C}_{11}$ =(2.03,2.05,2.08); $\tilde{C}_{12}$ =(2.15,2.20,2.3); $\tilde{C}_{31}$ =(60,65,75); $\tilde{C}_{32}$ =(45,50,55);

$\tilde{\theta}_1$ =(0.022,0.025,0.03) $\tilde{\theta}_2$ =(0.025,0.028,0.030); $\tilde{F}$ =(155,165,175), $\tilde{B}$ =(1700,1800,1900)

	PF	TC	WC
PF	**497.56036**	**494.54190**	**495.579783**
TC	**1994.711443**	**1994.3144**	**1994.356398**
WC	**16.6919147**	**16.273999**	**16.399390**

The optimum value of α, Q_1* and Q_2* are

α	β	Q_1*	Q_2*
0.56736239	**0.43263761**	**40.593676**	**180.49202**

Now the optimal values of the objective functions are

Max PF*	Min TC*	Min WC*
496.25445	**1994.4861**	**16.454804**

VIII. CONCLUSION

In this paper, a real life inventory problem under the investment and storage space constraints in fuzzy environment has been proposed. Various inventory costs, storage space and the maximum amount of investment are taken as triangular fuzzy numbers. Initially the model is defuzzified using Robust's ranking technique. Then using intuitionistic fuzzy optimization technique the optimum results are obtained. Linear membership and non membership functions are considered for the fuzzy objective functions

REFERENCES

[1] Angelov, P.P., "Intuitionistic Fuzzy Optimization". Notes on Intuitionistic Fuzzy sets; (1995); 123-129.

[2] Bellman R.E.Zadeh L.A., "Decision making in a fuzzy environment management sciences", 1970, 17,141-164.

[3] Chen, S. H., Wang (1996), "Backorder fuzzy inventory model under function principal", Information Sciences, 95, pp 71-79.

[4] Giri B.C., Pal S., Goswami A. and Chudhuri K.S., (1996), "An inventory model for deteriorating items with stock-dependent demand rate", European Journal of Operations Research, 95, 604-610.

[5] Katagiri,H.,Ishii,H.; "Some inventory problems with fuzzy shortage cost. Fuzzy Sets and Systems"; 2000;111(1),87-97.

[6] Mandal, M.T.K., Roy and Maiti, M.,(1998), "A fuzzy inventory model of deteriorating items with stock-dependent demand under limited storage space", Operations Research, 35[4], 323-337.

[7] Mandal, S. and Maiti, M.,(2002), "Multi-item fuzzy EOQ models", Using Genetic Algorithm, Computers and Industrial Engineering., 44, 105-117.

[8] Moore, R.E., "Method and application of interval analysis". SIAM, Philadelphia ;(1979).

[9] Nagarajan.R and Solairaju.A., "Computing improved Fuzzy Optimal Hungarian Assignment Problems with Fuzzy costs under Robust Ranking Techniques", International Journal of Computer Application,Vol.6,No.4(2010).

[10] Omprakash Jadhav and V.H. Bajaj: "Multi-objective inventory model of deteriorating items with fuzzy inventory cost and some fuzzy constraints". Int. Jr. Of Agri. And Stat. Sciences, vol.6, No.2. 2010, pp.529-538.

[11] Omprakash Jadhav and V.H. Bajaj: "multi-objective inventory model of deteriorating items with shortages in fuzzy environment".

[12] Roy,T.K. and Maiti, M.,(2000) " multi-item displayed EOQ model in fuzzy environment".

[13] Susovan Chakrabortty, Madhumangal Pal and Prasun Kumar Nayak, "Intuitionistic fuzzy optimization technique for Pareto Optimal solution of manufacturing Inventory models with shortages", European journal of Operational Research, 228(2013) 381-387.

[14] Umap. H.P. "Fuzzy EOQ model for deteriorating items with exponential membership function", Department of Statistics, Yashavantrao Chavan Institute of Science, Satara(M.S).

[15] Zadeh L.A."Fuzzy Sets, Information and Control",8,(1965).

[16] Zimmermann.H.J. "Fuzzy Set Theory and its Applications" ,Allied Publishers Limited ,New Delhi in association with Kluwar Academic Publishers 2nd Revised Edition, Boston 1996.

[17] Atanassov,K. Intuitionistic fuzzy sets. Fuzzy sets and Systems ;(1986) 20, 87-96.

[18] Durga Prasad Dash.P., Rajani B. Dash., "Solving Fuzzy multi objective non-linear programming problem using fuzzy programming technique".