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S-α Anti Fuzzy Semigroups

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Abstract: *The concept of S-α anti fuzzy semigroups is introduced and its properties are derived. S-α anti fuzzy left coset, S-α anti fuzzy right coset and S-α anti fuzzy normal subsemigroups are defined and some equivalent conditions on S-α fuzzy normal subsemigroups are proved.*

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I. INTRODUCTION AND PRELIMINARIES

In 1965, L.A. Zadeh introduced the concept of fuzzy set [15]. In [10] Rosenfeld used this concept to develop the theory of fuzzy groups. A.S. Mashour studied on normal fuzzy subgroups in [7]. In [13], W.B. Vasantha Kandhasamy studied about smarandache fuzzy semigroups. Biswas introduced the notion of anti fuzzy subgroups in [3]. In [4], R. Gowri and T.Rajeswari introduced the idea of S-α fuzzy semigroup and S-α fuzzy normal subsemigroups and discussed their properties. In [12], P.K. Sharma introduced the concept of α-anti fuzzy set, α-anti fuzzy groups, α-anti fuzzy cosets and α-anti fuzzy normal subgroups and derived their properties. In this paper, S-α anti fuzzy semigroups and S-α anti fuzzy normal subsemigroups are defined and their characterizations are discussed.

Definition 1.1 Let X be a non empty set. A **fuzzy subset** A of X is a function $A : X \rightarrow [0,1]$.

Definition 1.2 Let A and B be two fuzzy subsets of a set X. Then

(i) $A \subset B$ iff $A(x) \leq B(x)$, for all $x \in X$

(ii) $A = B$ iff $A \subset B$ and $B \subset A$

(iii)$(A \cup B)(x) = max\{A(x), B(x)\}$, for all $x \in X$.

Definition 1.3 A fuzzy subset A of a group G is called an **anti fuzzy** subgroup of G if

(i) $A(xy) \leq max\{A(x), A(y)\}$

(ii) $A(x^{-1}) = A(x)$, for all $x, y \in G$.

Definition 1.4 A fuzzy subset A of a semigroup S is called an **anti fuzzy semigroup** if $A(xy) \leq max\{A(x), A(y)\}$, for all $x, y \in S$.

Definition 1.5 A semigroup S is said to be a **Smarandache semigroup** (S-semigroup) if there exists a proper subset P of S which is a group under the same binary operation in S.

Result 1.6 *Ros* $A : G \rightarrow [0,1]$ is an anti fuzzy subgroup of a group G iff

$$A(xy^{-1}) \leq max\{A(x), A(y)\}, \text{ for all } x, y \in G.$$

Result 1.7 *Ros* If $A : G \rightarrow [0,1]$ is an anti fuzzy subgroup of a group G, then

(i) $A(x) \geq A(e)$, where e is the identity element of G.

(ii) $A(xy^{-1}) = A(e) \Rightarrow A(x) = A(y)$, for all $x, y \in G$.

Definition 1.8 Let S be an S-semigroup. A fuzzy subset A of S is said to be a **Smarandache anti fuzzy**

semigroup (S **-anti fuzzy semigroup)** if $A:S \to [0,1]$ is such that A is restricted to atleast one proper subset P of S which is a group and the restriction map $A_P : P \to [0,1]$ is an anti fuzzy group.

Definition 1.9 Let G be an S-semigroup. An S-anti fuzzy semigroup A of G is said to be a **Smarandache-anti fuzzy normal subsemigroup(** S **-anti fuzzy normal subsemigroup)** of G if $A_P(xy) = A_P(yx)$, for all $x, y \in P$, where P is a proper subset of G which is a group and the restriction map $A_P : P \to [0,1]$ is an anti fuzzy group.

Definition 1.10 Let A be a fuzzy subset of a group G and let $x \in G$. An **anti fuzzy left coset of** A, denoted by xA, is defined as $(xA)(y) = A(x^{-1}y)$, $y \in G$.Similarly, an **anti fuzzy right coset of** A, denoted by Ax, is defined as $(Ax)(y) = A(yx^{-1})$, $y \in G$.

Result 1.11 Let A be a fuzzy subset of a group G. Then the following conditions are equivalent for all $x, y \in G$

(i) $A(xyx^{-1}) \geq A(y)$

(ii) $A(xyx^{-1}) = A(y)$

(iii) $A(xy) = A(yx)$

(iv) $xA = Ax$

(v) $xAx^{-1} = A$

(vi) $A(y^{-1}xy) = A(x)$.

Remark 1.12

If A and B are two fuzzy subsets of a set G and H is a subset of G,then $(A \cap B)_H = A_H \cap B_H$,where A_H denotes the restriction of A to H.

Definition 1.13 Let A be a fuzzy subset of a group G. Let $\alpha \in [0,1]$. Then α -anti fuzzy subset of G (with respect to a fuzzy set A), denoted by A_α, is defined as $A_\alpha(x) = max\{A(x), 1-\alpha\}$, for all $x \in G$

Remark 1.14 (i) $A_1 = A$ and $A_0 = 1$ (ii) If A and B are two fuzzy subsets of a set X, then $(A \cup B)_\alpha = A_\alpha \cup B_\alpha$.

Definition 1.15 Let A be a fuzzy subset of a group G and $\alpha \in [0,1]$. Then A is called an α **-anti fuzzy subgroup** of G if A_α is an anti fuzzy group.

II. S - α ANTI FUZZY SEMIGROUP

In this section we define S - α anti fuzzy semigroup, S - α anti fuzzy normal subsemigroup and obtain their properties.

Definition 2.1 Let G be an S-semigroup. Let A be a fuzzy subset of G and let $\alpha \in [0,1]$. A is called a **Smarandache -** α **anti fuzzy semigroup (** S **-** α **anti fuzzy semigroup)** if there exists a proper subset P of G which is a group and the restriction of A to $P(A_P : P \to [0,1])$ is such that A_{P_α} is an anti fuzzy group. That is,

(i) $A_{P_\alpha}(xy) \leq max\{A_{P_\alpha}(x), A_{P_\alpha}(y)\}$

(ii) $A_{P_\alpha}(x^{-1}) = A_{P_\alpha}(x)$, for all $x, y \in P$

Example 2.2 Consider $S(3)$ which is an S-semigroup. Let $A : S(3) \to [0,1]$ be defined as

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$$A(x)=\begin{cases}0.4, & if\ x=\begin{pmatrix}1&2&3\\1&2&3\end{pmatrix}\\ 0.6, & if\ x=\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix} and \begin{pmatrix}1&2&3\\3&1&2\end{pmatrix}\\ 0.8, & otherwise\end{cases}$$

Let $P=\left\{\begin{pmatrix}1&2&3\\1&2&3\end{pmatrix},\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix},\begin{pmatrix}1&2&3\\3&1&2\end{pmatrix}\right\}$. Clearly P is a Proper subset of $S(3)$ and is also a group. Let $\alpha=0.8$.

Then $A_{P_\alpha}:P\rightarrow[0,1]$ is such that

$$A_{P_\alpha}=\begin{cases}0.4, & if\ x=\begin{pmatrix}1&2&3\\1&2&3\end{pmatrix}\\ 0.6, & if\ x=\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix} and \begin{pmatrix}1&2&3\\3&1&2\end{pmatrix}\end{cases}$$

It can be easily verified that A_{P_α} is an anti fuzzy group. Therefore A is an S-α anti fuzzy semigroup.

Remark 2.3 (i) Throught this paper α will always denote a member of $[0,1]$.

(ii) S-α anti fuzzy semigroup is also known as S-α anti fuzzy semigroup of level I.

Definition 2.4 Let G be an S-semigroup and $\alpha\in[0,1]$. An anti fuzzy semigroup $A:G\rightarrow[0,1]$ is called a **Smarandache-α anti fuzzy semigroup of level II (**S**-**α **anti fuzzy semigroup of level II)** if there exists a proper subset P of G which is a group and the restriction of A to P ($A_P:P\rightarrow[0,1]$) is such that A_{P_α} is an anti fuzzy group.

Example 2.5 Consider $S(3)$ which is an S-semigroup. Let $A:S(3)\rightarrow[0,1]$ be defined as

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$$A(x) = \begin{cases} 0.5, & if \ x = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix} \\ 0.6, & if \ x = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} \\ 0.7, & otherwise \end{cases}$$

Clearly A is an anti fuzzy semigroup.

If we take $P = \left\{ \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} \right\}$, then $P \subset S(3)$ which is a group. If $\alpha = 0.4$. We can easily see that A_{P_α} is an anti fuzzy group and hence A is an S-α anti fuzzy semigroup of level II.

Theorem 2.6 Every S-α anti fuzzy semigroup of level II is an S-α anti fuzzy semigroup of level I.

• Proof is trivial. W

Remark 2.7 Converse of the theorem 2.6 need not be ture. That is, an S-α anti fuzzy semigroup of level I need not be an S-α anti fuzzy semigroup of level II. For instance, define $A : S(3) \to [0,1]$ by

$$A(x) = \begin{cases} 0.4, & if \ x = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix} \\ 0.5, & if \ x = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} \\ 0.2, & otherwise \end{cases}$$

Take $P = \left\{ \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} \right\}$ and $\alpha = 0.4$.

Then it can be seen that A is an S-α anti fuzzy semigroup of level I, but not an S-α anti fuzzy semigroup of level II. For if $x = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$ and $y = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$ then $A(xy) \quad max\{A(x), A(y)\}$ and hence A is not an S-α anti fuzzy semigroup of level II.

Definition 2.8 Let G be an S-semigroup and $\alpha \in [0,1]$. Let X be a proper subset of G which is a subsemigroup of G and X contains the largest group in G. A fuzzy subset $A : G \to [0,1]$ is called **Smarandache-α anti fuzzy hyper semigroup (S-α anti fuzzy hyper semigroup)** if A is restricted to P

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which is the largest group in X such that A_{P_α} is an anti fuzzy group.

Example 2.9 Consider $S(3)$ which is an S-semigroup. Let $A:S(3)\to[0,1]$ be defined as

$$A(x)=\begin{cases}0.45, & if\ x=\begin{pmatrix}1&2&3\\1&2&3\end{pmatrix}\\ 0.65, & if\ x=\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix} and \begin{pmatrix}1&2&3\\3&1&2\end{pmatrix}\\ 0.35, & otherwise\end{cases}$$

Let $P=S_3=\left\{\begin{pmatrix}1&2&3\\1&2&3\end{pmatrix},\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix},\begin{pmatrix}1&2&3\\3&1&2\end{pmatrix}\right.$
$\left.\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix},\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix},\begin{pmatrix}1&2&3\\3&2&1\end{pmatrix}\right\}$

and $X=\left\{\begin{pmatrix}1&2&3\\1&1&1\end{pmatrix},\begin{pmatrix}1&2&3\\2&2&2\end{pmatrix},\begin{pmatrix}1&2&3\\3&3&3\end{pmatrix}\right\}\cup S_3$

Clearly X is a proper subset of $S(3)$ which is also a subsemigroup of $S(3)$ and X contains the largest group P. If $\alpha=0.35$, then we can see that A_{P_α} is an anti fuzzy group and hence A is an S-α anti fuzzy hyper semigroup.

Theorem 2.10 Every S-α anti fuzzy hyper semigroup of an S-semigroup G is an S-α anti fuzzy semigroup.

• Proof is trivial. W

Remark 2.11 Converse of the theorem 2.10 need not be ture. That is, an S-α anti fuzzy semigroup need not be an S-α anti fuzzy hyper semigroup. For instance, if we define $A:S(3)\to[0,1]$ by

$$A(x)=\begin{cases}0.5, & if\ x=\begin{pmatrix}1&2&3\\1&2&3\end{pmatrix}\\ 0.7, & if\ x=\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix}\\ 0.4, & otherwise\end{cases}$$

then we can easily see that A is an S-α anti fuzzy semigroup whereas it is not an S-α anti fuzzy hyper semigroup.

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Theorem 2.12 If $A: G \to [0,1]$ is an S-α anti fuzzy semigroup of an S-semigroup G relative to a group P which is a proper subset of G, then

(i) $A_{P_\alpha}(x) \geq A_{P_\alpha}(e)$, where e is the identity element of P.

(ii) $A_{P_\alpha}(xy^{-1}) = A_{P_\alpha}(e) \Rightarrow A_{P_\alpha}(x) = A_{P_\alpha}(y)$, for all $x, y \in P$.

• It can be easily proved by using theorem 1.7. W

Theorem 2.13 If A is an S-anti fuzzy semigroup of an S-semigroup G, then A is an S-α anti fuzzy semigroup.

• Since A is an S-anti fuzzy semigroup of G, A is restricted to a proper subset P of G which is a group such that the restriction map A_P is an anti fuzzy group. Let $x, y \in P$. Now,

$$(i) \quad A_{P_\alpha}(xy) = max\{A_P(xy), 1-\alpha\}$$

$$\leq max\{max\{A_P(x), A_P(y)\}, 1-\alpha\}$$

$$= max\{max\{A_P(x), 1-\alpha\}, max\{A_P(y), 1-\alpha\}\}.$$

$$A_{P_\alpha}(xy) \leq max\{A_{P_\alpha}(x), A_{P_\alpha}(y)\}, \quad \text{forall} \quad x, y \in P$$

$$(ii) \quad A_{P_\alpha}(x^{-1}) = max\{A_P(x), 1-\alpha\}$$

$$= A_{P_\alpha}(x), \quad \text{forall} \quad x \in P.$$

Therefore A is an S-α anti fuzzy semigroup. W

Theorem 2.14 Let G be an S-semigroup and P be a proper subset of G which is a group. Then $A: G \to [0,1]$ is an S-α anti fuzzy semigroup of G relative to P iff $A_{P_\alpha}(xy^{-1}) \leq max\{A_{P_\alpha}(x), A_{P_\alpha}(y)\}$, for all $x, y \in P$.

• $A: G \to [0,1]$ is an S-α anti fuzzy semigroup iff $A_{P_\alpha}: P \to [0,1]$ is an anti fuzzy group iff $A_{P_\alpha}(xy^{-1}) \leq max\{A_{P_\alpha}(x), A_{P_{\alpha}(y)}\}$, for all $x, y \in P$.

[By theorem 1.6] W

Theorem 2.15 Let A be a fuzzy subset of an S-semigroup G. Let P be a proper subset of G which is a group and let $q = \sup\{A(x) / x \in P\}$. If $\alpha \leq 1-q$, then A is an S-α anti fuzzy semigroup.

• By the definition of restriction of A to P, we have

$q = \sup\{A_P(x) / x \in P\}$. Since $\alpha \leq 1-q$, $\sup\{A_P(x) / x \in P\} \leq 1-\alpha$ which implies that $A_P(x) \leq 1-\alpha$. Therefore $A_{P_\alpha}(x) = max\{A_P(x), 1-\alpha\} = 1-\alpha$, for all $x \in P$. Clearly, $A_{P_\alpha}(xy) = max\{A_{P_\alpha}(x), A_{P_\alpha}(y)\}$ and $A_{P_\alpha}(x^{-1}) = A_{P_\alpha}(x)$. Therefore A_{P_α} is an anti fuzzy group and hence A is an S-α anti fuzzy semigroup. W

Theorem 2.16 If A and B are S-α anti fuzzy semigroups of an S-semigroup G relative to two groups P and Q in G respesctively and $R = P \cap Q$ is a group in G, then $A \cup B$ is also an S-α anti fuzzy semigroup of G.

• Since A and B are S-α anti fuzzy semigroups of G relative to P and Q respectively, A_{P_α} and B_{Q_α} are anti fuzzy groups.

Consider the restriction of $A \cup B$ to R. That is $(A \cup B)_R : R \to [0,1]$ is defined as $(A \cup B)_R(x) = (A \cup B)(x)$, $x \in R$. Let $x, y \in R$.

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Now $A_{P_\alpha}(x) = max\{A_P(x), 1-\alpha\} = max\{A_R(x), 1-\alpha\}$ (since $R \subset P$). Thus $A_{P_\alpha}(x) = A_{R_\alpha}(x)$. Similarly $B_{Q_\alpha}(x) = B_{R_\alpha}(x)$. Since A_{P_α} and B_{Q_α} are anti fuzzy groups, $A_{R_\alpha}(xy) \le max\{A_{R_\alpha}(x), A_{R_\alpha}(y)\}$ and $A_{R_\alpha}(x^{-1}) = A_{R_\alpha}(x)$. $B_{R_\alpha}(xy) \le max\{B_{R_\alpha}(x), B_{R_\alpha}(y)\}$ and $B_{R_\alpha}(x^{-1}) = B_{R_\alpha}(x)$.

By remarks 1.12, 1.14

$$(i)\quad (A \cup B)_{R_\alpha}(xy) = (A_R \cup B_R)_\alpha(xy)$$
$$= (A_{R_\alpha} \cup B_{R_\alpha})(xy)$$
$$= max\{A_{R_\alpha}(xy), B_{R_\alpha}(xy)\}$$
$$\le max\{max\{A_{R_\alpha}(x), A_{R_\alpha}(y)\},$$
$$max\{B_{R_\alpha}(x), B_{R_\alpha}(y)\}\}$$
$$= max\{(A_{R_\alpha} \cup B_{R_\alpha})(x), (A_{R_\alpha} \cup B_{R_\alpha})(y)\}.$$
$$(A \cup B)_{R_\alpha}(xy) \le max\{(A \cup B)_{R_\alpha}(x), ((A \cup B)_{R_\alpha}(y)\}$$
$$(ii)\quad (A \cup B)_{R_\alpha}(x^{-1}) = (A_R \cup B_R)_\alpha(x^{-1})$$
$$= (A_{R_\alpha} \cup B_{R_\alpha})(x^{-1})$$
$$= max\{A_{R_\alpha}(x^{-1}), B_{R_\alpha}(x^{-1})\}$$
$$= max\{A_{R_\alpha}(x), B_{R_\alpha}(x)\}$$
$$= (A_{R_\alpha} \cup B_{R_\alpha})(x).$$

$(A \cup B)_{R_\alpha}(x^{-1}) = (A \cup B)_{R_\alpha}(x)$. By (i) and (ii) $(A \cup B)_{R_\alpha}$ is an anti fuzzy group and hence $(A \cup B)$ is an S-α anti fuzzy semigroup. W

Theorem 2.17 The union of two S-α anti fuzzy semigroups of an S-semigroup G relative to a group $P \subset G$, is also an S-α anti fuzzy semigroup of G.

• Since $P \cap P = P$ is a group in G, by theorem 2.16 result follows. W

Corollary 2.18 Any finite union of S-α anti fuzzy semigroups of an S-semigroup G relative to a group $P \subset G$ is also an S-α anti fuzzy semigroup of G.

• The result is straightforward by mathematical induction. W

Definition 2.19 Let G be an S-semigroup and $\alpha \in [0,1]$. Let A be an S-α anti fuzzy semigroup of G relative to a group P in G and let $x \in P$.

A smarandache-α anti fuzzy right coset (S-α **anti fuzzy right coset)** of A in G, denoted by $A_{P_\alpha}x$, is defined as

$(A_{P_\alpha}x)(g) = max\{A_P(gx^{-1}), 1-\alpha\}$, for all $g \in P$.

Example 2.20 Consider $S(3)$ which is an S-semigroup. Let $A : S(3) \to [0,1]$ be defined as

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$$A(x) = \begin{cases} 0.4, & if\ x = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix} \\ 0.5, & if\ x = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} and \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} \\ 0.6, & otherwise \end{cases}$$

Let $P = \left\{ \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} \right\}$ and $\alpha = 0.6$.

For $x = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$, an S-α anti fuzzy right coset of A in G is given by

$$(A_{P_\alpha} x)(g) = \begin{cases} 0.5, & if\ g = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix} \\ 0.4, & if\ g = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} \\ 0.5, & if\ g = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} \end{cases}$$

Definition 2.21 Let G be an S-semigroup and $\alpha \in [0,1]$. Let A be an S-α anti fuzzy semigroup of G relative to a group P in G and let $x \in P$.

A smarandache-α anti fuzzy left coset (S-α anti fuzzy left coset) of A in G, denoted by xA_{P_α}, is defined as $(xA_{P_\alpha})(g) = max\{A_P(x^{-1}g), 1-\alpha\}$, for all $g \in P$.

Example 2.22 In example 2.20, for $x = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$, an S-α fuzzy anti left coset of A in G is given by

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$$(xA_{P_\alpha})(g) = \begin{cases} 0.5, & if\ g = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix} \\ 0.4, & if\ g = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} \\ 0.5, & if\ g = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} \end{cases}$$

Definition 2.23 Let G be an S-semigroup and $\alpha \in [0,1]$. An S-α anti fuzzy semigroup A of G relative to a group P in G is said to be a **Smarandache-α anti fuzzy normal subsemigroup (S-α anti fuzzy normal subsemigroup)** of G if $xA_{P_\alpha} = A_{P_\alpha}x$, for all $x \in P$.

Example 2.24 From examples 2.20 & 2.22, we can easily see that

$xA_{P_\alpha} = A_{P_\alpha}x$, for all $x \in P$ and hence A is an S-α anti fuzzy normal subsemigroup of G.

Remark 2.25 If $\alpha = 1$, then clearly every S-α anti fuzzy normal subsemigroup of an S-semigroup G is S- anti fuzzy normal subsemigroup of G.

Theorem 2.26 If A is an S-anti fuzzy normal subsemigroup of an S-semigroup G, then A is also an S-α anti fuzzy normal subsemigroup of G.

• By definition, there exists a proper subset P of G which is a group in G such that the restriction map $A_P : P \to [0,1]$ is an anti fuzzy group and $A_P(xy) = A_P(yx)$, for all $x, y \in P$. By theorem 1.11, $xA_P = A_P x$, for all $x \in P$. Therefore for any $g \in P$, $(xA_P)(g) = (A_P x)(g) \Rightarrow A_P(x^{-1}g) = A_P(gx^{-1})$. Let $\alpha \in [0,1]$. Then $max\{A_P(x^{-1}g), 1-\alpha\} = max\{A_P(gx^{-1}), 1-\alpha\}$. Therefore $(xA_{P_\alpha})(g) = (A_{P_\alpha}x)(g)$, for all $g \in P$. Thus $(xA_{P_\alpha}) = (A_{P_\alpha}x)$, $x \in P$ and hence A is an S-α anti fuzzy normal subsemigroup of G. W

Theorem 2.27 Let A be an S-α anti fuzzy semigroup of an S-semigroup G relative to a group P in G. Then the following conditions are equivalent for all $x, y \in P$

(i) A is an S-α anti fuzzy normal subsemigroup.

(ii) $A_{P_\alpha}(xyx^{-1}) \geq A_{P_\alpha}(y)$

(iii) $A_{P_\alpha}(xyx^{-1}) = A_{P_\alpha}(y)$

(iv) $A_{P_\alpha}(xy) = A_{P_\alpha}(yx)$

(v) $xA_{P_\alpha}x^{-1} = A_{P_\alpha}$

(vi) $A_{P_\alpha}(y^{-1}xy) = A_{P_\alpha}(x)$.

• By theorem 1.11, proof is obvious. W

Theorem 2.28 Let A be an S-α anti fuzzy semigroup of an S-semigroup G relative to a group P in G. Let $q = \sup\{A(x)/x \in P\}$ and $\alpha \leq 1-q$. Then A is also an S-α anti fuzzy normal subsemigroup of G.

• Since $q = \sup\{A(x)/x \in P\}, A_P(x) \leq q$, for all $x \in P$. But $\alpha \leq 1-q$. This implies

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$A_P(x) \le 1-\alpha$, for all $x \in P$. For $y \in P$, $(A_{P_\alpha} y)(g) = max\{A_P(gy^{-1}), 1-\alpha\} = 1-\alpha = max\{A_P(y^{-1}g), 1-\alpha\} = (yA_{P_\alpha})(g)$.

Therefore $A_{P_\alpha} y = yA_{P_\alpha}$, for all $y \in P$ and hence A is an S-α anti fuzzy normal subsemigroup of G.

W

Theorem 2.29 Let A be an S-α anti fuzzy normal subsemigroup of an S-semigroup G relative to a group P in G. Define a set $G_{A_{P_\alpha}} = \{x \in P / A_{P_\alpha}(x) = A_{P_\alpha}(e)$, e is the identity of P$\}$. Then $G_{A_{P_\alpha}}$ is a normal subgroup of P.

• Let $G_{A_{P_\alpha}} = \{x \in P / A_{P_\alpha}(x) = A_{P_\alpha}(e)\}$. Since $e \in G_{A_{P_\alpha}}$, $G_{A_{P_\alpha}}$ is non empty subset of P.

Let $x, y \in G_{A_{P_\alpha}}$.

Now $A_{P_\alpha}(xy^{-1}) \le max\{A_{P_\alpha}(x), A_{P_\alpha}(y)\} = max\{A_{P_\alpha}(e), A_{P_\alpha}(e)\} = A_{P_\alpha}(e)$. Thus by Theorem 2.12, $A_{P_\alpha}(xy^{-1}) = A_{P_\alpha}(e)$ which implies $xy^{-1} \in G_{A_{P_\alpha}}$. Thus $G_{A_{P_\alpha}}$ is a subgroup of P. Now let $x \in G_{A_{P_\alpha}}$ and $y \in P$. Since A is an S-α anti fuzzy normal subsemigroup of G, $A_{P_\alpha}(y^{-1}xy) = A_{P_\alpha}(x) = A_{P_\alpha}(e)$.

Therefore $y^{-1}xy \in G_{A_{P_\alpha}}$ and hence $G_{A_{P_\alpha}}$ is a normal subgroup of P. W

Theorem 2.30 Let A be an S-α anti fuzzy normal subsemigroup of an S-semigroup G relative to a group P in G. Then for all $x, y \in P$

(i) $xA_{P_\alpha} = yA_{P_\alpha} \Leftrightarrow x^{-1}y \in G_{A_{P_\alpha}}$

(ii) $A_{P_\alpha} x = A_{P_\alpha} y \Leftrightarrow xy^{-1} \in G_{A_{P_\alpha}}$.

• Let $x, y \in P$.

(i) Let $xA_{P_\alpha} = yA_{P_\alpha}$. Now

$$A_{P_\alpha}(x^{-1}y) = max\{A_P(x^{-1}y), 1-\alpha\} = (xA_{P_\alpha})(y) = (yA_{P_\alpha})(y) = max\{A_P(y^{-1}y), 1-\alpha\} = A_{P_\alpha}(e)$$

. This implies $x^{-1}y \in G_{A_{P_\alpha}}$.

Conversely, assume that $x^{-1}y \in G_{A_{P_\alpha}}$. Therefore $A_{P_\alpha}(x^{-1}y) = A_{P_\alpha}(e)$.

Let $z \in P$. Now

$$(xA_{P_\alpha})(z) = max\{A_P(x^{-1}z), 1-\alpha\} = A_{P_\alpha}(x^{-1}z) = A_{P_\alpha}((x^{-1}y)(y^{-1}z)) \le max\{A_{P_\alpha}(x^{-1}y), A_{P_\alpha}(y^{-1}z)\} = A_{P_\alpha}(y^{-1}z) =$$

. Thus $(xA_{P_\alpha})(z) \le (yA_{P_\alpha})(z)$. If we interchange x and y,

we have $(yA_{P_\alpha})(z) \le (xA_{P_\alpha})(z)$ for all $z \in P$. Hence $xA_{P_\alpha} = yA_{P_\alpha}$.

(ii) This follows as in part(i) W

Theorem 2.31 Let A be an S-α anti fuzzy normal subsemigroup of an S-semigroup G relative to a group P in G and let x, y, u, v be elements of P. If $xA_{P_\alpha} = uA_{P_\alpha}$ and $yA_{P_\alpha} = vA_{P_\alpha}$, then $(xy)A_{P_\alpha} = (uv)A_{P_\alpha}$.

• Let $x, y, u, v \in P$. since $xA_{P_\alpha} = uA_{P_\alpha}$ and $yA_{P_\alpha} = vA_{P_\alpha}$, $x^{-1}u$ and $y^{-1}v$ are elements of $G_{A_{P_\alpha}}$.

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Now $(xy)^{-1}(uv) = y^{-1}(x^{-1}u)v = (y^{-1}(x^{-1}u)y)(y^{-1}v) \in G_{A_{P_\alpha}}$ (since $G_{A_{P_\alpha}}$ is a normal subgroup of P).

This implies that $(xy)(A_P)_\alpha = (uv)A_{P_\alpha}$.

Hence the theorem. W

III. CONCLUSION

The idea of S-α anti fuzzy semigroup has been defined and its properties have been obtained with some suitable examples. It has also been studied about S-α anti fuzzy left coset, S-α anti fuzzy right coset and S-α anti fuzzy normal subsemigroups. Basic properties of group theory may be carried over to S-α anti fuzzy semigroups.

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