ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 4, Issue 5, September 2015

Unsteady Magnetohydrodynamic Fluid Flow in a Collapsible Tube

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***Abstract:** The dynamics of fluid flow in collapsible tube are vital in understanding the behavior and analysis of flow phenomenon in veins, arteries, airways, urethra, etc. These fluid-conveying vessels in human body are highly flexible and collapsible, and accommodate deformation that result to a highly noncircular cross sectional area. That is, when the external pressure exceeds the internal pressure, the tube cross-sectional area can be significantly reduced, if not fully diminished. In this research, unsteady magneto hydrodynamic fluid flow in a collapsible tube is investigated. The effects of non-dimensional numbers on temperature and velocity of the fluid are determined. The tube is considered to be collapsible in the transverse direction, taken to be perpendicular to the main flow direction. The fixed magnetic field is perpendicular to the direction of flow of the conducting fluid. Partial differential equations governing the flow are transformed into non dimensional form and solved numerically using the finite difference method. A computer program is used to generate the results which are presented in form of graphs. The results obtained on the effect of non-dimensional numbers on temperature and velocity of the fluid is discussed. The results show that increase in Reynolds number and magnetic parameter leads to an increase in velocity profiles and an increase in Eckert and Hartmann numbers leads to an increase in temperature profiles while increase in prandtl number leads to a decrease in temperature profiles. These results are useful in medicine and also in industries where collapsible tubes are used.*

Index Terms: **Biomagnetic fluid dynamics, collapsible tube, incompressible viscous fluid, Magneto fluid dynamics.**

I. INTRODUCTION

A biomagnetic fluid is a fluid that exists in a living creature and its flow is influenced by the presence of a magnetic field. The most characteristic biomagnetic fluid is blood, which behaves as a magnetic fluid, due to the complex interaction of the intercellular protein, cell membrane and the hemoglobin, a form of iron oxides, which is present at a uniquely high concentration in the mature red blood cells. Flow through collapsible tubes has been extensively studied in the laboratory. In experimental studies flexible tubing is employed. According to Edward [1] these experiments are based upon the assumption that the difference between the flexible tubing and veins are quantitative in nature rather than qualitative. Marzo et al. [2] studied three-dimensional collapse of a steady flow through finite-length elastic tubes numerically. Odejide et al. [3] examined an incompressible viscous fluid flow and heat transfer in a collapsible tube. It was noted that increase in Reynolds number led to an increase in the fluid temperature with maximum magnitude at the pipe center and minimum at the wall. The fluid velocity profile was noted to be parabolic in nature. Andrew et al. [4] described the role of venous valves in pressure shielding. A one-dimensional mathematical model of a collapsible tube with the facility to introduce valves at any position was used. They observed that a valve decreased the dynamic pressures applied to a vein when gravity is applied by a considerable amount. Liu et al. [5] explained that the wall stiffness is dominated by the axial tension. Emilie and Patrice [6] developed a simple and effective numerical physiological tool to help clinicians and researchers in the understanding of flow phenomena. One-dimensional Runge–Kutta discontinuous Galerkin (RK-DG) method coupled with lumped parameter models for the boundary conditions was used.Various benchmark problems that showed the flexibility and applicability of the numerical method were presented. The emptying process in a calf vein squeezed by contracting skeletal muscle in a normal and pathological subject was also studied and the results compared with experimental simulations. After the comparison of the results it was noted that the efficiency of muscular calf pump is strongly dependent on the valves pathology and the walking frequency. Eleuterio F.T. & Annunziato S. [7] formulated a one-dimensional time-dependent non-linear mathematical model for physiological fluid flow in collapsible tubes with discontinuous material properties. He observed that although the solution algorithm dealt with idealized cases, it is uniquely well-suited for assessing the performance of numerical methods intended for simulating more general situations. Siviglia and M. Toffolon [8] performed a steady analysis of transcritical flows in collapsible tubes with discontinuous mechanical properties. They found transcritical curves that describe the conditions under which the incoming flow inside a collapsible tube may pass through the critical state due to an abrupt change of the mechanical properties of the wall vessel. They investigated two different cases. In the first case they assumed a simplified tube law (i.e. $\beta_2 = 0$), and in the second they considered a complete tube law (i.e.

$\beta_1 \geq 0$ and $0 < \beta_2 \geq 2$) .For both cases they described the full range of conditions for which transition can occur. In particular they identified the values of the speed indexes, one supercritical and one subcritical, which are the limits of the transcritical region, and the associated values of the area ratio α which are related to the values of the transmural pressure. Kanyiri et al. [9] studied the effects of flow parameters (tube stiffness and longitudinal tension) on the flow variables of a Newtonian fluid flowing through a cylindrical collapsible tube. The results show that the flow parameters considered are directly proportional to both the cross sectional area and internal pressure and inversely proportional to the flow velocity. Boileau *et al.* [10] modeled the numerical schemes for one-dimensional arterial blood flow. They showed that arterial pulse wave hæmodynamics can be accurately simulated using finite element, finite volume or finite difference methods. Extensive researches have been done, including those cited above, on fluid flow in collapsible tubes. However, no emphasis has been given in consideration of magnetic field. The aim of this work is therefore to study unsteady magnetohydrodynamic fluid flow in a collapsible tube.

II. MATHEMATICAL FORMULATION

Let's consider the transient flow of a viscous incompressible fluid in a collapsible tube. We take a cylindrical polar coordinate system r, θ, z where *oz* lies along the center of the tube; *r* is the distance measured radially and θ is the azimuthal angle. Let *u* and *v* be the velocity components in the directions of *z* and *r* increasing respectively as shown in the figure below.

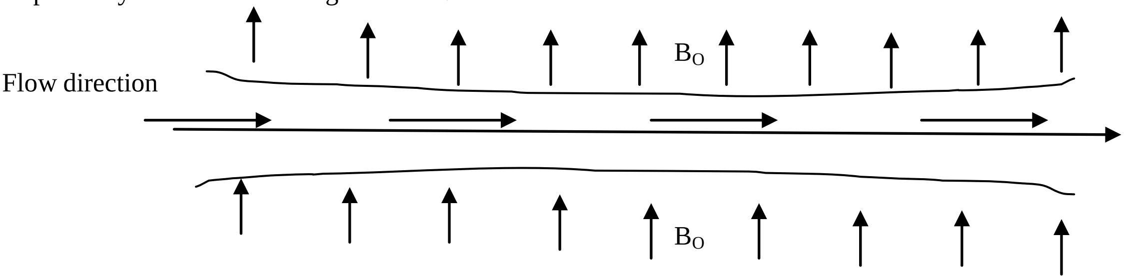

Fig 1: Geometry of the problem.

For axisymmetric unsteady viscous incompressible flow, the governing equations are as follows:

Continuity equation

In cylindrical polar co-ordinates form and considering two dimensional fluid flow equation is expressed as

$$\frac{\partial u}{\partial z} + \frac{1}{r}\frac{\partial (rv)}{\partial r} = 0 \qquad (1)$$

Considering the velocity in radial distance to be negligible the equation reduces to

$$\frac{\partial u}{\partial z} = 0 \qquad (2)$$

Equation of conservation of momentum

Equation of momentum in cylindrical coordinates for a two dimensional flow is given as

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial z} + v\frac{\partial u}{\partial r} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial z^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial r^2}\right) + F \qquad (3)$$

In this study, body forces considered are electromagnetic force and gravitational force thus

$$F = g + J \times B \qquad (4)$$

but since the tube is assumed to be horizontal the only body force to be considered is the electromagnetic force.

ISSN: 2319-5967
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Hence $F = J \times B$ (5)

Generalized Ohm's law, neglecting Hall effect, is expressed as

$$\vec{J} = \sigma\left(\vec{E} + \vec{q} \times \vec{B}\right) \quad (6)$$

The term $\vec{q} \times \vec{B}$ in equation (6) yields

$$\vec{q} \times \vec{B} = \begin{vmatrix} \hat{r} & \hat{\theta} & \hat{z} \\ v & 0 & u \\ B_0 & 0 & 0 \end{vmatrix} = uB_0\hat{\theta} \quad (7)$$

Assuming that the induced electric current is negligible the Ohm's law reduces to

$$\vec{J} = \sigma u B_0 \hat{\theta} \quad (8)$$

Hence $J \times B = \begin{vmatrix} \hat{r} & \hat{\theta} & \hat{z} \\ 0 & \sigma u B_0 & 0 \\ B_0 & 0 & 0 \end{vmatrix} = -\sigma B_0{}^2 u\hat{z}$ (9)

Considering the Lorenz force and the assumption that the fluid velocity in radial direction is negligible the momentum equation (3) in z direction reduces to

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + v\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) - \frac{\sigma B_0{}^2}{\rho} u \quad (10)$$

On nondimensionalizing the equation above yields

$$\frac{\partial u}{\partial t} = -\frac{\partial P}{\partial z} + \frac{1}{Re}\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) - Mu \quad (11)$$

Equation of energy

For the flow of an incompressible fluid with constant fluid conductivity k ,the energy equation is given by

$$\rho C_P \frac{DT}{Dt} = k\nabla^2 T + \mu\phi \quad (12)$$

Considering ohmic heating due to electrical resistance of the fluid equation above becomes

$$\rho C_P\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial z} + v\frac{\partial T}{\partial r}\right) = k\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2}\right) + \mu\left(\frac{\partial u}{\partial r}\right)^2 + \frac{J^2}{\sigma} \quad (13)$$

Simplifying the second term on the left hand side of this equation yields

$u\frac{\partial T}{\partial z} + v\frac{\partial T}{\partial r} = 0$ since $\frac{\partial T}{\partial z} = 0$ and $v = 0$. Consequently the third term on the right hand side

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$$\frac{\partial^2 T}{\partial z^2} = 0$$

Therefore equation (13) becomes

$$\rho C_P \frac{\partial T}{\partial t} = k\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) + \mu\left(\frac{\partial u}{\partial r}\right)^2 + \frac{J^2}{\sigma} \tag{14}$$

Since from equation (8) $\vec{J} = \sigma u B_0 \hat{\theta}$ then $\frac{J^2}{\sigma} = \sigma u^2 B_0^2$ (15)

The energy equation (14) becomes

$$\rho C_P \frac{\partial T}{\partial t} = k\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) + \mu\left(\frac{\partial u}{\partial r}\right)^2 + \sigma u^2 B_0^2 \tag{16}$$

On nondimensionalizing the equation above yields

$$\frac{u_0 a_0}{\mu}\frac{\partial T}{\partial t} = \frac{1}{\mu \Pr}\left(\frac{\partial^2 T}{\partial r^2} + \frac{\partial T}{\partial r}\right) + \frac{Ec}{\rho}\left(\frac{\partial u}{\partial r}\right)^2 + \frac{Hu^2}{(T_w - T_\infty)} \tag{17}$$

The solution procedure

The equations governing the flow problem were written in finite difference form and then reorganized. The governing equations describing the unsteady, incompressible laminar fluid flow through a cylindrical collapsible tube in presence of magnetic field, in finite difference form are given as:

$$U_j^{k+1} = \left(U_j^k + \Delta t\left(-\frac{\partial P}{\partial z} + \frac{1}{Re}\left(\frac{1}{2}\left(\frac{U_{j+1}^k - 2U_j^k + U_{j-1}^k + U_{j+1}^{k+1} + U_{j-1}^{k+1}}{(\Delta r)^2}\right) + \frac{1}{2r}\left(\frac{U_j^k - U_{j-1}^k - U_{j-1}^{k+1}}{(\Delta r)}\right)\right) - \frac{M}{2}U_j^k\right)\right) / \left(1 + \frac{\Delta t}{Re(\Delta r)^2} + \frac{\Delta t}{2r\,Re(\Delta r)} + \frac{M\Delta t}{2}\right) \tag{18}$$

and

$$\theta_j^{k+1} = \left(\theta_j^k + \frac{\mu \Delta t}{u_0 a_0}\left(\frac{\Pr}{\mu}\left(\left(\frac{\theta_{j+1}^k - 2\theta_j^k + \theta_{j-1}^k \theta_{j+1}^{k+1} + \theta_{j-1}^{k+1}}{2(\Delta r)^2}\right)\right) + \left(\frac{\theta_j^k - \theta_{j-1}^k - \theta_{j-1}^{k+1}}{2\Delta r}\right)\right) + \frac{Ec}{\rho}\left(\frac{U_j^k - U_{j-1}^k + U_j^{k+1} - U_{j-1}^{k+1}}{2(\Delta r)}\right) + \frac{H}{T_w - T_\infty}\left(\frac{U_j^k + U_j^{k+1}}{2}\right)\right) / \left(1 + \frac{\Delta t \Pr}{u_0 a_0 (\Delta r)^2} - \frac{\Delta t \Pr}{2u_0 a_o r(\Delta r)}\right) \tag{19}$$

subject to the initial and boundary conditions given below;

$$t \le 0: \quad u = 0 \quad T = 0$$
$$t > 0: \quad u = 0.45 \quad T = 37 \quad \textit{at the center of the tube } (r = 0)$$
$$t > 0: \quad u = 0 \quad T = 0 \quad \textit{at the wall}$$

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III. RESULTS AND DISCUSSION

A glimpse on the outlook of the numerical solutions of velocity and temperature is provided below.

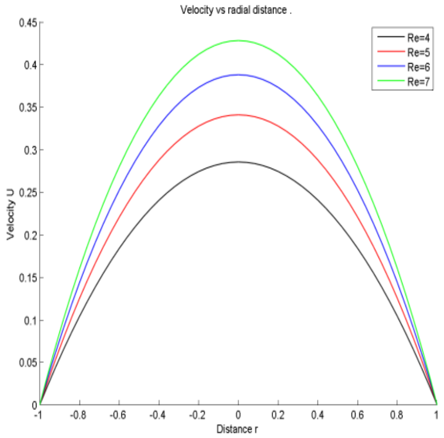

Fig 2: M=4 ,t=100

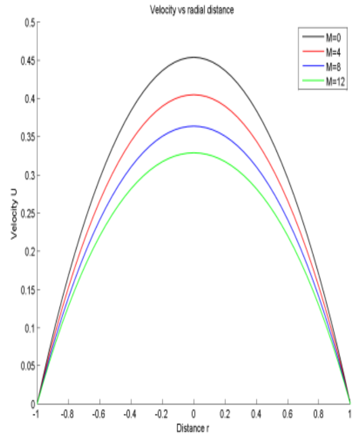

Fig 3: Re=7, t=100

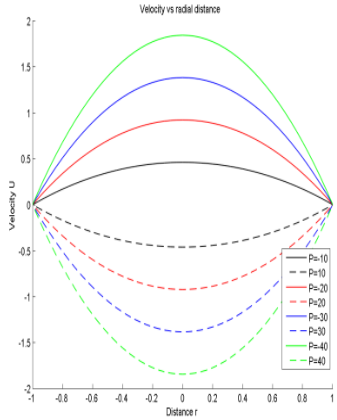

Fig 4: Re=7,M=4,t=100

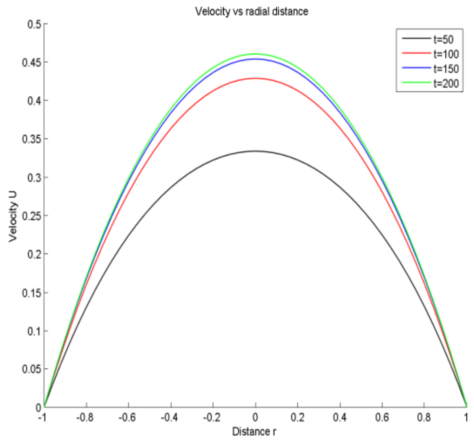

Fig 5: Re=7, M=4

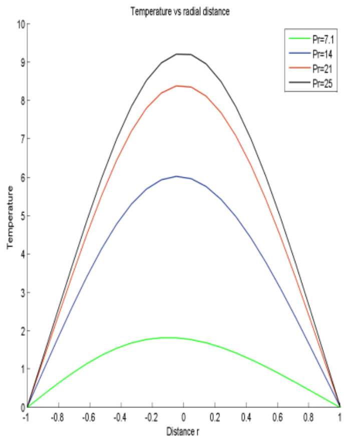

Fig 6: Re=7,M=4,Ec=0.5,H=4

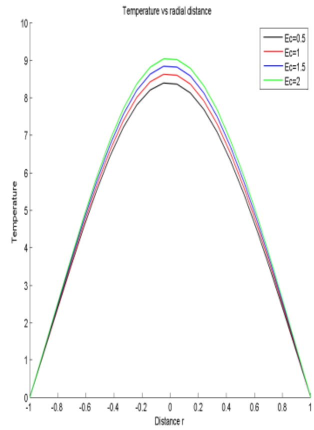

Fig 7: Re=7,M=4,Pr=21

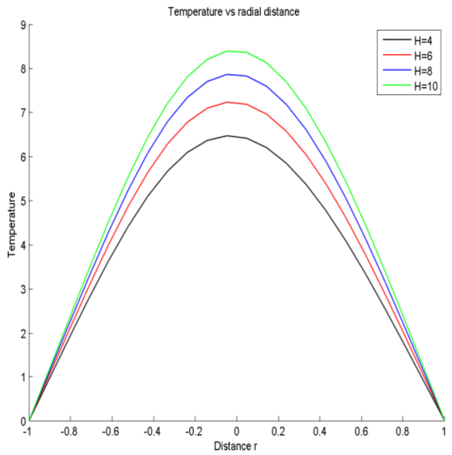

Fig 8: Re=7,M=4,Pr=21

From Fig 2 it is observed that when the Reynolds number is increased holding M and P constant the magnitude of the primary velocity profiles increased with maximum value at the center. This can be explained by, as the Reynolds number increase the viscous forces decrease hence the increase in fluid velocity.

From Fig 3 it is observed that when the magnetic parameter is increased holding Re and P constant the magnitude of the primary velocity profiles decreased. This is because Introduction of strong magnetic field normal to the direction of electrically conducting fluid results to emergence of a resistive force to the main flow. This resistive force called Lorentz force resists the flow of the fluid resulting to deceleration of the fluid and thus the fluid velocity profiles reduces.

From Fig 4 it is observed that when the pressure gradient is negative it leads to an increase in the primary velocity profiles in the positive direction. When the pressure gradient is positive it leads to an increase in the primary velocity profiles in the negative direction .This is because when pressure gradient is positive, then the pressure force term acts in the opposite direction of the direction of the fluid flow. On the other hand, when it is negative, then the pressure force term acts in the same direction as that of the fluid flow hence aiding the fluid flow.

From Fig 5 it is observed that when time was increased holding Reynolds number and magnetic parameter constant the primary velocity profiles increased. This is because with time the flow gets to the free stream and therefore its velocity increases. From Fig 6 it is observed that when prandtl number was increased holding Eckert number and Hartmann number constant the temperature profiles decreased. A rise in Prandtl number leads to a decrease in temperature distribution because an increase in prandtl number means a slow rate in thermal diffusion.

From Fig 7 it is observed that when Eckert number was increased holding prandtl number and Hartmann number constant the temperature profiles increased. The temperature profile is parabolic with maximum temperature at the center and minimum at the wall of the collapsible tube. This result shows that as the Eckert number is increased, the rate at which fluid loses heat decreases and hence there will be an increase in the temperature of the fluid. This behavior is attributed to the decrease of viscous dissipation as Eckert increases. Increase in Eckert number reduces the temperature gradient thus increasing the temperature boundary layer and therefore the increase in temperature.

From Fig 8 it is observed that when Hartmann number was increased Prandtl number and Eckert number constant the temperature profiles increased. The rise in temperature is as a result of interaction between the atomic ions that constitute conductor and the moving particles that form the current. The charged particles in the electric circuit are speeded up by the electric field but they lose some kinetic energy whenever they collide with ions. The increase in vibration energy of the ions manifests itself as heat that is depicted by the rise of temperature of the fluid. The increased fluid temperature results to non- uniform changes in fluid properties like fluid density and conductivity. Thus energy is converted from electrical power supply to the fluid or any other medium that is in thermal contact. This heat is referred to as joules heating.

IV. CONCLUSION

Unsteady magnetohydrodynamic fluid flow in a collapsible tube has been carried out with the resulting partial differential equations solved to obtain both the velocity and temperature profiles of the fluid flow respectively. The results show that increase in Reynolds number and magnetic parameter leads to an increase in velocity profiles and an increase in Eckert and Hartmann numbers leads to an increase in temperature profiles while increase in prandtl number leads to a decrease in temperature profiles. When magnetic field is not considered in the flow the results agree with those of Odejide S.A (2008). This proves the validity of the developed governing equations as well as the method of solution. The approach here could be extended to factor in varying magnetic field.

ACKNOWLEDGEMENT

The authors gratefully acknowledge the support of Pure and Applied Mathematics department of Jomo Kenyatta University of Agriculture and Technology Mr. Njagi, Mr.Mburu and Mr. Kihuga for their assistance and guidance.

ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
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NOMENCLATURE

B	Magnetic field strength vector, [wbm^{-2}]
J	Current density, [Am^{-2}]
E	Electric field, [v]
M	Magnetic parameter
Re	Reynolds number
P	Pressure force, [nm^{-2}]
Pr	Prandtl number
H	Hartman number
L	Peripheral length, [m]
r, θ, z	Cylindrical coordinates
F	Body force [N]
υ	Kinematic viscocity of the fluid, [$m^2 s^{-1}$]
σ	ELECTRICAL CONDUCTIVITY [$\Omega^{-1} m^{-1}$]
κ	Thermal conductivity [$Wm^{-1}K^{-1}$]
ϕ	Viscous dissipation function [s^{-2}]

REFERENCES

[1] Edward K. & Abraham N, "Cross-Sectional Shape of Collapsible Tube," *Biophysical Journal*,vol **12**, pp 274-294, 1972.

[2] Marzo A., Luo X.Y. and Bertram C.D, "Three-dimensional collapse and steady flow in thick-walled flexible tubes," *Journal of Fluids and Structures*, vol **20** pp 817–835, 2005.

[3] Odejide S.A, Aregbesola Y.A and MakindeO. D,"Fluid Flow and Heat Transfer in a Collapsible Tube," *Romanian Journal of Physics*, vol **53** pp 499–506,*2008.*

[4] Andrew J.N., Patricia V. L. and David R. H, "The role of venous valves in pressure shielding," *Journal of the American society for mass spectrometry, vol***7** pp 8-20, 2008.

[5] Liu H.F., Luo X.Y., Cai Z.X and Pedley T.J, Sensitivity of unsteady collapsible channel flows to modeling assumptions, *Communications in Numerical Methods in Engineering,* vol **25** pp 483-504, 2009.

[6] Emilie M. and Patrice F, "Accurate modeling of unsteady flows in collapsible tubes,*"Computer Methods in Biomechanics and Biomedical Engineering*, vol **13** pp 279–290, 2010.

[7] Eleuterio F.T. & Annunziato S, "Flow in Collapsible tubes with Discontinuous mechanical properties:

[8] Mathematical model and exact solutions," *Communications in Computational Physics Journal*, vol **13** pp 361-385, 2013.

[9] Siviglia, & Toffolon, M, "Steady analysis of transcritical flows in collapsible tubes with discontinuous mechanical properties: implications for arteries and veins," *J. Fluid Mech*, vol ***736.*** pp195-215, 2013.

[10] Kanyiri et al, Effects of flow parameters on flow variables of a Newtonian fluid through a cylindrical collapsible tube. *Mathematical Theory and Modeling*, vol ***4,*** pp7-17, 2014.

[11] Boileau et al ,A benchmark study of numerical schemes for one-dimensional arterial blood flow modeling. *International Journal for Numerical Methods in Biomedical Engineering*, vol **10** 16-24,201117.

ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 4, Issue 5, September 2015

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