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# Finite Element Analysis of Doubly Curved Thin Concrete Shells

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***Abstract:*** ***A thin shell is a "Three-dimensional spatial structure made up of one or more curved surfaces whose thickness is small compared to their other dimensions". Shells belong to the class of stressed skin structures which, because of their geometry and small flexural rigidity of the skin, tend to carry loads primarily by direct stresses acting in their plane. The shells are subjected to pure membrane state of stress, under appropriate loading and boundary condition the resulting bending and twisting moments are either zero or small which may be neglected. The coordinates of funicular shells are determined by masonry mould method by developing a computer program. In this study doubly curved thin shells are analysed using finite element software SAP 2000. Doubly curved shells which are in square plan having 10mX10m and 15mX15m are considered and shells in rectangular plan having dimensions 10mX15m and 15mX20m are considered. The behavior of shells under self-weight, live load varying from 0-20KN/m (UDL) is obtained. In this case study deflection curves, membrane stress and stress contour diagram are obtained. It is observed that with the increase in rise and thickness of funicular shell the deflection are reduced. The membrane stresses decreases with the increase in rise and thickness of concrete funicular shell. The aim of this study is to develop shells of different sizes and investigation is done on the shells by finite element analysis under given uniformly distributed load, to find out the behavior of shells in various cases using standard software, Structural Analysis Package (SAP 2000).***

*Keywords:* **Funicular shell, Membrane theory, Finite element models, Discretization, SAP 2000.**

## I. INTRODUCTION

A shell structure is a three dimensional structure, thin in one direction and long in the other two directions. Such structures are abundantly found in nature, the shell of an egg is an impressive example. Concrete shells include single curved geometries such as cylinders and cones and double curved geometries such as domes, arches, vaults. Although shell structure is thin and lightweight, hence they span over relatively large areas, and support applied loads in a very effective way. It seems that with shell structures, nature have maximized the ability to span over large areas with a minimum amount of material. Thin shells provide in an advantageous low consumption of material. The low consumption of material in shell structures follows from the unique character of the shell i.e. the geometry which is in curvature in spatial form. This unique character the shell structures are very efficient in carrying loads acting perpendicular to their surface by so-called membrane action. Modern shell structures can span larger column-free areas and, with thinner thicknesses than the traditional domes. The desire to reduce the thickness is understandable as the dead weight of the shell represents the major portion of the total load. Moreover, the preference for membrane action arises as a consequence of being thin. Most of the different types of superstructures we commonly used for present day building are only a modification of the age old system of column, beam and roof covering arrangements. They fulfil their function by two separate systems. One is the space covering system to cover the space, such as concrete slab or roof covering sheets in steel building. These are supported by a second system of beams and columns which we may call the supporting system. In many steel buildings they are obviously separate and in R.C buildings also, they are treated as two separate systems. In reinforced concrete shells, however, the two functions of covering the space and supporting the covering system are integrated into one. The structure covers the space without beams and columns within the buildings. There are several different styles of concrete shell, including the dome, the barrel shell, and the folded plate. They can be elaborate or utilitarian, depending on the intended use, construction budget, and space requirements. For the most part, these structures do not have buttresses on the exterior or interior supporting columns. As the structure often must provide its own support, it is important that concrete shells be carefully designed for proper weight distribution, durability, and stability. A typical view of funicular shells used for a roof construction is shown in figure 1.

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**Fig.1:- Typical view of funicular shell**

## II. LITERATURE REVIEW

The selection of available documents (both published and un published) on the topic, which contain information, ideas, data and evidence written from a particular standpoint to ful fill certain aims or express certain views on the nature of the topic and how it is to be investigated, and the effective evaluation of these documents in relation to the research being proposed. A literature review is a survey of already existing writings (usually published) on a given topic or area with a view to assessing their relevance to a proposed project. The behavior of prefabricated shell units has been investigated, both theoretically and experimentally, by many researchers. Apart from construction schemes, the use of pre-fabricated shell units creates certain problems which have to be investigated from the research point of view. One of the most important is the structural behavior of the units when subjected to the concentrated loads at various points along their span. Another consideration of interest is the ultimate strength and pattern of failure for these types of shells. The shells of double curvature usually offer a higher ultimate strength than the shells of single curvature, such as cylindrical shells. The funicular shell roof is one such compression structure, which ensures conservation of natural resources by utilizing waste materials effectively and optimizing the use of expensive steel and cement. Further, the arch distributes the point load in all direction equally thus, is able to withstand impact loading at any point. Diagonal grid of funicular shell gives the illusion of a larger space.

## III. REVIEW OF SELECTED TECHNICAL PAPERS

**Odello, Robert J, Allgood J. R. (1970)** Research is described in which a concrete funicular shell, 35 x 40 feet in plan with a 2-inch thickness and a 30-inch rise, was tested to define its behaviour under various loadings. The shell sustained a uniformly distributed load of 135 psf before failing in local buckling. In a subsequent test on an undamaged portion, it sustained a concentrated load of 10,800 pounds over an area 6 inches square before failing in punching shear. In addition to the test, pertinent construction and analysis techniques are discussed. It was found that double-curved shallow shells may be easily cast over an earth mound. When combined with the lift-slab technique, this mode of construction is expected to provide an inexpensive method for fabricating large shells for floors and roofs. Approximate limit analyses can be used to proportion shallow shells for ordinary purposes; however, no completely satisfactory method is available for treating such members. Elastic analysis provides a reasonable representation of behaviour only at low loads. Despite the limitations in current analysis, the technology has developed sufficiently to permit use of shallow shells in military and civilian construction. Naval Shore Establishment uses include decks of docks and floors and roofs of warehouses.

**Zacharia George, V.S. Parameshwaran and B.R. Rama Murthy (1971)** the paper describes in detail the salient features in the design construction of the building. A structure testing laboratory has been constructed recently for the Structural Engineering Research (Regional) Centre in madras. It has three large funicular shells built of brick for its roof. The structural frame is designed using the ultimate strength design procedure and high strength deformed bars. The concrete used has an as cast‟ finish. Three shells, each 13.50 m X 12.00 m in size, form the roof of the building. The funicular shell developed by the SERC has been used for a large variety of structural floors and roofs in the form of precast elements for roofs and floors, and also as large cast in situ shells for roofs. Because of the funicular shape, the shell is practically free from flexural moments, and is subjected mainly to membrane compressive stresses. Moreover, the compressive stresses are very low. This property

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renders the shells suitable for being built with materials of low compressive strength, such as bricks. When used for roofing industrial buildings, the higher thermal insulation value of the bricks is an added advantage. The roof is designed to carry a live load of 75 kg/m2, and the weight of bitumen felt waterproofing on top, in addition to its self-weight. Although the shell does not need any reinforcement in its body to carry static loads, reinforced concrete ribs, spaced at 1m centres both ways, are incorporated in the 10cm thickness of the shell. These ribs are provided as a safeguard against vibrations which may be transmitted to the roof by the pulsator on the test floor and by the movement of the EOT crane over the supporting columns. They are inset 1cm from the soffit of the shell to emphasise the grid pattern.

**Albolhassan Vafai and Mehdi Farshad (1979)** have done a research work on theoretical and experimental study of prefabricated funicular shell units. This paper presents the results of a theoretical and experimental study of funicular shell structures. A particular design is given for the geometry, the form, and the type of reinforcement of the units. Ten models are constructed based on this design. First, six samples supported at two edges are loaded to a specified load within the elastic region. Electrical resistance gauges are mounted both inside and outside at several locations on the surface of the shell on two different specimens. Also, dial gauges are installed at several locations on the surface of four shells. Following these non-destructive tests, all ten samples are loaded to failure, subjected to a concentrated load at the center. To relate experimental results to theory, the finite technique is utilized to analyse a similar model. The experimental values of membrane stresses along the central section in the direction of the supports are calculated and compared with the theory. The results are in close agreement at some distance away from the supports, but the difference becomes noticeable closer to the support. The same phenomenon appears to be true when the experimental values of vertical deflections along the longitudinal and transverse sections of the shells are compared with the theory. Also, the experimental failure loads are found to be directly related to the amount of reinforcement, and the age of the concrete shells.

**John W. Weber, Kwong-chi Wu and AdholhassanVafai (1984)** have done an evaluation of ultimate loads for shallow funicular concrete shells. Shells belong to the class of stressed skin structures which, because of their geometry and small flexural rigidity of the skin, tend to carry loads primarily by direct stresses acting in their plane. Ten shallow funicular concrete shells were loaded to failure with a concentrated central force. Five shells were randomly reinforced with steel wires and the reminder with a wire mesh through the middle surface. All of the shells were 90 cm x 90 cm in plan form. Strain gauges showed a linear relationship between load and strain in the elastic range of the concrete, whereas measured deflections were larger than those determined analytically by small deflection theory, would be more appropriate for theoretical investigations of shallow funicular shells subjected to large concentrated loads. Ultimate loads were not clearly related to type of reinforcement, but were a function of the rise and thickness of the shell in general the larger the rise parameter (square of the ratio of rise to thickness), the larger the ultimate load. Failure patterns for shells with both kinds of reinforcement were the same. They observed that the mathematical investigations of shallow funicular shells with large concentrated loads should be based on large deflection theory and the deflection characteristics of a shell vary closely with its rise parameter.

**Suresh, Desai and G.S. Ramaswamy (1985)** This paper gives the details like theory of funicular shell, method of construction and shape generation of thin, shallow, funicular concrete shells by the finite element technique. Funicular shells are thin shallow concrete shells in which a state of equal biaxial compression develops under vertical loading. Large funicular shells may be cast on accurately-cut forms profiled to the funicular shape. This procedure requires a computation of accurate values of 'Z' for the commonly encountered aspect ratios. The table is prepared by using both the finite element method and the energy method. The two sets of values are in close agreement. The concept of funicular shells was initiated by Ramaswamy. A simple technique of letting a thin shell cast itself to the funicular shape has been developed and patented in India. The technique involves loading a flexible fabric stretched across a mould with wet concrete. The concrete, being wet, carries no load at this point. The fabric carries the load in tension and sags to generate the funicular shell over a ground plan whose form is determined by the shape of the mould. Twenty four hours after casting, the shell is inverted and taken off the mould. It is easy to see that the weight of the shell will now be carried by the concrete in compression. The same reasoning would apply to other vertical loads applied to the shell. No distinction need be made between uniform vertical loads and snow loads, if the shell is shallow. Finally they concluded that Reinforced concrete will continue to dominate the construction scenario in coming decades on a scale larger than has been witnessed at the present. This gives the vast scope to the engineers in the academic and the construction field to

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endeavour, with concerted efforts, to make the material more economic and more versatile in its presented by Ramaswamy. These values are enclosed in parentheses in Table1. The agreement between the two sets of values is very close and it is inferred that the finite element values are accurate enough for use in the field for the manufacture of forms.

**Ramaswamy (1986)** this article provides the information about innovative applications of funicular shells. In Civil Engineering practice, concrete thin shells are normally used only as roofs to carry relatively light loads, because most shell forms develop sizable bending stresses when submitted to heavy loads, making them unsuitable as intermediate floors of industrial and institutional buildings. The funicular shell described in this paper is an exception. Its use in Civil Engineering practice can extend the frontiers of application of concrete thin shells to intermediate floors, loading platforms to carry heavy warehouse loading, foundation footings of tall chimneys, foundation rafts, and bridge decks and water tanks. In this paper, some of these possibilities are briefly outlined. The normal sequence followed in designing concrete shells is to assume their geometry and topology to start with. A stress analysis is next carried out. There are definite advantages to be gained in reversing this sequence. Thus, we may assume a desired state of stress under a specified condition of loading, to begin with and then proceed to arrive at the appropriate shape. Shapes so found have been designated as funicular shells by Ramaswamy. The desired state of stress in concrete thin shells is equal biaxial compression, unaccompanied by shear, so that tensile stresses do not develop. The allowable compressive stress in both the directions is limited to what the material can safely carry. It is easy to see that such a prescription of stresses lead to a fully stressed design. Consequently, funicular shells are optimal structural forms. Shapes of funicular shells may be found by analytical or experimental means.

**S.Elangovan and A.R.Santhakumar (1988)** have done a research work on parametric study of funicular shells. The funicular shell was analysed by the finite element method using isoparamertic elements with five degrees of freedom at each node. A computer programme developed by the authors was used in the analysis of funicular shells with clamped boundaries. The behavior of the shell under uniformly distributed load, for various ratios of spans and rise/span was studied analytically. Approximate expressions for the calculation of bending moment at the edge of the shell, in-plane force at crown, and deflection at crown had been proposed. The funicular shell is a shallow shell of double curvature. The shape of such shells to suit any boundary geometry can be found by analytical or experimental methods. The behavior of the shell under uniformly distributed load, for various ratios of spans and rise/span was studied analytically. The load applied was within the elastic range and hence, the deflections, forces, and moments are proportional to the load. So the intensity of the uniformly distributed load was taken equal in all cases.

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## IV. METHODOLOGY

Funicular shells in square and rectangular ground plan are considered for the investigation. Numerical approach using advanced finite element analysis based software called as SAP 2000 is adopted. Based on reported literature of similar nature a FE model is developed and a study is conducted to investigate the behaviour of funicular shells under uniformly distributed load.

## V. EXECUTION OF PROGRAM

The model consisted of a funicular shell uniformly loaded at its top. The finite element technique and a related computer program (SAP 2000) were utilized to analyze structure. The finite element model used to analyse the shell is based on the assumption that the material is linearly elastic. Hence, the theoretical prediction is expected to be valid only in the elastic region of the shell behavior. The plan of the shell, its dimensions and other dimensions are represented in the Table No.1, and fig 2 shows a four noded quadrilateral element.

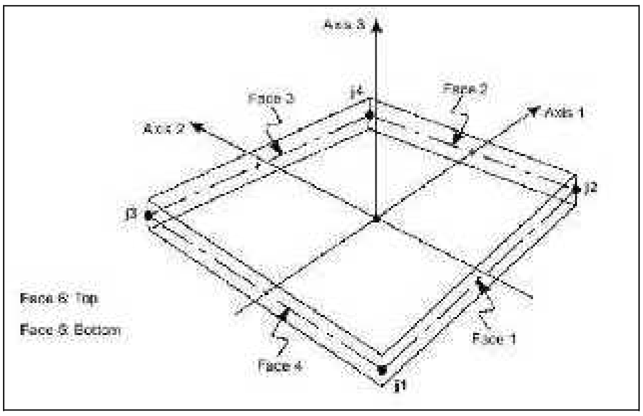


**Fig.2:- Four noded quadrilateral shell element**

**Table No.1:- Shell Dimensions**

| Geometry | Designation | Plan (m) | Rise (mm) | Thickness (mm) |
|---|---|---|---|---|
| Square | FS I | 10X10 | 1000 | 50 |
| | FS II | 10X10 | 1500 | 50 |
| | FS III | 10X10 | 2000 | 50 |
| | FS IV | 15X15 | 1000 | 50 |
| | FS V | 15X15 | 1500 | 50 |
| | FS VI | 15X15 | 2000 | 50 |
| Rectangle | FS VII | 15X10 | 1000 | 50 |
| | FS VIII | 15X10 | 1500 | 50 |
| | FS IX | 15X10 | 2000 | 50 |
| | FS X | 20X15 | 1000 | 50 |
| | FS XI | 20X15 | 1500 | 50 |
| | FSV XII | 20X15 | 2000 | 50 |

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The finite element model of funicular shell with different ground plan is developed by using SAP 2000 finite element package is shown in Figure 3 to 5.

**Fig.3:- Square funicular shell**

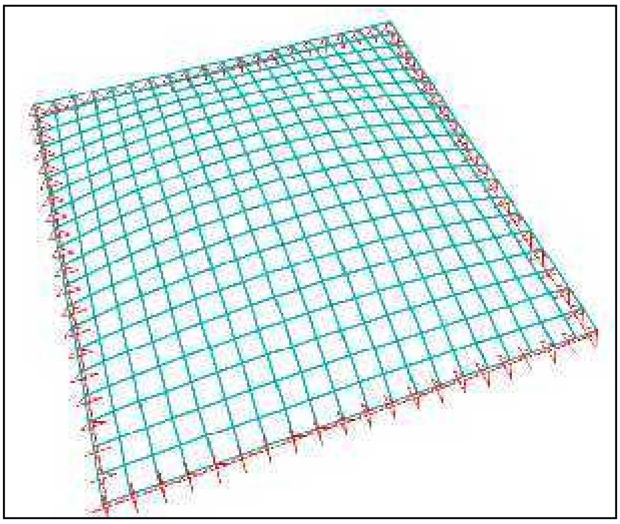

**Fig.4:- Hinge support at all the edges**

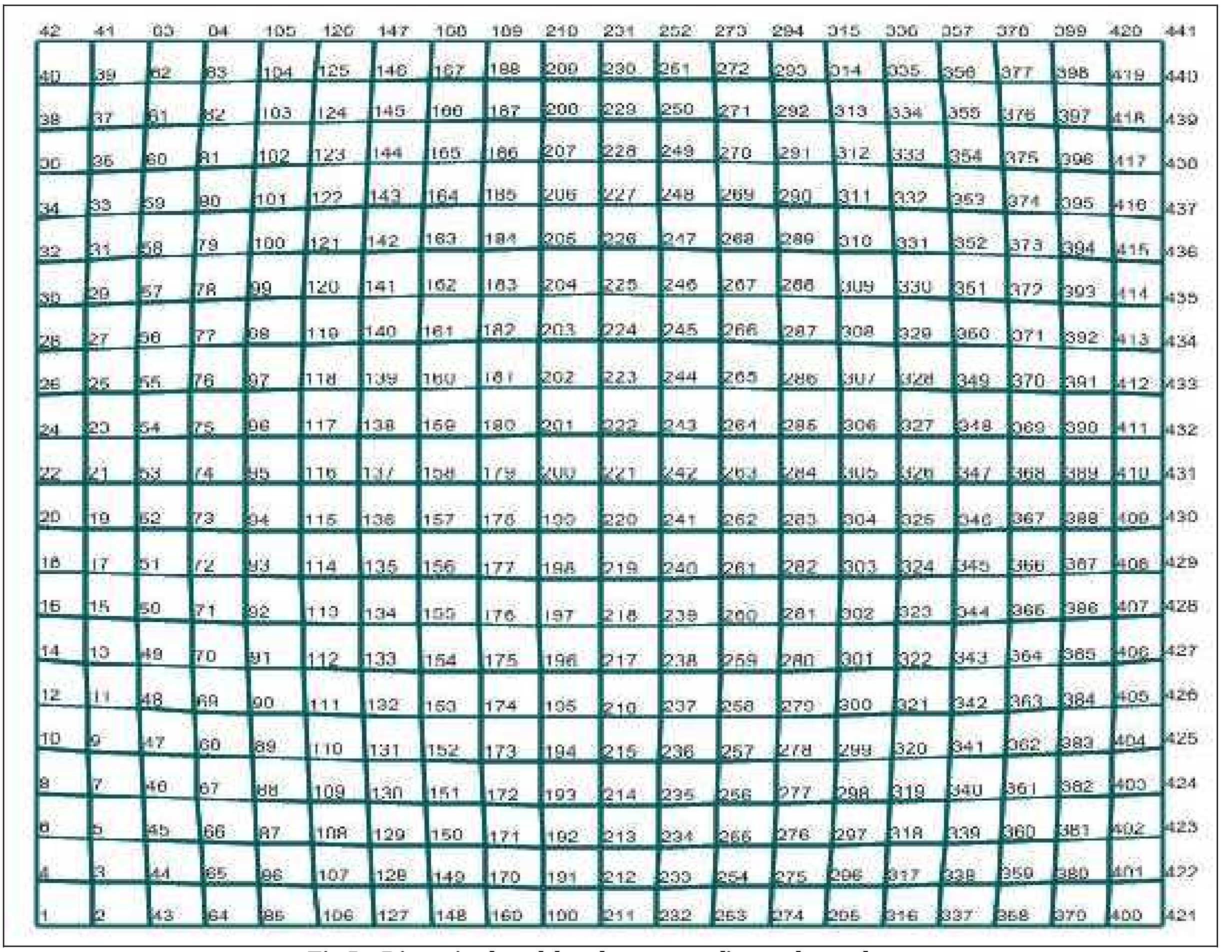

**Fig.5:- Discretized model and corresponding node numbers.**

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## VI. RESULTS AND DISCUSSION

Shells of various sizes were modelled and analyzed for various conditions. The shell is simply supported at all the four edges. Since the stresses are more at the edges, edge beam is place at the edge to reduce stress. The shells are subjected to given uniformly distributed load. From test specimens values of stresses and deflections for different loadings were recorded. From the stress and deflection values graph is potted, funicular shells are loaded up to 20 KN/m. It is observed that the deflection of the shell due to applied loads is small. From results obtained it is found out that when the shell thickness is reduced the deflection of the increases for the same applied load. Also the deflection reduces with increase in rise (R) of the shell. Deflection is more in shell center compared to other points in the shell membrane.

*Analysis of funicular shells with varying rises*

**Square shell of size 10X10 profile:**

The Funicular shell over square ground plan were modelled with the corresponding varying rises 1m, 1.5m, 2m. The shell is supported along its all four faces. The thickness of the shell is uniform throughout, with an edge beam along the edges of four faces. A plot is made with size of the shell to varying sizes of the funicular shell to know the shell profile of size 10m X10m. Figure.6 shows the profile of square funicular shell of size 10mX10m.

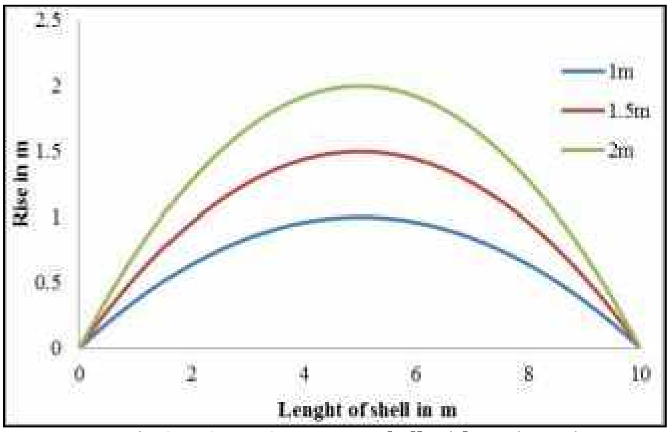


**Fig.6:- 10mX10m square shell with various rise**

**Table No.2:- Max Stresses and Deflection for FS I**

| Load (KN/m) | Stress ($KN/m^2$) | Deflection (mm) |
|---|---|---|
| 0 | 0.156 | -0.1123 |
| 1 | 0.2432 | -0.1997 |
| 2 | 0.3304 | -0.2871 |
| 3 | 0.4176 | -0.3745 |
| 4 | 0.5048 | -0.4619 |
| 5 | 0.592 | -0.5493 |
| 6 | 0.6792 | -0.6367 |
| 7 | 0.7664 | -0.7241 |
| 8 | 0.8536 | -0.8115 |
| 9 | 0.9408 | -0.8989 |
| 10 | 1.0127 | -0.9864 |
| 11 | 1.09828 | -1.0737 |
| 12 | 1.18379 | -1.1611 |
| 13 | 1.2693 | -1.2485 |
| 14 | 1.35481 | -1.3359 |
| 15 | 1.4403 | -1.4233 |
| 16 | 1.5258 | -1.5107 |
| 17 | 1.6113 | -1.5981 |
| 18 | 1.6968 | -1.6855 |
| 19 | 1.7823 | -1.7729 |
| 20 | 1.8678 | -1.8606 |

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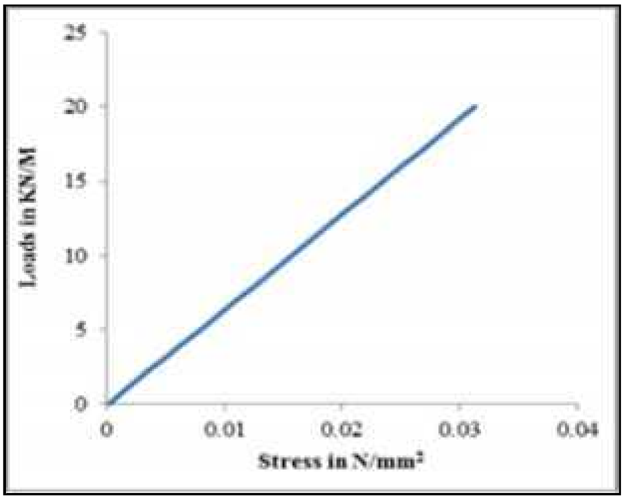


Fig.7:- Maximum stress for FS I

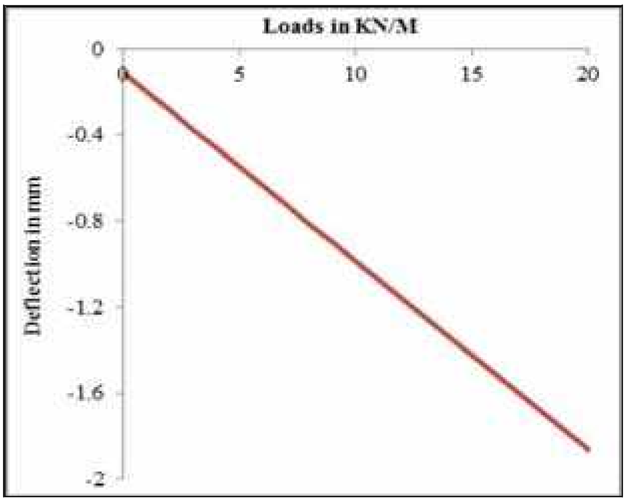


Fig.8:- Maximum deflection for FS I

From the analysis of funicular shell over square ground plan FS I is made and the results of the analysis a plot is made between the distance from the center of the shell and membrane stress distribution along the direction i,j for S23 as shown in fig 9. From the analysis results, the deflections of the concrete funicular shell over square ground plan along vertical direction are calculated and a plot is made between the various points considered along the span of the shell to the corresponding deflections as shown in Fig 10.

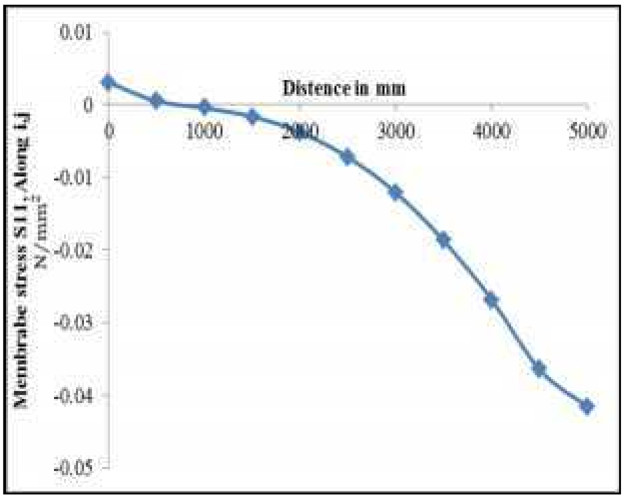


Fig.9:- Membrane stress curve along i,j directions

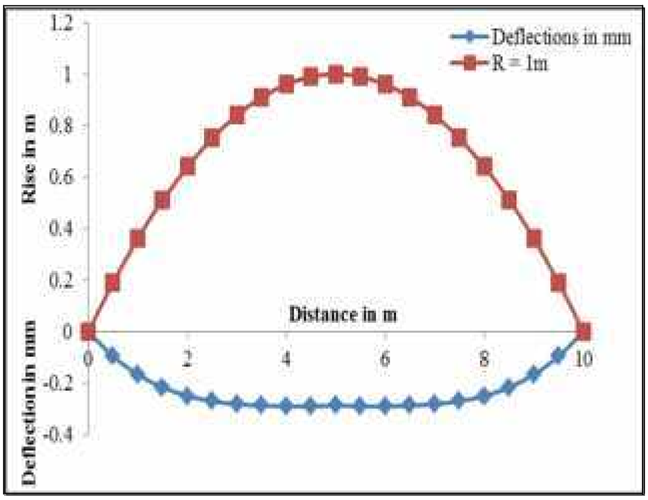


Fig.10:- Deflection curve along i &j

A stress contour for maximum tensile stress of Funicular shell I over square ground plan is shown in fig 11. The maximum value of tensile stress is observed at the corners.

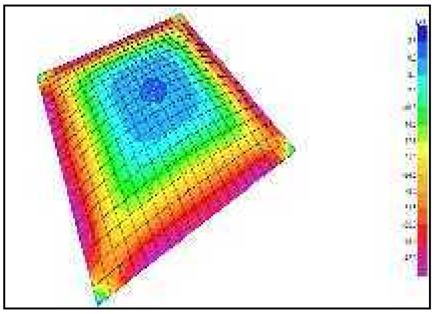

Fig.11:- Stress contour, FS I

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***Analysis of square Funicular shell IV, Funicular shell V, Funicular shell VI (FS IV, FS V, FS VI)***

Finite element analysis is carried out on three various maximum rises by uniformly distributed load. From the analysis results, it is observed that the membrane stresses S11 and S22 along the direction i and j, decreases with the increase in rise of the shallow funicular shell over square ground plans. From the results of displacements due to the uniformly distributed load applied on shallow funicular shell over square ground plans of FS IV, FS V and FS VI for the same magnitude, the deflections are decreased with the increase in rise. From the table a plot is made between, deflections and span / rise ratio (λ). The span to rise ratio has considerable influence in the maximum deflection of shell. The increase in shell rise is result in the reduction of maximum deflection. The reduction in deflection is also means the stiffness increase in the funicular shell. From figure.12 it is observed that deflection increases with the increase in span / rise ratio.

**Table No.3:- Results of FS IV, FS V, FS VI**

| Designation of the shell | Span in mm | Rise (R) in mm | Span/rise ratio (λ) | Deflection in mm |
|---|---|---|---|---|
| FS IV | 15000 | 1000 | 15 | 1.374769 |
| FS V | 15000 | 1500 | 10 | 0.630883 |
| FS VI | 15000 | 2000 | 7.5 | 0.364606 |

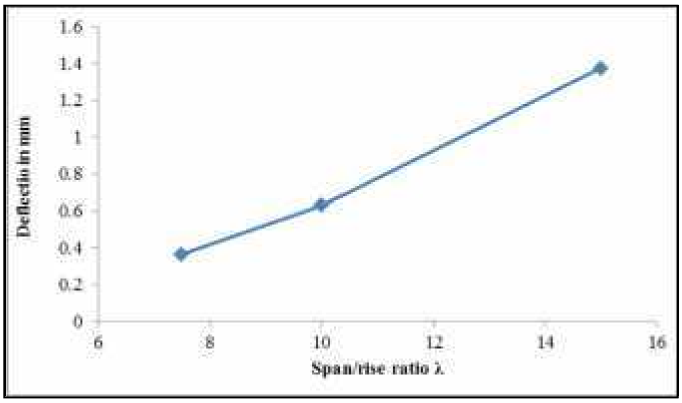


**Fig.12:- Relation between Deflection and Span/rise ratio (λ)**

## VII. SUMMARY AND CONCLUSIONS

An analytical investigation has been carried out to study the behaviour of funicular shells with varying sizes, rises and thickness using SAP 2000 software. Various parameters like variation of deflection, stress distribution have been obtained in this study. The behaviour of funicular shell under uniformly distributed load is presented in this study. The following conclusions are drawn from the test results:

1. It is observed that with the increase in rise of funicular shell the deflection of shell are reduced.
2. The membrane stresses decreases with the increase in rise of concrete funicular shell
3. It is observed that with the increase in thickness of funicular shell the deflection of shell are reduced.
4. Membrane stresses decreases with the increase in thickness of concrete funicular shell.
5. The maximum tensile stresses are developed at shell corners.
6. The maximum compressive stresses are observed at the edges of the shell.
7. The deflections and stress are reduced with use of reinforcement in shells.
8. From the present case study it is observed that the span to rise ratio should be minimum as possible to obtain better performance.

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