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Vertical and Longitudinal Control of a Hopping System by Controlled Limit Cycle

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***Abstract:* This paper deals with the vertical and longitudinal control of a mechanical system which interacts intermittently with the environment. The control approach is based on Controlled LimitCycle for performing the hopping motion, whereas a nonlinear energy based control law is used for longitudinal motion control. The study of this simple system permits a better comprehension of more complicated legged robots. This approach may be used for gait generation, nominal stance stabilization, energy shaping and optimization.**

Keywords: **control of limit cycle, hopping robots, robotics, vertical and longitudinal motion.**

I. INTRODUCTION

Control of legged locomotion systems has aroused an increasing interest and a number of researcher's efforts are concentrated on modelling, control, trajectory generation, interaction with environment and energy optimization. Legged robots present numerous advantages over wheeled robots: capability to walk and run in rugged terrain, climb obstacles...Dynamic model of legged robots is nonlinear and interaction with unknown and varying environment, phase transformations (flight and stance phases...) complicate a great deal the modelling task. Control of fast gait robots necessitates an on-line resolution of model equations and trajectory generation. This uses a heavy procedure and the time of calculation may be prohibitive. Our approach is based on Control of Limit Cycle *'CLC'* for energy optimization in vertical motion control, whereas the translation motion is controlled by an energy-based nonlinear control law.

Raibert [8] has demonstrated that a robot's leg can be modelled by a spring and mass system. The study of simple mechanical systems offers a precious tool for analysis, design and control of legged robots.

The classical methods described in the literature for obtaining the dynamic fast gaits have appeared to be not performing [5]-[7] due to system complexity, varying nature of the ground and the constraints imposed to the system. According to the control approach proposed in this article, the system operates periodic motions which can be characterized by a limit cycle in the phase plane. The control approach is based on energetic optimization which makes the system trajectory tend towards the limit cycle. The evaluation of the energy reference model does not necessitate a great deal of computation. So the trajectory evaluation can be done on-line to allow fast dynamic gaits.

The studied cases in literature can be divided into two categories: passive dynamics (unactuated) and active dynamics. In the first category, we can enumerate:
- Raibert's passive robot [8] for vertical motion in which the stiffness of the leg engender an oscillatory behaviour.
- Mc Geer's [17] non-actuated (gravity powered) biped capable to walk down in inclines.
- Espiau *et al*. [5] have studied a compass-like biped robot and the stability in a passive biped gait.

In the second category, we can enumerate a great number of robots electrically, pneumatically and hydraulically actuated.
- SAP: Pneumatically Actuated System [1]-[2] is a biped, constructed in our laboratory which constitutes our investigations and prospects.
- K. Y. Yi [15] presents a biped robot with compliant ankle joints, all the joints being actuated by DC motors.

S. Arimoto and F. Myazaki describe the Stability and robustness of PID feedback control for robot manipulators of sensory capability [3].

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P. Channon, S. Hopkins and D. Pham, 1. study the optimal walking motions of a bipedal robot [4].

Some aspects of modelling and kinematics of robots are analysed in [6]-[14]-[16]. The walking of robots is treated in[9]-[12]-[15], whereas the hopping is dealt with in [10]-[11]. A robot is not in general a simple linear dynamic system, [13] is interested in non-linear dynamic systems.

The paper is organized as follows: Section II presents a simple mechanical system of two degrees of freedom. Section III describes the control of vertical motion (hopping) by *limit cycle control* approach, and illustrates the forward motion control as well. Section IV provides some simulation results. The final section is devoted to discussions and conclusions.

II. MODEL OF MASS-SPRING-DAMPER SYSTEM

Fig. 1 illustrates the masse-spring-damper system with mass M, viscous friction coefficient β and stiffness k. The latter represents, in the general case, the equivalent stiffness of spring and environment:

$k = \frac{k_s k_g}{k_s + k_g}$ where k_s and k_g designate the stiffness of spring and groundrespectively. If the ground is infinitely rigid: $k \approx k_s$.

The vertical and horizontal coordinates of the mass in a fixed frame are noted z and x respectively. This system can lead either a hopping motion in z-direction or a forward motion in x-direction.

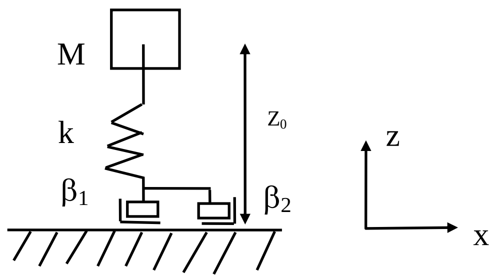

Fig. 1 Masse-spring-damper system

z_0 denotes the spring length at rest.

A. *Vertical free motion*

Let, at first, consider the free vertical motion of this system. This motion is composed of two phases: flight phase and stance phase. During these phases the energy is converted succeedingly into potential and kinetic forms. Let z_m, v_l and v_t be maximum flight height, lift off velocity and touchdown velocity respectively. The energy flow and tranformations in the system have been illustrated in Fig 2.

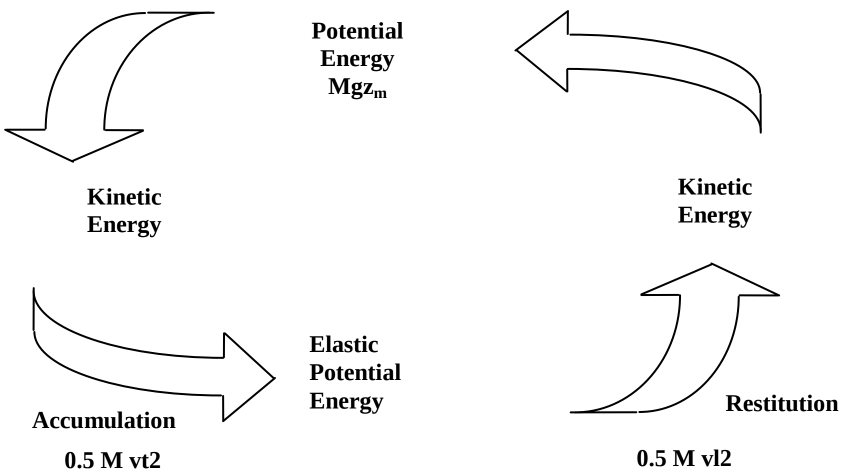

Fig.2 Energy flow and transformations in the system

1 - Flight phase

During this phase the system undergoes a ballistic trajectory depending on initial conditions. In this phase, we take into account the friction. Let z_i and $\dot{z}_i$ be initial position and velocity respectively. The equation of the motion can be written:

$$M\ddot{z} + \beta_1\ \dot{z} = -Mg \quad (1)$$

where β_1 represents the viscous friction coefficient. The general solution of the "(1)" can be expressed:

$$z = z_i + \frac{M}{\beta_1}(\frac{Mg}{\beta_1} + \dot{z}_i) - \frac{M}{\beta_1}(\frac{Mg}{\beta_1} + \dot{z}_i)e^{-\frac{\beta_1}{M}t} - \frac{Mg}{\beta_1}t \qquad (2)$$

The equations of velocity and acceleration are found by differentiating "(2)":

$$\dot{z} = (\frac{Mg}{\beta_1} + \dot{z}_i)e^{-\frac{\beta_1}{M}t} - \frac{Mg}{\beta_1}$$

$$\ddot{z} = -(g + \frac{\beta_1}{M}\ \dot{z}_i)e^{-\frac{\beta_1}{M}t}$$

At maximum height, the velocity is zero:

$$\dot{z} = (\frac{Mg}{\beta_1} + \dot{z}_i)e^{-\frac{\beta_1}{M}t} - \frac{Mg}{\beta_1} = 0$$

This occurs at time t_m.

$$t_m = \frac{M}{\beta_1} Ln\ (1 + \frac{\beta_1}{Mg}\dot{z}_i)$$

The time t_m permits to calculate the maximum height:

$$z_m = z_i + \frac{M}{\beta_1}(\frac{Mg}{\beta_1} + \dot{z}_i) - \frac{M}{\beta_1}(\frac{Mg}{\beta_1} + \dot{z}_i)e^{-\frac{\beta_1}{M}t_m} - \frac{Mg}{\beta_1}t_m$$

On the assumption that upward and downward flight duration is equal; the total flight duration would be evaluated by:

$$t_f \cong 2\ t_m = 2\frac{M}{\beta_1} Ln\ (1 + \frac{\beta_1}{Mg}\dot{z}_i)$$

Using the Taylor development, the exponential term, in "(2)" can be written:

$$e^{-\frac{\beta_1}{M}t} = 1 - \frac{\beta_1}{M}t + \frac{1}{2}\frac{\beta_1^2}{M^2}t^2 - \ldots \quad (3)$$

Substituting "(3)" in the "(2)", we obtain:

$$z = z_i + \frac{M}{\beta_1}(\frac{Mg}{\beta_1} + \dot{z}_i) - \frac{M}{\beta_1}(\frac{Mg}{\beta_1} + \dot{z}_i)(1 - \frac{\beta_1}{M}t + \frac{1}{2}\frac{\beta_1^2}{M^2}t^2 - \ldots) - \frac{Mg}{\beta_1}t$$

In frictionless motion ($\beta_1 = 0$), the well-known classical vertical motion equations can be found:

$$z(t) = z_i + \dot{z}_i t - \frac{1}{2}gt^2$$

$$\dot{z}(t) = \dot{z}_i - gt$$

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$$\ddot{z}(t) = -g$$

The maximum height is $z_m = z_i - \frac{1}{2} g t_m^2$ with $t_m = \frac{\dot{z}_i}{g}$

Thus, the flight phase duration is: $t_f = 2\, t_m = 2\, \frac{\dot{z}_i}{g}$

Recall the assumption that $\beta_1 = 0$, in the flight phase, so the motion equation, in the phase plane can be written as:

$$z = z_i + \frac{1}{2}\frac{z_i^2}{g} - \frac{1}{2}\frac{\dot{z}^2}{g}$$

2 - Stance phase

Contact with ground occurs when $z - z_0 \le 0$; the contact is assumed energyless. The equation in this phase can be expressed by:

$$M\ddot{z} + k(z - z_0) + \beta_2 \dot{z} = -Mg \quad (3)$$

where β_2 designates the viscous friction coefficient at stance phase.

Let us define a contact function $\xi(z)$ equal to zero during flight phase and equal to one duringstance phase:

$$\xi(z) = \frac{1}{2}(1 - sign\ (z - z_0))$$

Introducing this function in “(3)”, we can write the general equation, valid either on flight phase or on stance phase:

$$M\ddot{z} + \xi(z)[k(z - z_0) + \beta_2 \dot{z}] + [1 - \xi(z)]\beta_1 \dot{z} = -Mg \quad (4)$$

The equation (4) admits, at stance phase, the following general solution:

$$z(t) = \frac{1}{2\sqrt{(\frac{\beta_2}{2M})^2 - \omega_0^2}}[\dot{z}_i + (\frac{\beta_2}{2M} + \sqrt{(\frac{\beta_2}{2M})^2 - \omega_0^2})(z_i - C)]\exp(s_1 t)$$
$$- \frac{1}{2\sqrt{(\frac{\beta_2}{2M})^2 - \omega_0^2}}[\dot{z}_i + (\frac{\beta_2}{2M} - \sqrt{(\frac{\beta_2}{2M})^2 - \omega_0^2})(z_i - C)]\exp(s_2 t) + C \quad (5)$$

s_1 and s_2 are the roots of the characteristic equation of (3): $s_1 = -\frac{\beta_2}{2M} + \sqrt{(\frac{\beta_2}{2M})^2 - \omega_0^2}$ and

$$s_2 = -\frac{\beta_2}{2M} - \sqrt{(\frac{\beta_2}{2M})^2 - \omega_0^2}$$

where $\omega_0 = \sqrt{\frac{k}{M}}$ designates the system natural frequency and $C = -\frac{g}{\omega^2{}_0} + z_0$.

The velocity attains maximum value at touch down and lift off positions. At these points the acceleration is zero:

$$\ddot{z}(t) = K_1 s_1^2 \exp(s_1 t) + K_2 s_2^2 \exp(s_2 t) = 0 \quad (6)$$

with

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$$K_1 = \frac{1}{2\sqrt{(\frac{\beta_2}{2M})^2 - \omega_0^{\ 2}}}[\dot{z}_i + (\frac{\beta_2}{2M} + \sqrt{(\frac{\beta_2}{2M})^2 - \omega_0^{\ 2}})(z_i - C)]$$

$$K_2 = \frac{1}{2\sqrt{(\frac{\beta_2}{2M})^2 - \omega_0^{\ 2}}}[\dot{z}_i + (\frac{\beta_2}{2M} - \sqrt{(\frac{\beta_2}{2M})^2 - \omega_0^{\ 2}})(z_i - C)]$$

Thus, the equation (6) permits to evaluate the time at the end of which the acceleration is zero:

$$t_f = \frac{1}{2\sqrt{(\frac{\beta_2}{2M})^2 - \omega_0^{\ 2}}} Ln(\frac{K_2 s_2^{\ 2}}{K_1 s_1^{\ 2}})$$

Consequently, the stance duration is equal to:

$$T_C = 2\ t_l = \frac{1}{\sqrt{(\frac{\beta_2}{2M})^2 - \omega_0^{\ 2}}} Ln(\frac{K_2 s_2^{\ 2}}{K_1 s_1^{\ 2}})$$

On the assumption that $(\beta_2 = 0)$, “(5)” can be written:

$$z(t) = \frac{1}{2j\ \omega_0}[\dot{z}_i + j\omega_0(z_i - C)](\cos\omega_0 t + j\sin\omega_0 t) - \frac{1}{2j\ \omega_0}[\dot{z}_i - j\omega_0(z_i - C)](\cos\omega_0 t - j\sin\omega_0 t) + C$$

Finally, this yields to:

$$z(t) = \frac{\dot{z}_i}{\omega_0}\sin\omega_0 t - \frac{g}{\omega_0^{\ 2}}(1-\cos\omega_0 t) + z_i\cos\omega_0 t + z_0\ (1-\cos\omega_0 t)$$

and the contact duration is:

$$T_C = \frac{2}{\omega_0}[\pi - \arctan(-\frac{\dot{z}_i\omega_0}{g})]$$

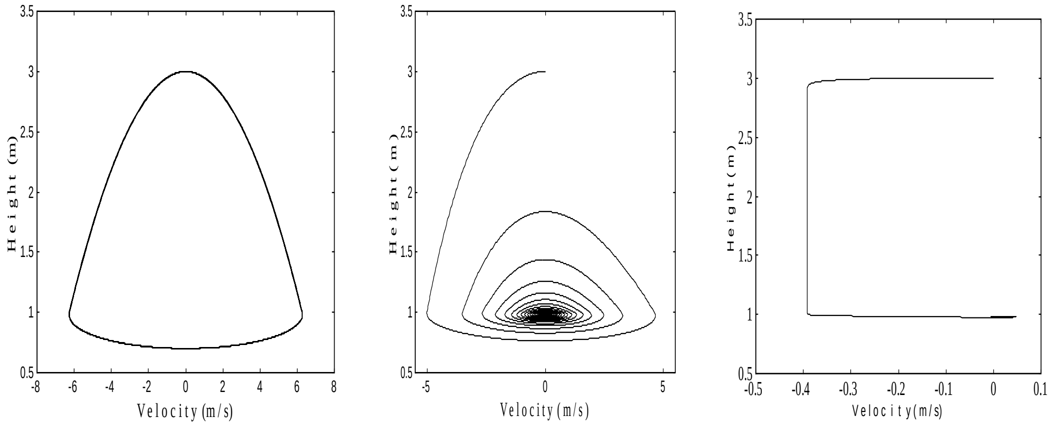
The form of the trajectory in the phase plane depends greatly on the value of β_2. Simulation results in the phase plane are depicted in Fig. 3 for several cases. The plots start at top, point $(\dot{z}_i, z_i)$, go successively through touch down and lift off in the counter-clockwise direction (Fig. 3 a and b). We can remark the presence of singular points:

-With no friction, the system is conservative, it continues to oscillate indefinitely. There is a summit, the stability is non asymptotical, Fig. 3.a.

-With non-negligible viscous friction, the system is damped and we find a stable focus, Figure 3.b.

-With high level of viscous friction, the system is greatly damped; it appears a stable node, Figure 3.c.

In each case, the gait is composed of a flight phase and a stance phase.



$(a)\, z_i = 3m; \dot{z}_i = 0\,; \beta = 0 \;\; (b)\, z_i = 3m; \dot{z}_i = 0\,; \beta = 2 \;\; (c)\, z_i = 3m; \dot{z}_i = 0\,; \beta = 4$

Fig. 3 Trajectories in the phase plane

B. Forward free motion

In forward motion, we interest at the velocity. The motion equation at x-direction can be written as:

$$M\dot{v}_x + \beta_x v_x = 0 \qquad (7)$$

where v_x and β_x designate respectively the velocity and the viscous friction coefficient at x-direction. The first order equation (7) admits as solution:

$v_x = v_{0x} \exp(-\beta_x\ t / M)$ (8) Where v_{Ox} denotes the initial forward velocity.

III. CONTROL STRATEGY

We suppose that vertical and forward motions are decoupled.

A. Control of vertical motion by CLC approach

The equation of motion at flight phase is the same as in free motion due to uncontrollability of the system:“(1)”.

Let us consider the motion equation in the stance phase, during which the control, u, is applied.

$$M\ddot{z} + k(z - z_0 - u) + \beta_2 \dot{z} = -Mg \qquad (9)$$

u is a non-linear feedback control whose multiplication by the stiffness is equivalent to force.

$$u = -\frac{\Lambda}{k} sign(V - V_0)\dot{z} \text{ with } \Lambda > 0$$

We define, V_0 as energy reference, as follows:

$V_0 = gz_m$ where z_m represents the desired height.

Let us select the following Lyapunov function candidate:

$$V = \frac{1}{2}\dot{z}^2 + gz + \frac{k}{2M}(z - z_0)^2$$

Hence, the orbit to stabilize would be:

$$\Omega_0 = \{(z, \dot{z}) \in R^2, V = V_0 = gz_m\}$$

The control must act so as Ω_0 be attractive.

The control has for main task to increase the system energy if $V < V_0$ and decrease it if $V > V_0$.

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Differentiating the proposed Lyapunov function:

$$\dot{V} = \dot{z}\ddot{z} + g\dot{z} + \frac{k}{M}\dot{z}(z-\dot{z})$$

Calculating $\ddot{z}$ from “(4)” and substituting it in the above equation, we obtain:

$$\dot{V} = -\dot{z}\xi(z-u)\frac{k}{M}(z-\dot{z}-u) - \frac{\beta_2}{M}\dot{z} + \frac{k}{M}\dot{z}(z-\dot{z})$$

with $\xi(z-u)=1$ the control being active at stance phase.

We can finally deduce :

$$\dot{V} = -\dot{z}\Lambda\frac{k}{M}sign(V-V_0)\dot{z}\xi(z-u) - \frac{\beta_2}{M}\dot{z}$$

- If $z-z_0-u>0 \Rightarrow \xi(z-u)=0$: The control has no effect, the system is uncontrollable during the flight phase.
- If $z-z_0-u<0 \Rightarrow \xi(z-u)=1$ The control being applied during stance phase.

$$\dot{V}(V-V_0) = (-\dot{z}\Lambda\frac{K}{M}sign(V-V_0)\dot{z} - \dot{z}\frac{\beta_2}{M})(\frac{1}{2}\dot{z}^2 + gz + \frac{K}{2M}(z-z_0)^2 - gz_m)$$

* If $V>V_0 \Rightarrow \dot{V}<0$ The control having a passive (dissipative) action, the global energy of the system is decreased.

* If $V<V_0 \Rightarrow \dot{V}>0$ The control has an active role, the global energy of the system is increased.

The function $sign(V-V_0)$ does admit two values 0 and 1.

- If $z<z_m \Rightarrow V<V0 \Rightarrow sign(V-V_0)=-1 \Rightarrow u=\frac{\Lambda}{k}\dot{z}$

The equation (9) can be expressed:

$$M\ddot{z} + k(z-z_0) + (\beta_2 - \frac{\Lambda}{k})\dot{z} = -Mg \quad (10)$$

The equation (10) admits as solution:

$$z(t) = \frac{1}{2\sqrt{(\frac{\alpha}{2M})^2-\omega_0^{\ 2}}}[\dot{z}_i + (\frac{\alpha}{2M} + \sqrt{(\frac{\alpha}{2M})^2-\omega_0^{\ 2}})(z_i - C)]\exp(r_1 t)$$
$$- \frac{1}{2\sqrt{(\frac{\alpha}{2M})^2-\omega_0^{\ 2}}}[\dot{z}_i + (\frac{\alpha}{2M} - \sqrt{(\frac{\alpha}{2M})^2-\omega_0^{\ 2}})(z_i - C)]\exp(r_2 t) + C \quad (11)$$

with $\alpha = \beta_2 - \frac{\Lambda}{k}$; $r_1 = -\frac{\alpha}{2M} + \sqrt{(\frac{\alpha}{2M})^2 - \omega_0^2}$ and $r_2 = -\frac{\alpha}{2M} - \sqrt{(\frac{\alpha}{2M})^2 - \omega_0^2}$

ω_0 and C defined as for “(5)”.

- If $z>z_m \Rightarrow V>V0 \Rightarrow sign(V-V_0)=1 \Rightarrow u=-\frac{\Lambda}{k}\dot{z}$

The equation of motion, in this case can be expressed by:

$$M\ddot{z} + k(z-z_0) + (\beta_2 + \frac{\Lambda}{k})\dot{z} = -Mg$$

The solution of this equation is identical to that of (10) but with $\alpha = \beta_2 + \frac{\Lambda}{k}$.

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Poincaré return maps

We are going now to demonstrate the existence of stable limit cycles, as a result of application of the above control, by Poincaré return maps, Fig.4.

We consider the intersections of a constant section $\dot{z} = 0$ with the $(z, \dot{z})$ plot in the phase plane. These consecutive intersections constitute a set of discrete points: $[z_n \in R,\ \dot{z}_n = 0,\ z_n \neq \min]$

Two consecutive points are related by $z_{n+1} = f(z_n)$. Such a system corresponds to a discrete dynamic system.

At the first time, we suppose that: $sign(V - V_0) = -1$

- If the viscous coefficient is supposed null neither at flight phase nor at stance phase, we can elaborate the function f as follows :

* $z_n \to z_{nt}$: z_n permits to calculate z_{nt} (height of touch down)

* $z_{nt} \to z_{nl}$: z_{nt} allows computing z_{nl} (height of lift off)

* $z_{nl} \to z_{n+1}$: z_{nl} permits to calculate z_{n+1} (consecutive apex)

Calculation of z_{nt} **:** Equation (2) $\to z_{nt} = z_n + \frac{M}{\beta_1}[\frac{Mg}{\beta_1}(1 - \exp(-\frac{\beta_1}{M} t_f)) - t_f]$ (12.a)

$$\dot{z}_{nt} = -\frac{Mg}{\beta_1}(1 - \exp(-\frac{\beta_1}{M} t_f))$$

Calculation of z_{nl} **:** Equation (5)

$$z_{nl} = \frac{1}{2\sqrt{(\frac{\alpha}{2M})^2 - \omega_0^{\ 2}}}[\dot{z}_{nt} + (\frac{\alpha}{2M} + \sqrt{(\frac{\alpha}{2M})^2 - \omega_0^{\ 2}})(z_{nt} - C)]\exp(s_1 T_c)$$

$$\to - \frac{1}{2\sqrt{(\frac{\alpha}{2M})^2 - \omega_0^{\ 2}}}[\dot{z}_{nt} + (\frac{\alpha}{2M} - \sqrt{(\frac{\alpha}{2M})^2 - \omega_0^{\ 2}})(z_{nt} - C)]\exp(s_2 T_c) + C \quad (12.b)$$

with $\alpha = \beta_2 - \frac{\Lambda}{k}$; $r_1 = -\frac{\alpha}{2M} + \sqrt{(\frac{\alpha}{2M})^2 - \omega_0^{\ 2}}$ and $r_2 = -\frac{\alpha}{2M} - \sqrt{(\frac{\alpha}{2M})^2 - \omega_0^{\ 2}}$

$$T_C = 2\, t_l = \frac{1}{\sqrt{(\frac{\alpha}{2M})^2 - \omega_0^{\ 2}}} Ln(\frac{K_2 s_2^{\ 2}}{K_1 s_1^{\ 2}})$$

$$\dot{z}_{nt} = -\frac{Mg}{\beta_1}(1 - \exp(-\frac{\beta_1}{M} t_f))$$

Calculation of z_{n+1} **:** equation (2) $\to$

$$z_{n+1} = z_{nl} + \frac{M}{\beta_1}(\frac{Mg}{\beta_1} + \dot{z}_{nl})(1 - \exp(-\frac{\beta_1}{M} t_f)) - \frac{Mg}{\beta_1} t_m \quad (12.c)$$

The equations (12.a), (12.b) and (12.c) permit to calculate the function f.

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$$z_{n+1} = \frac{1}{2\sqrt{(\frac{\alpha}{2M})^2 - \omega_0{}^2}}[(\frac{\alpha}{2M} + \sqrt{(\frac{\alpha}{2M})^2 - \omega_0{}^2})\exp(s_1 T_c)$$

$$-(\frac{\alpha}{2M} - \sqrt{(\frac{\alpha}{2M})^2 - \omega_0{}^2})\exp(s_2 T_c)]z_n + R_1$$

where R_1 represents the sum of the terms independent of z_n.

$$\frac{dz_{n+1}}{dz_n} = \frac{1}{2}[(\frac{\frac{\alpha}{2M}}{\sqrt{(\frac{\alpha}{2M})^2 - \omega_0{}^2}} + 1)\exp(s_1 T_c) - (\frac{\frac{\alpha}{2M}}{\sqrt{(\frac{\alpha}{2M})^2 - \omega_0{}^2}} - 1)\exp(s_2 T_c)] \quad (13)$$

The limit cycle is stable if the following condition is satisfied:

$$\frac{dz_{n+1}}{dz_n} < 1$$

The above condition could be obtained if : $\alpha = \beta_2 - \frac{\Lambda}{k} > 0 \Rightarrow \Lambda < k\beta_2$

At the second time, we suppose that $sign(V - V_0) = -1$

In this case, equation (13) is valid as well but with:

$$\alpha = \beta_2 + \frac{\Lambda}{k} > 0$$

With the following parameters, we obtain $\frac{dz_{n+1}}{dz_n} = .9$

$$M = 2\,kg;\, k = 1000\,N/m;\, \Lambda = 10;\, \beta_1 = 0.05;\, \beta_2 = 0.05$$

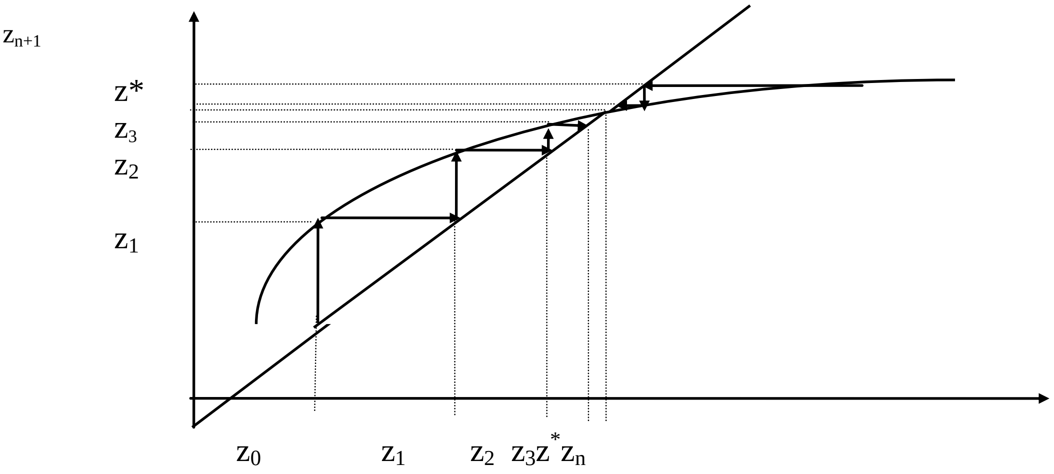

Fig.4 Stability of limit cycle

B. Forward motion control

Recall that the system is not controllable in flight phase. So the control acts both on vertical and forward motion solely at stance phase. We apply two different control laws and compare the obtained results by simulation.

1. PD control for forward motion

The control law is of the form:

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$F = k_{px}(v_{xd} - v_x) + k_{vx}(\dot{v}_{xd} - \dot{v}_x)$ With :

k_{px} : proportional gain

k_{vx} : derivative gain

v_{xd} : desired x-velocity

$\dot{v}_{xd}$: desired x-acceleration

Introducing the above defined control, the forward motion equation can be written:

$$M\dot{v}_x = \xi(z-u)[k_{px}(v_{xd} - v_x) + k_{vx}(\dot{v}_{xd} - \dot{v}_x) - \beta_{x2} v_x] - [1 - \xi(z-u)]\beta_{x1} v_x \qquad (14)$$

where β_{x1} *and* β_{x2} designate viscous friction coefficient, in x-direction, during flight and stance phase respectively.

2. Non-linear control for forward motion

We apply a non-linear control law:

$F = -\Lambda_x sign(V_x - V_{x0})$ with

$V_x = \frac{1}{2} M v_x^{\ 2}$

$V_{xO} = \frac{1}{2} M v_{xd}^{\ 2}$: Reference energy; v_{xd} : desired velocity at x-direction

Let us select a Lyapunov function candidate V_{LX} :

$V_{LX} = \frac{1}{2} v_x^{\ 2}$

In order to demonstrate the stability of the proposed law, let us differentiate the above lyapunov function:

$\dot{V}_{LX} = v_x \dot{v}_x$

The control is solely operational at stance phase: $\xi(z-u) = 1$

Substituting the above derivative in the forward motion equation, we find:

$\dot{V}_{LX} = -v_x \frac{\Lambda_x}{M} sign(V_x - V_{x0}) - \frac{\beta_x}{M} v_x^{\ 2}$

- If $v_x > v_{xd} \Rightarrow sign(V_x - V_{x0}) = 1 \Rightarrow \dot{V}_{LX} < 0$: The feedback would have a passive (dissipative) role in order to diminish v_x

- If $v_x < v_{xd} \Rightarrow sign(V_x - V_{x0}) = -1 \Rightarrow \dot{V}_{LX} > 0$: The feeedback would have an active contribution in order to increase v_x ; in this case, the following condition should be satisfied: $\Lambda_x > \beta_x v_{xm}$

IV. SIMULATION RESULTS

The system parameters are: M = 2 Kg, k = 1000 N/m. Fig. 5 illustrates the simulation results for $\beta_1 = 0.05$ and $\beta_2 = 0.1$. The control gain for hopping is $\Lambda = 1$. Fig. 5.a represents the limit cycle for an initial height z(0) =2 m and a desired jump height z_m =1.5 m; a *saturation* function is used instead of a *sign* function (for saving simulation time). Simulation results illustrate that the same limit cycle would be obtained for different initial heights for a preselected desired height. In Fig. 5.b is presented the endpoint position. In Fig. 5.c is depicted the endpoint vertical velocity. We remark that the period is nearly 1.2 s (depending on control gain). The stance phase duration is very short compared to that of the flight phase. Fig. 5.d illustrates the contact function. For higher values of control gain, Λ=100, the limit cycle is obtained within one iteration, as shows Fig. 5.e. In Fig. 5.f are depicted forward motion velocity both by PD control and the proposed non-linear energy reference control, for a desired velocity of 1 m / s. We note that the response time is markedly shorter in the latter case, 3

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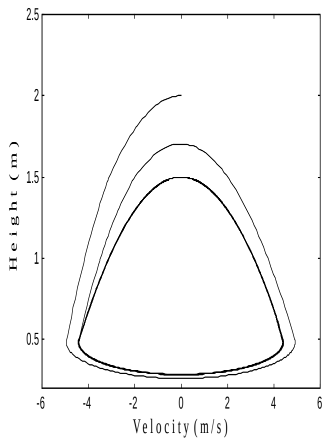
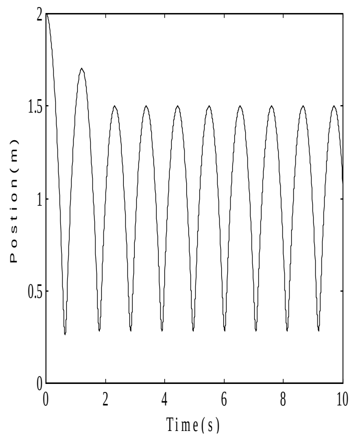
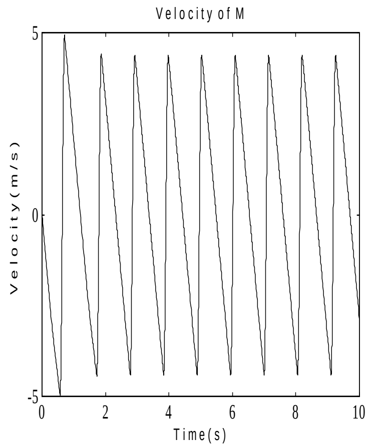
seconds, against to more than 15 seconds for the former case. In each case, the velocity evolves as a step-like-function before attaining the desired value. This is due to the fact that the control is frozen during flight phase.

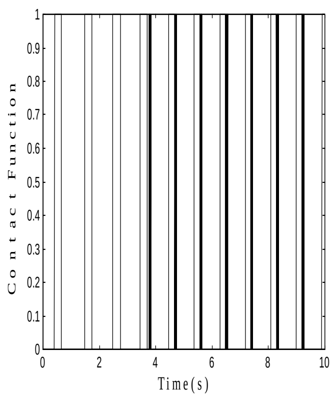


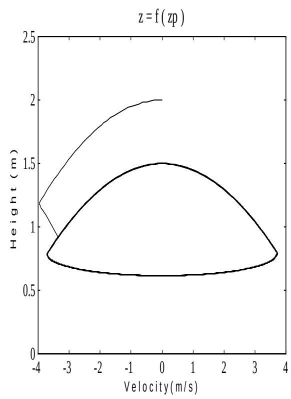




Λ = 1

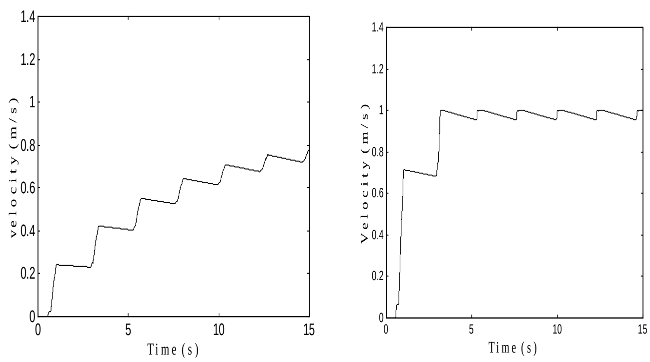
Fig. 5.a **Fig. 5.b** **Fig. 5.c**





Λ = 100

Fig. 5.d **Fig. 5.e**



β_{1x} = 0.05 ; β_{2x} = 0.05

PD control **Energy reference control**

k_{px} = 50 ; k_{vx} = 50 **Λ = 5**

Fig.5 .f

V. CONCLUSION

In this paper we presented a control approach, based on energy reference adjusting, for realizing hopping and bouncing gaits in the case of a mass-spring-damper system. This simple system constitutes a good basis for carrying out analytical studies in the field of legged robots. This control approach can be easily generalized to hopping or running of legged robots, as our current experiments confirm. The phase trajectory of the hopping motion is composed of a closed orbit that tends towards a controlled limit cycle, whose stability has been demonstrated by Poincaré return maps. We use an energy reference model, associated to a feedback, which is non-linear in regard to positions and velocities. The control of longitudinal translation is obtained as well, by a non-linear energy-based law, whose response time is less than that of a PD control. This approach is appropriate for implicit on-line trajectory generation, gait stabilization and energy optimisation. The control is robust versus model parameter uncertainties and disturbances. This method being applied, in our laboratory, successfully to a pneumatic monopod robot and is generalized to a multi-legged robot.

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