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Hydromagnetic Fluid Flow between Porous Parallel Plates in Presence of Variable Transverse Magnetic Field

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Abstract*— Magneto-hydrodynamic fluid flow between two parallel plates the top plate being porous with a constant suction and variable magnetic field lines are fixed relative to the top moving plate has been investigated.The governing equation for the fluid flow includes continuity equation, momentum equation and the energy equation. Finite difference technique is used to solve the non-linear partial differencial equations. The numerical scheme used is implemented in MATLAB software and the solutions are presented graphically. Observations and discussions on the effect of varying various parameters such as magnetic parameter M, pressure gradient, prandtl number Pr, Eckert number Ec, joules heating parameter R and suction parameter So on the velocity profiles and temperature profiles is done. A change in the value of the parameters mentioned above was observed to either increase, decrease or to have no effect on the velocity and Temperature profiles respectively. The result obtained in this research can be applied by engineers in designing and improving efficiency and performance of machines in the dyeing industry and in extraction of metal industry.

Index Terms—**Magneto-hydrodynamic, velocity, temperature.**

I. INTRODUCTION

Magneto-hydrodynamic (MHD) fluid flow is the study of flow of an electrically conducting fluid in presence of a magnetic field. MHD studies the dynamics of the interaction of electrically conducting fluids and electromagnetic field. The electrically conducting fluid in presence of variable magnetic fields is an important phenomenon in our day-to-day lives. Fluid is a type of matter which under given thermodynamic conditions and in absence of external forces takes the shape of the container. Hoogendorn et al [1] investigated the coquette flow in presence of uniform suction and injection between porous walls when one of the walls is uniformly accelerated. He found that when the magnetic field is considered to be fixed relative to the plate, the flow is accelerated by the magnetic field. However, the magnetic field retards the fluid flow when the magnetic field is fixed relative to the flow. .Henry and Notee et al [2] considered the unsteady MHD free convection coquette flow between two vertical parallel porous plates with uniform suction and injection. The velocity and temperature distributions were obtained using the Laplace transform technique. The results revealed that both temperature and velocity decrease with increasing Prandtl number and with increasing suction/injection parameter. The velocity was also found to increase with increasing Grashof number. Ozoe and Okada et al [3] studied the unsteady MHD flow bounded by an infinite vertical porous plate in presence of thermal radiations. The radiative heat flux was described using the Rossel and approximation, the temperature and suction velocity at the plate were taken to be time-dependent. The velocity and temperature distributions were obtained using an asymptotic expansion of velocity for small magnetic number, as well as a similarity transformation. The results showed that a decrease in both velocity and temperature with increasing radiation and suction parameter. Murakami, Kato and Siyamara et al [4] studied the problem considered when the fluid flow is confined to porous boundaries with suction and injection considering two cases of interest namely the impulsive movement of the lower plate and the uniformly accelerated movement of the lower plate .Seth concluded that the suction exerted a retarding influence on the flow while the magnetic field, time and injection reduce shear stress at lower plate in both the cases while suction increases shear stress at the lower plate. Kwanza, Kinyanjui and Uppal et al [5] investigated MHD Stokes free convection flow past an infinite vertical porous plate subject to constant heat flux with ion-slip current and radiation absorption. The magnetic field lines were assumed to be fixed relative to the moving plate. It was found that the effect of radiation parameter and Prandtl number each reduce the velocity and the temperature of the fluid. .Zikanov and Thess et al [6] investigated unsteady MHD Couette flow of a viscous, incompressible and electrically conducting fluid near an accelerated plate of the channel under constant magnetic field. He compared the unsteady free convection coquette flow at large values of time with the corresponding steady-state problem and found that they are in good agreement. It was also observed that the flow velocity decreases with increasing Prandtl number. Jain et al [7] concentrated on the

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effect of wall porosity on the stability of hydro-magnetic flow between parallel plates under transverse magnetic field and expressed the idea that the flow is largely influenced by porosity and the flow parameters such as velocity, pressure and temperature. Joseph et al [8] investigated unsteady MHD couette flow between two infinite parallel porous plates in an inclined magnetic field with heat transfer. The lower plate was considered to be porous and stationary. He found out that at high Hartmann number the velocity as well as the skin friction decreases. He also found that large Nusselt number corresponds to more active convection and also when the Prandlt number increases, the temperature distribution decreases. Although many investigations have been made in the past two decades, only a few published papers take into account Magneto-hydrodynamic fluid flow between two parallel plates, the top plate being porous with a constant suction in presence of magnetic field lines fixed relative to the top moving plate has been investigated. This has varied applications in dyeing industries.

II. MATHEMATICAL ANALYSIS

Flow configuration

This study has consideredthe analysis of the unsteady fluid flow between parallel porous plates with a variable magnetic field fixed to the upper plate.The upper plate is porous and moves in the opposite direction to the main flow with a constant suction. The two parallel plates are located at a distance y=-h and y=h away from the x axis .The plates are of infinite length in both x and z directions and the variable transverse magnetic fields is applied parallel to the y-axis as shown in figure 1 below.

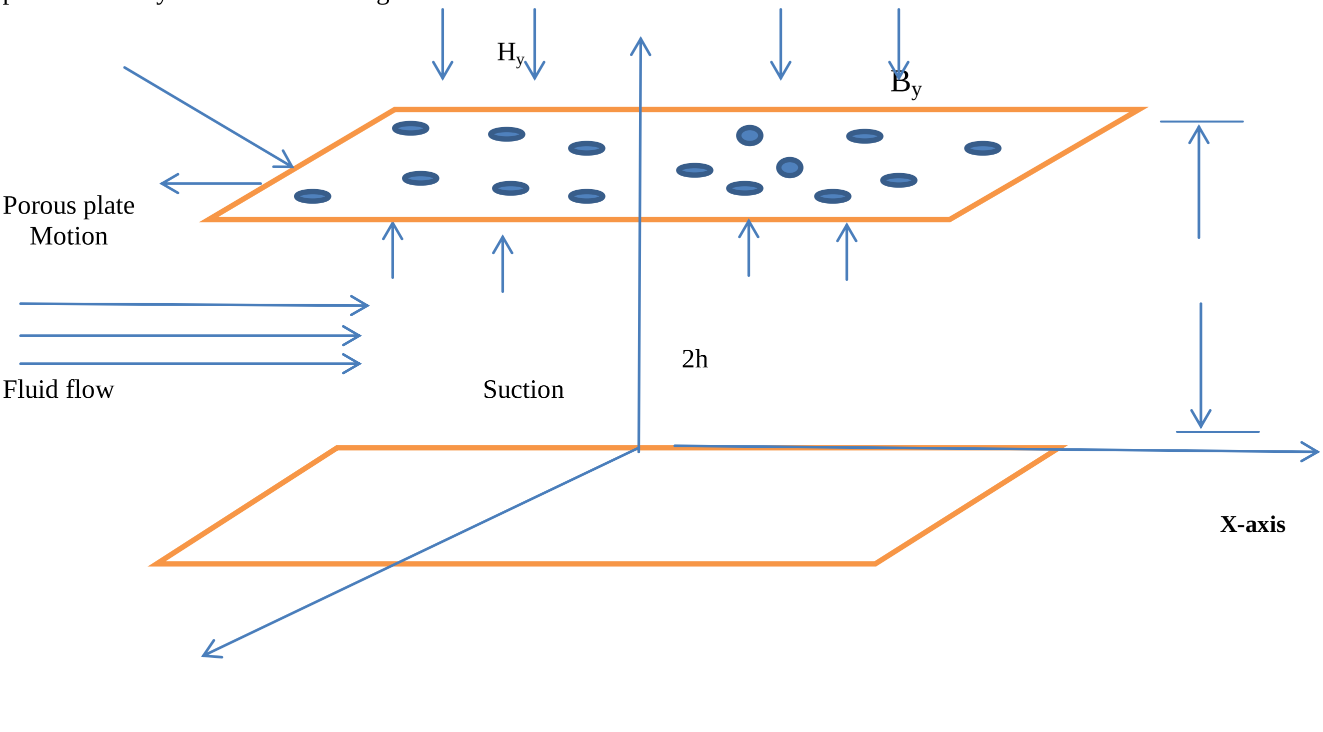

Fig 1. Flow configuration

The lower plate located at y=-h distance away from x-axis is held stationary. The top plate starts moving in the opposite direction of the main flow at a constant velocity $-U_0$. Initially (when time $t \leq 0$) it is assumed thatthe velocity of the fluid and the two parallel plates are assumed to be at rest. When $t > 0$, suction takes place through the upper porous plate with a uniform velocity v_0 and the upper plates starts moving at a constant velocity $-u_0$.

Governing equations

Equation of conservation of mass

The law of conservation of mass states that under normal conditions, mass can neither be created nor destroyed, i.e. given a steady flow process, the stored mass in a control volume does not change. For an unsteady fluid flow, from Kinyanjui et al. [8], the tensor form of the equation of continuity is;

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$$\frac{\partial \rho}{\partial t}+\frac{\partial(\rho . u_i)}{\partial x_i}=0 \quad (1)$$

Where i =1, 2, 3 represent the x, y and z directions respectively. The flow depends on y axis only and assumed to be incompressible and due to these assumption made, equation (1) reduces to;

$$\frac{\partial \rho}{\partial t}+\frac{\rho \partial v}{\partial y}=0 \quad (2)$$

Since the density of the fluid is a constant differentiating equation (2) reduces to;

$$\frac{\partial v}{\partial y}=0 \quad (3)$$

Integratingequation (3) reduces to v=constant. This constant is equal to the suction velocity Vo.

Equations of conservation of momentum

The equation of conservation of momentum states that the time rate of change of momentum of a body is equal to the external force applied to the body. These internal forces include surface forces (e.g. viscous force) and body forces (e.g. gravitational, centrifugal and magnetic force). The equation of conservation of momentum is given by;

$$\frac{\partial \mathbf{q}}{\partial t}+\mathbf{q}(\nabla . \mathbf{q})=-\frac{1}{\rho} \nabla P+\frac{\mu}{\rho} \nabla^2 \mathbf{q}+\mathbf{F}_e \quad (4)$$

The electromagnetic force Fe can be expressed as $Fe = \mathbf{J}\times\mathbf{B}$,where $\mathbf{J}=\sigma(\mathbf{q}\times\mathbf{B})$ and $\mathbf{B}=B_0\mathbf{j}$ solving $\mathbf{J}\times\mathbf{B}=-\sigma\mu_e^2H_0^2u\mathbf{i}$ .The velocity component**q=**u(y,t) and the pressure gradient depends on x only. Equation (4) can now be rewritten as;

$$\frac{\partial u}{\partial t}+v_0 \frac{\partial u}{\partial y}=-\frac{1}{\rho} \frac{dP}{dx}+\frac{\mu}{\rho} \frac{\partial^2 u}{\partial y^2}-\frac{1}{\rho} \sigma \mu_e{}^2 H_o{}^2 u \quad (5)$$

Equation of conservation of energy

Equation of conservation of energystates thatunder normal conditions energy cannot be created nor destroyed but can be transformed from one form to another. It is given by;

$$\rho C_p \frac{DT}{Dt}=k \nabla^2 T+\mu \phi+\frac{J^2}{\sigma} \quad (6)$$

From equation (6) above,

$$\mu\phi = 2\mu\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2\right]+\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2$$ and $$\frac{\mathbf{J}^2}{\sigma}=\sigma\mu_e^2H_y^2u^2$$

substituting these in (6) above yields;

$$\rho C_p\left[\left(\frac{\partial T}{\partial t}\right)+V_o \frac{\partial T}{\partial y}\right]=k \frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2+\sigma \mu_e^2 H_y^2 u^2 \quad (7)$$

The solution procedure

The following transformationshave been used to non-dimensionalize the equations. $X=LX^*, Y=LY^*, u=Uu^*, P=Pp^*, t=Tt^*, T^*=\frac{T-T_w}{T}$

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The governing equations (5) and (7), are solved using a finite difference scheme as proposed by Jain et al [7].

After performing the non –dimensionalization process and substituting the non-dimensional parameters, the two equations i.e. (5) and (7) reduced to

$$U_j^{k+1} = (U_j^k - \frac{dP}{dx} - \Delta t S_0 \frac{U_j^{k+1} - U_{j-1}^{k+1} + U_j^k}{2\Delta y} + \frac{\Delta t}{R_e}\frac{U_{j+1}^{k+1} - 2U_j^{k+1} + U_{j-1}^{k+1} + U_{j+1}^k - 2U_j^k + U_{j-1}^k}{2(\Delta y)^2} - \Delta t M U_j^k) / (1 - \frac{\Delta t S_0}{2\Delta y} + \frac{\Delta t}{2(\Delta y)^2 R_e}) \quad (8)$$

$$T_j^{k+1} = (T_j^k + \frac{\Delta t}{R_e P_r}\frac{T_{j+1}^{k+1} - 2T_j^{k+1} + T_{j-1}^{k+1} + T_{j+1}^k - 2T_j^k + T_{j-1}^k}{2(\Delta y)^2} + \Delta t S_0 \frac{T_j^{k+1} - T_{j-1}^{k+1} + T_j^k - T_{j-1}^k}{2\Delta y} + \frac{\Delta t E_c}{R_e}\left(\frac{U_j^{k+1} - U_{j-1}^{k+1} + U_j^k - U_{j-1}^k}{2\Delta y}\right)^2 + R\left(U_j^{k+1}\right)^2) / (1 + \frac{\Delta t S_0}{2\Delta y} + \frac{\Delta t}{R_e P_r (\Delta y)^2}) \quad (9)$$

The initial and boundary conditions are as given below;

$$t \le 0: \quad u = 0, \quad T = 0 \quad at \quad -h \le y \ge h$$
$$t > 0: \quad u = 0, \quad T = 1 \quad at \quad y = -h$$
$$t > 0: \quad u = -1 \quad T = 0 \quad at \quad y = h$$

III. RESULTS AND DISCUSSION

The following are the results obtained for equations (8) and (9) and are presented graphically as shown below;

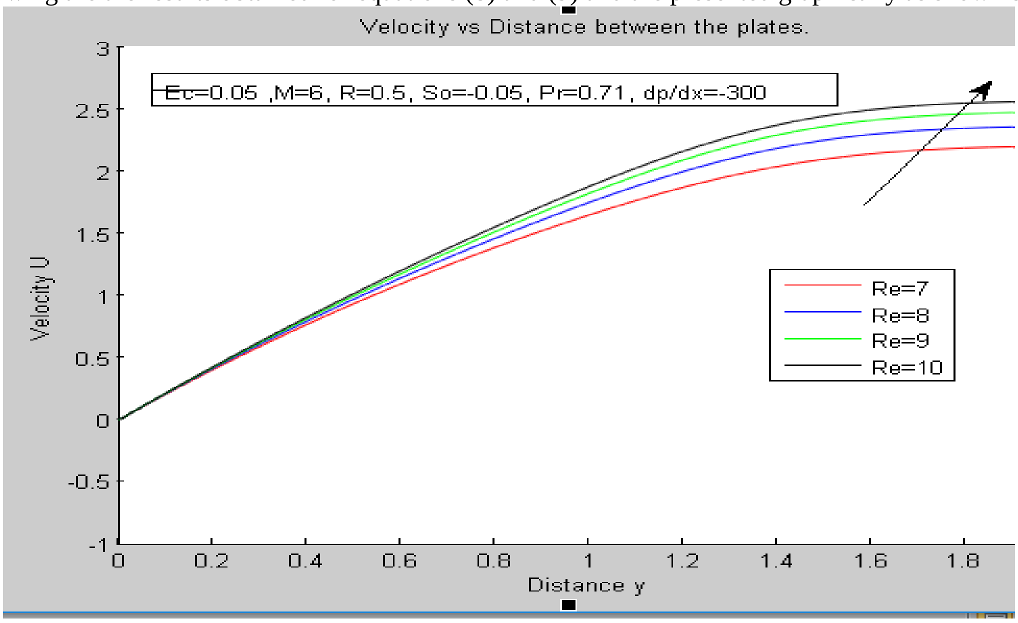

Fig 2: Velocity profiles for different values of hydrodynamic Reynolds number (Re).

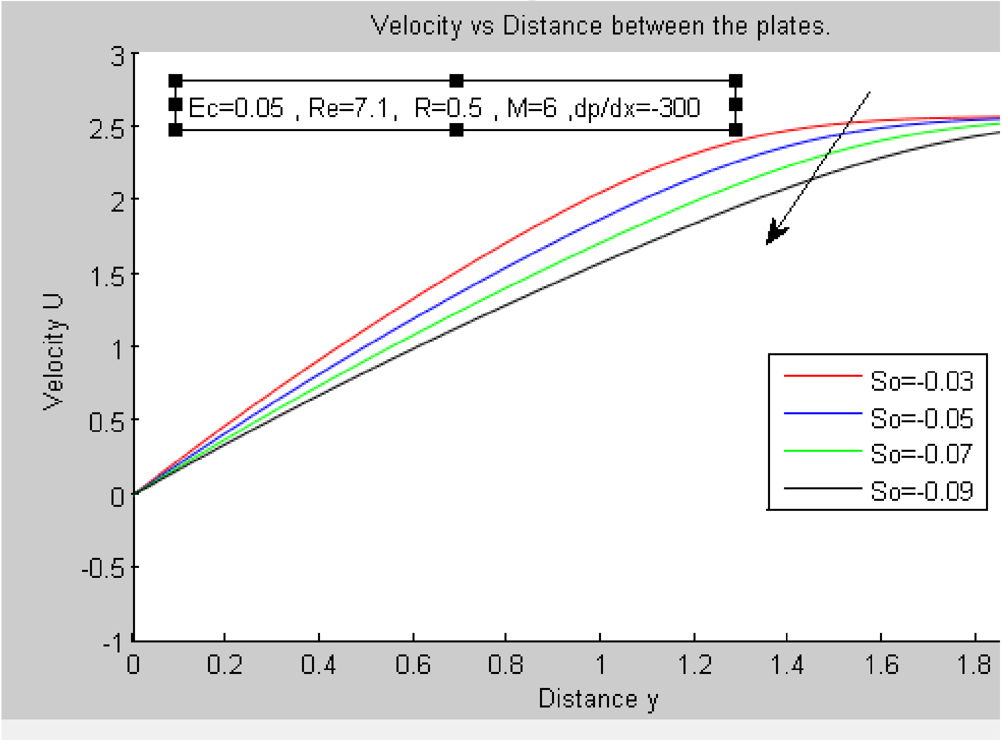

Fig 3: Velocity profiles for different values of So

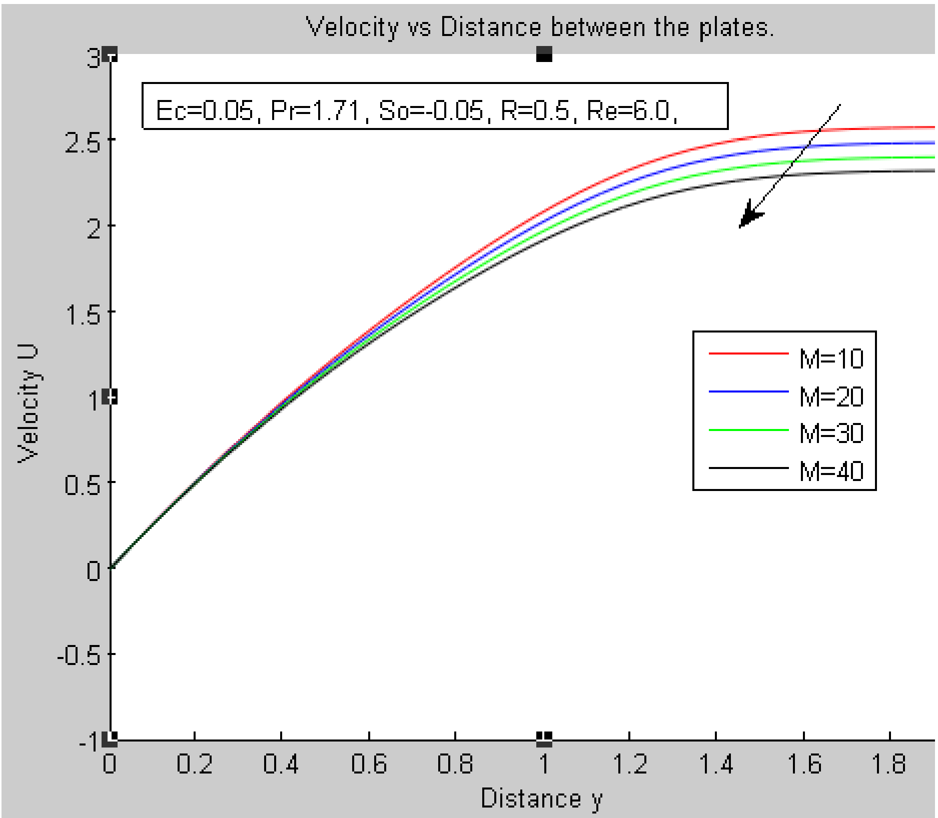

Fig 4. Velocity profiles for different values of Magnetic parameter (M)

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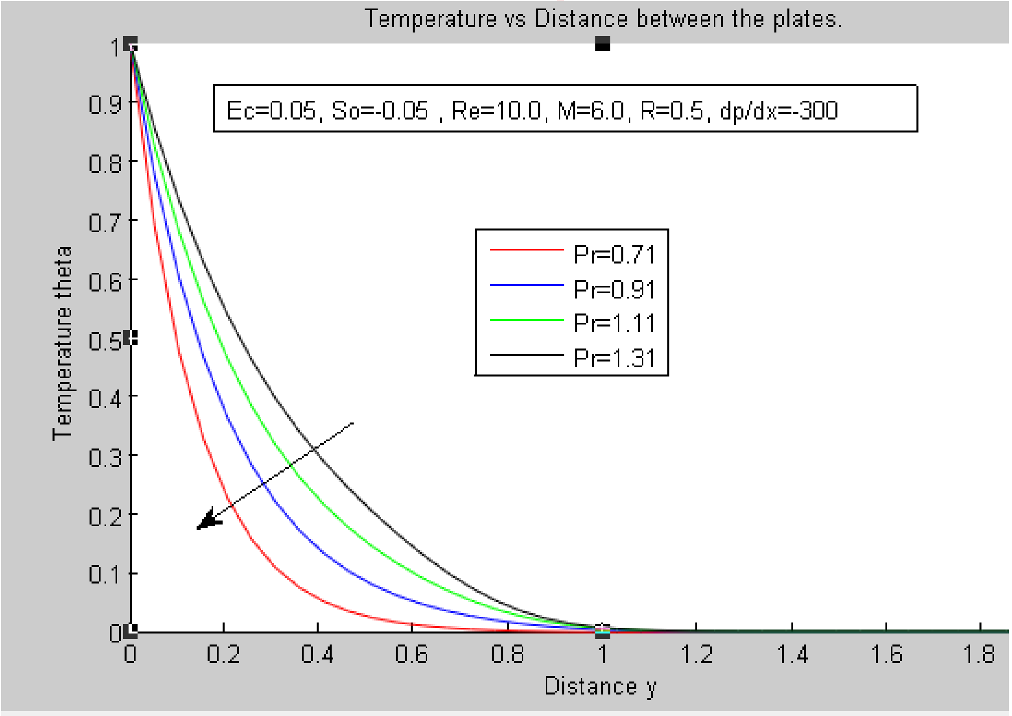

Fig 5: Temperature profiles for different values of prandtl (Pr)

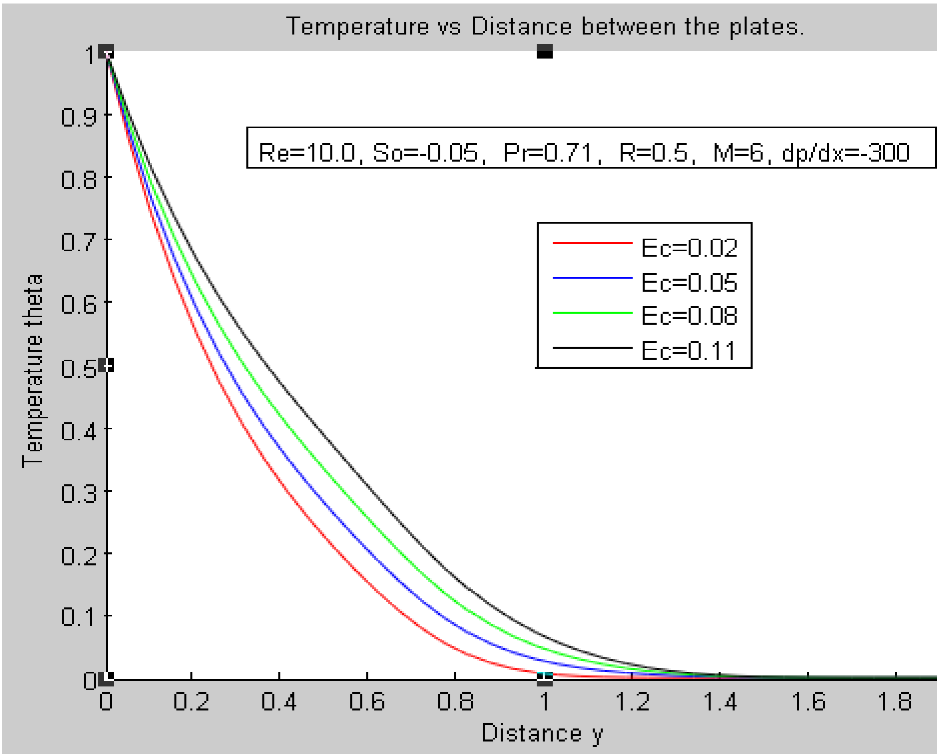

Fig 6 : Temperature profiles for different values of Eckert (Ec)

Figure1 above shows that holding other parameters fixed an increase in Reynolds number (Re) leads to an increase in the velocity profiles.An increase in the Reynolds number will lead to a decrease in the viscous forces which is the force that opposes the motion of the fluid which leads to an increase in velocity profiles of the fluid.

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It an increase in the Reynolds number leads to an increase in the flow velocity within the boundary layer due to the dominance of inertia forces over the viscous forces.

Figure2 shows that keeping other parameters fixed an increase in Suction parameter leads to a decrease in the velocity profile of the fluid as shown above indicating the usual fact that suction stabilizes the boundary layer growth**.** This means that the effect of increasing suction parameter retards the fluid flow which can be attributed to the convection of the fluid across the plates.

Figure 3 shows that keeping other parameters fixed an increase in magnetic parameter (M) leads to a decrease in the velocity profile of the fluid. The presence of a magnetic field in an electrically conducting fluid introduces a force called Lorentz force which acts against the flow if the magnetic field is applied in the normal direction as considered in the present problem. This type of resistive force tends to slow down the flow field.

Figure 4 above shows that keeping other parameters fixed an increase in Prandtl (Pr) causes a decrease in temperature profiles. This is due to the fact that a fluid with high Prandtl number has a relatively low thermal conductivity which results in the reduction of the thermal boundary layer thickness.

Figure 5 above shows that keeping other parameters fixed an increase in Eckert number (Ec) leads to an increase temperature profiles. This is because for an increase in Eckert number, it implies that the kinetic energy is large and hence the velocities are higher hence when this particles attained high velocity, the vibrations also increases and this leads to increased collision of the particles. This increased collision of particles brings about dissipation of heat in the boundary layer region hence an increase in temperature profiles.

IV. CONCLUSION

Magneto-hydrodynamic fluid flow between two parallel plates the top plate being porous with a constant suction and variable magnetic field lines are fixed relative to the top moving plate has been established. It is foundan increase in Suction parameter leads to a decrease in the velocity profile and an increase in the temperature profiles respectively. When suction is not considered in the flow the results agreed with those of Joseph et al (2014).The results obtained in these research can be applied by engineers in designing machines used in dyeing industry to produced quality dyed clothes.

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NOMENCLATURE

B	Magnetic flux density Wbm^{-2}
E	Electric field strength, Vm^{-1}
Ec	Eckert number
H	Magnetic field strength, Wbm^{-2}
J	Electric current density, Am^{-2}
M	Magnetic parameter
Pr	Prandtl number
ρ	Density of the fluid, Kg/m^3
Re	Reynold's number
R	Joules heating parameter
μ	Coefficient of Viscosity, $kgm^{-1}s^{-1}$
s_o	Dimensional Suction velocity, m/s
t	Dimensional time, s
u_o	Velocity of the moving plate, ms^{-1}

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