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Improved Exponential Ratio–Product Type Estimator for finite Population Mean

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Abstract- In this paper, an exponential ratio-product type estimator has been proposed for population under simple random sampling scheme. Expression for the bias and MSE of the proposed estimator have been derived up to first order approximation. The optimum MSE of the proposed estimator has been obtained. The efficiency of the proposed estimator has been compared theoretically and empirically with existing estimators. The empirical comparison shows that the proposed estimator is more efficient than others for both when the correlations between the study and auxiliary variables are positive and negative.

Keywords: **Auxiliary variable, Bias, Exponential estimator, Mean Squared Error, Precision.**

I. INTRODUCTION

It is well known fact that the use of auxiliary information at the estimation stage improves the precision of estimates of the population mean of characteristic under study. Classical ratio, product and linear regression estimators are good examples in this context. If the study variable Y is positively correlated with auxiliary variable X, the ratio method of estimation introduced by [1] is more applicable in practice while the product estimator introduced by [2] is more useful when the study variable Y is negatively correlated with auxiliary variable X. Later on, statisticians concentrated their attention to develop modified ratio and product type estimators. Such modified estimators are generally developed either using one or more unknown constants or introducing a convex linear combination of sample and population means of auxiliary characteristic with unknown weights. In both the cases, optimum choices of unknown parameters are made by minimizing the mean square error of modified estimators so that they become superior than the conventional one. In the sequence of suggesting modification over classical ratio and product estimators [4] - [9] considered a ratio –type estimators with the use of weighted mean of $\bar{X}$ and $\bar{x}$ in place of $\bar{x}$ in classical ratio and product estimators. [10]- [12] do some other remarkable works in this direction.[13]proposed new exponential ratio and product types estimators for estimating the mean of the finite population using information on single auxiliary variable.[14] have suggested improved estimators for estimating unknown population mean of study variable Y. In this paper, a exponential ratio-product type estimator has been suggested for estimating finite population mean of characteristic under study.

II. NOTATION

Let Y_i denotes the value of characteristic under study for the i^{th} unit in population of size N (i=1,2…N). and X_i, The value of auxiliary characteristic for the i^{th} unit in population. Then

$\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i$; The population mean of characteristic under study.

$\bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i$; The population mean of auxiliary characteristic.

$R = \frac{\bar{Y}}{\bar{X}}$;The ratio of population means

$S_Y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i - \bar{Y})^2$; The population mean square of characteristic under study.

$S_X^2 = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \bar{X})^2$; The population mean square of auxiliary characteristic.

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$C_Y = \frac{S_Y}{\bar{Y}}$; The coefficient of variation of characteristic under study.

$C_X = \frac{S_X}{\bar{X}}$; The coefficient of variation of auxiliary characteristic.

$\rho = \frac{S_{XY}}{S_X S_Y}$; The Correlation coefficient between the value of auxiliary variable and value of characteristic under study.

Let a sample of size n has been drawn by method of simple random sampling without replacement. Further let y_i and x_i denote the values of characteristic under study and auxiliary characteristic respectively which is included in the sample at i^{th} draw (i= 1,2, 3, . . . ,n). Now,

$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$; The sample mean of study characteristic.

$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$; The sample mean of auxiliary characteristic.

III. EXISTING ESTIMATORS

The sample mean estimator $\bar{y}$ is an unbiased estimator of population mean $\bar{Y}$,

The classical ratio and product estimators of population mean of the study variable Y are respectively, defined as

$$\bar{y}_r = \bar{y}\frac{\bar{X}}{\bar{x}} \text{ and } \bar{y}_p = \bar{y}\frac{\bar{x}}{\bar{X}} \quad (1)$$

[13] suggested exponential ratio-type and product –type estimators for population mean $\bar{Y}$, respectively, as

$$\bar{y}_{\text{Re}} = \bar{y}\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right) \text{ and } \bar{y}_{Pe} = \bar{y}\exp\left(\frac{\bar{x}-\bar{X}}{\bar{x}+\bar{X}}\right) \quad (2)$$

[15] suggested a class of ratio–type estimators for population mean $\bar{Y}$ as

$$\bar{y}^{g}_{RHA} = \bar{y}\left[2-\left(\frac{\bar{x}}{\bar{X}}\right)^{\alpha}\exp\left(\frac{\delta\left(\bar{x}-\bar{X}\right)}{\left(\bar{x}-\bar{X}\right)}\right)\right] \quad (3)$$

To the first degree of approximation, the variances/mean square errors of the estimators $\bar{y}, \bar{y}_r, \bar{y}_p, \bar{y}_{\text{Re}}, \bar{y}_{Pe}$ and $\bar{y}^{g}_{RHA}$ respectively, given as

$$V(\bar{y}) = \left(\frac{1}{n}-\frac{1}{N}\right)S_Y^2 \quad (4)$$

$$MSE(\bar{y}_r) = \left(\frac{1}{n}-\frac{1}{N}\right)(S_Y^2 + R^2S_X^2 - 2RS_{XY}) \quad (5)$$

$$MSE(\bar{y}_p) = \left(\frac{1}{n}-\frac{1}{N}\right)(S_Y^2 + R^2S_X^2 + 2RS_{XY}) \quad (6)$$

$$MSE(\bar{y}_{Re}) = \left(\frac{1}{n}-\frac{1}{N}\right)\left(S_Y^2 + \frac{1}{4}R^2S_X^2 - RS_{XY}\right) \quad (7)$$

$$MSE(\bar{y}_{Pe}) = \left(\frac{1}{n}-\frac{1}{N}\right)\left(S_Y^2 + \frac{1}{4}R^2S_X^2 + RS_{XY}\right) \quad (8)$$

$$MSE(\bar{y}^{g}_{RHA}) = \left(\frac{1}{n}-\frac{1}{N}\right)\left(S_Y^2 + \frac{(2\alpha+\delta)^2}{4}R^2S_X^2 - (2\alpha+\delta)RS_{XY}\right) \quad (9)$$

$$MSE(\bar{y}^{1}_{RHA}) = \left(\frac{1}{n}-\frac{1}{N}\right)\left(S_Y^2 + \frac{9}{4}R^2S_X^2 - 3RS_{XY}\right) \quad (10)$$

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IV. PROPOSED ESTIMATOR

Motivated by work done by [13] and others exponential ratio-product type estimator for population mean has been proposed as:

$$\bar{y}^{e}_{RP}=\bar{y}\left[k\exp\left(\frac{\sqrt{\bar{X}}-\sqrt{\bar{x}}}{\sqrt{\bar{X}}+\sqrt{\bar{x}}}\right)+(1-k)\exp\left(\frac{\sqrt{\bar{x}}-\sqrt{\bar{X}}}{\sqrt{\bar{x}}+\sqrt{\bar{X}}}\right)\right] \quad (11)$$

Where k is constant.

When k=1, the proposed estimator reduces to an estimator which is modified over exponential ratio type estimator suggested by [13].

When k=0, the proposed estimator reduces to an estimator which is modified over exponential product type estimator suggested by [13].

V. BIAS AND MEAN SQUARE ERROR (MSE) OF $\bar{y}^{e}_{RP}$

To obtain the bias and mean square error, let us suppose

$$\bar{y}=\bar{Y}(1+e_0) \text{ and } \bar{x}=X(1+e_1) \text{ thus,} \quad (12)$$

$$E(e_0)=E(e_0)=0,\ \mathrm{E}(e_0^2)=\left(\frac{1}{n}-\frac{1}{N}\right)\frac{S_Y^2}{\bar{Y}^2},\ \mathrm{E}(e_1^2)=\left(\frac{1}{n}-\frac{1}{N}\right)\frac{S_X^2}{\bar{X}^2},\ \mathrm{E}(e_0e_1)=\left(\frac{1}{n}-\frac{1}{N}\right)\frac{S_{yx}}{\bar{Y}\bar{X}} \quad (13)$$

From (11) and (12) we have,

$$\bar{y}^{e}_{RP}-\bar{Y}=\bar{Y}\left(e_0+\frac{e_1}{4}(1-2k)-\frac{e_1^2}{16}(1-2k)+\frac{e_0e_1}{4}(1-2k)\right) \quad (14)$$

The bias of the proposed estimator $\bar{y}^{e}_{RP}$ to terms of order n^{-1} can be obtained by taking the expectation of (14) and substituting results obtained in (13) as

$$B(\bar{y}^{e}_{RP})=\left(\frac{1}{n}-\frac{1}{N}\right)\frac{(1-2k)}{4}\left[\frac{S_{XY}}{\bar{X}}-\frac{RS^2_X}{4\bar{X}}\right] \quad (15)$$

Squaring both sides of (14), then taking expectation and using results in (13), we obtain the MSE of the estimator $\bar{y}^{e}_{RP}$ to terms of order n^{-1} as

$$MSE(\bar{y}^{e}_{RP})=\left(\frac{1}{n}-\frac{1}{N}\right)\left[S^2_Y+\left(k-\frac{1}{2}\right)^2\frac{R^2S^2_X}{4}-\left(k-\frac{1}{2}\right)RS_{XY}\right] \quad (16)$$

From (16) the optimum value of k is obtained as

$$k=\frac{2S_{XY}}{RS^2_X}+\frac{1}{2} \quad (17)$$

By substituting the optimum value of k in (16), we get the minimum MSE of $\bar{y}^{c}_{RP}$ as

$$MSE(\bar{y}^{e}_{RP})_{\min}=\left(\frac{1}{n}-\frac{1}{N}\right)S^2_Y(1-\rho^2_{XY}) \quad (18)$$

This is the MSE of the linear regression estimator for population mean. Hence for optimum value of k the proposed estimator $\bar{y}^{e}_{RP}$ is equally efficient as the linear regression estimator.

VI. EFFICIENCY COMPARISON

In this section efficiency of the proposed estimator is compared with that of existing estimators and conditions are obtained under which the proposed estimator is more efficient.

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Volume 4, Issue 3, May 2015

$$(i)\ \mathrm{var}\left(\bar{y}\right)-MSE\left(\bar{y}_{RP}^{e}\right)\geq 0$$

$$R\left(k-\frac{1}{2}\right)\left[S_{XY}-\left(k-\frac{1}{2}\right)\frac{RS_X^2}{4}\right]\geq 0$$

$$if\ \min\left(\frac{1}{2},\frac{1}{2}+\frac{4S_{XY}}{RS_X^2}\right)<k<\max\left(\frac{1}{2},\frac{1}{2}+\frac{4S_{XY}}{RS_X^2}\right) \quad (19)$$

For the range of k given in (19) the proposed estimator $\bar{y}_{RP}^{e}$ is better than $\bar{y}$.

$$(ii) MSE\left(\bar{y}_r\right)-MSE\left(\bar{y}_{RP}^{e}\right)\geq 0$$

$$R\left(k-\frac{5}{2}\right)\left[S_{XY}-\left(k+\frac{3}{2}\right)\frac{RS_X^2}{4}\right]\geq 0$$

$$if\ \min\left(\frac{5}{2},-\frac{3}{2}+\frac{4S_{XY}}{RS_X^2}\right)<k<\max\left(\frac{5}{2},-\frac{3}{2}+\frac{4S_{XY}}{RS_X^2}\right) \quad (20)$$

For the range of k given in (20) the proposed estimator $\bar{y}_{RP}^{e}$ is better than ratio estimator $\bar{y}_r$.

$$(iii) MSE\left(\bar{y}_p\right)-MSE\left(\bar{y}_{RP}^{e}\right)\geq 0$$

$$R\left(k+\frac{3}{2}\right)\left[S_{XY}-\left(k-\frac{5}{2}\right)\frac{RS_X^2}{4}\right]\geq 0$$

$$if\ \min\left(-\frac{3}{2},\frac{5}{2}+\frac{4S_{XY}}{RS_X^2}\right)<k<\max\left(-\frac{3}{2},\frac{5}{2}+\frac{4S_{XY}}{RS_X^2}\right) \quad (21)$$

For the interval of k given in (21) the proposed estimator $\bar{y}_{RP}^{e}$ is better than product estimator $\bar{y}_p$.

$$(iv) MSE\left(\bar{y}_{\mathrm{Re}}\right)-MSE\left(\bar{y}_{RP}^{e}\right)\geq 0$$

$$R\left(k-\frac{3}{2}\right)\left[S_{XY}-\left(k+\frac{1}{2}\right)\frac{RS_X^2}{4}\right]\geq 0$$

$$if\ \min\left(\frac{3}{2},-\frac{1}{2}+\frac{4S_{XY}}{RS_X^2}\right)<k<\max\left(\frac{3}{2},-\frac{1}{2}+\frac{4S_{XY}}{RS_X^2}\right) \quad (22)$$

For the interval of k given in (22) the proposed estimator $\bar{y}_{RP}^{e}$ is more efficient than the ratio –type estimator $\bar{y}_{\mathrm{Re}}$ suggested by [13].

$$(v) MSE\left(\bar{y}_{Pe}\right)-MSE\left(\bar{y}_{RP}^{e}\right)\geq 0$$

$$R\left(k+\frac{1}{2}\right)\left[S_{XY}-\left(k-\frac{3}{2}\right)\frac{RS_X^2}{4}\right]\geq 0$$

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Volume 4, Issue 3, May 2015

$$if\ \min\left(-\frac{1}{2},\frac{3}{2}+\frac{4S_{xy}}{RS_x^2}\right)<k<\max\left(-\frac{1}{2},\frac{3}{2}+\frac{4S_{xy}}{RS_x^2}\right) \qquad (23)$$

For the interval of k given in (23) the proposed estimator $\bar{y}_{RP}^{e}$ is more efficient than the product –type estimator $\bar{y}_{Pe}$ suggested by [13].

$$if\ \min\left(\frac{7}{2},-\frac{5}{2}+\frac{4S_{XY}}{RS_X^2}\right)<k<\max\left(\frac{7}{2},-\frac{5}{2}+\frac{4S_{XY}}{RS_X^2}\right) \qquad (24)$$

For the interval of k given in (24) the proposed estimator $\bar{y}_{RP}^{e}$ is more efficient than the ratio –type estimator $\bar{y}_{RHA}^{1}$ ($\alpha = 1$ and $\delta = 1$) suggested by [15].

VII. EMPIRICAL STUDY

The efficiency of the proposed estimator has been compared over sample mean estimator, classical ratio and product estimator, exponential ratio type estimator suggested by [13] and class of ratio-type estimator suggested by [15] using three population data sets. The Descriptions of the populations are given in Table I.

Table I: Description and parameters of the populations

Parameters	Population I	Population II	Population III
	Source: Cochran 1977, page 186 Y: Total no. of Members X: No. of Children	Source: Cochran 1977, page 34 Y: Food Cost X: Family Size	Source: US Environmental Protection Agency 1991 Y: Average miles per gallon X:Engine horsepower
N	21	33	80
n	5	5	5
S_x^2	1.537415	111.880682	3223.011
S_{xy}	1.5646	27.0207386	-438.445
S_y^2	1.678005	102.632727	92.96819
ρ_{xy}	0.97413	0.25216031	-0.801
R	2.222	0.37894737	0.283655982
a	1.091805503	0.784525	-1.04257815
$\bar{X}$	1.714285714	72.5454545	118.1625
$\bar{Y}$	3.80952381	27.4909091	33.5175
k	0.457963115	0.63732832	-0.47959745
$k_{h,a}$	2.183611005	1.56905	-2.08515629

Table II: Effective ranges and optimum value of k of $\bar{y}_{RP}^{e}$

Population	Ranges of k for which $\bar{y}_{RP}^{e}$ is better than						
	$\bar{y}$	$\bar{y}_r$	$\bar{y}_p$	$\bar{y}_{Re}$	$\bar{y}_{Pe}$	$\bar{y}_{RHA}^{1}$	k_{opt}
I	(0.50, 2.33)	(0.33, 2.50)	(-1.50,4.33)	(1.33, 1.50)	(-0.5, 3.33)	(-0.67,3.5)	1.42
II	(0.50, 3.05)	(1.05, 2.50)	(-1.50,5.05)	(1.50, 2.05)	(-0.5, 1.05)	(0.05, 3.5)	1.78
III	(0.50, 2.42)	(0.42, 2.50)	(-1.50,4.42)	(1.42, 1.50)	(-0.5, 3.42)	(-0.58,3.5)	1.46

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Table III: Percentage relative efficiencies of different estimators with respect to $\bar{y}$

Estimator	$\bar{y}$	$\bar{y}_r$	$\bar{y}_p$	$\bar{y}_{\mathrm{Re}}$	$\bar{y}_{Pe}$	$\bar{y}^1_{RHA}$	$\bar{y}^e_{RP}$
Population I	100	72.47	10.34	1692.78	23.79	20.15	1958.01
Population II	100	104.49	73.74	106.45	87.80	94.97	106.79
Population III	100	15.47	89.77	32.95	278.08	8.86	279.02

VIII. CONCLUSION

The conditions, under which the proposed estimator is more efficient than the considered estimators are given in section 6 but table II provides the ranges of k for which $\bar{y}^e_{RP}$ is better than others . The study of Table 3 reveals that the proposed estimator $\bar{y}^c_{RP}$ is more efficient than the other estimators as $\bar{y}$, $\bar{y}_r$, $\bar{y}_p$, $\bar{y}_{\mathrm{Re}}$, $\bar{y}_{Pe}$ and $\bar{y}^1_{RHA}$ under optimal condition for all the three set of Populations.

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