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A Study on Root Mean Square Labelings in Graphs

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Abstract*—In this paper, the new concept Root Mean Square Labeling has been introduced. A function f is called a root mean square labeling of a graph G with q edge, if f is a injective function from the vertices of G to the set {0, 1, 2, 3, . . . ,2p } such that when each edge uv is assigned the label $f(uv) = \sqrt{\frac{[f(u)]^2 + [f(v)]^2}{2}}$ then the resulting edge labels are distinct numbers. In this paper, the graph such as path, star, flower pot, Bistar, split which satisfy Root Mean Square labeling are being illustrated.

***Keywords*—Labelings in graphs, mean labeling, square sum labeling, root mean square graph.**

I. INTRODUCTION

The graphs considered in this paper are finite and undirected. The symbols V(G) and E(G) denote the vertex set and edge set of a graph G. The cardinality of the vertex set is called the order of G denoted by P. The cardinality of the edge set is called the size of G denoted by q. Hence (p,q)is called a graph G.

A graph labeling is an assignment of integers to the vertices or edges. Some basic definitions and notations are taken from Bondy and Murthy [2],[5]. Different types of graph labelings in directed graphs also been applied in various fields[6]. A dynamic survey on graph labeling is regularly updated by Gallian [4] and it is published by Electronic Journal of Combinatory. In this paper, we introduced the new concept Root Mean Square labeling.

Definition 1.1

Let G=(V(G),E(G)) be a graph G. A graph G is said to be root mean square labeling if there exist a injective mapping from the vertices of G to set {0, 1, 2, 3, ...,2p } such that when each edge uv is assigned the label $f(uv) = \sqrt{\frac{[f(u)]^2 + [f(v)]^2}{2}}$, then the resulting edge labels are distinct numbers.

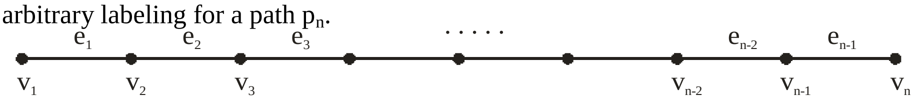
II. MAIN RESULTS

Theorem 2.1

For every positive integer n,the path $p_n (n \geq 2)$ is a Root Mean Square graph.

Proof :

Let G be a graph of path P_n.
Let $\{v_1, v_2, v_3, \ldots\ldots\ldots v_n\}$ be the vertices of path p_n and $\{e_1, e_2, e_3, \ldots\ldots\ldots e_{n-1}\}$ be the edges of path p_n.
The path p_n consists of n vertices and n-1 edges.
$E(G)=\{v_i v_{i+1}; 1 \leq i \leq n-1\}$
Let us set an arbitrary labeling for a path p_n.



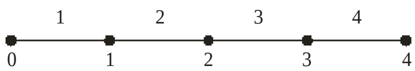
Define $f : V(G) \to \{0, 1, 2, 3, \ldots \ldots 2p\}$ as follows

$f(v_i)=i-1 \; ; 1 \leq i \leq n$

$f(e_i)=i \quad ; 1 \leq i \leq n-1$

Hence the path p_n is a Root Mean Square graph.

Example : 2.1



Path P_5 is RMS

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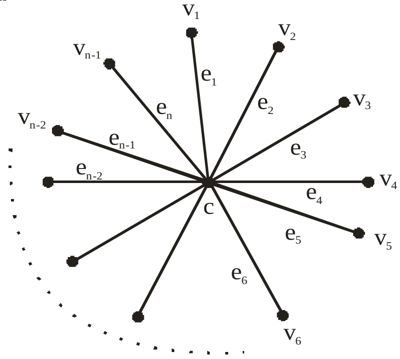
Theorem :2.2

For every positive integer n, the star graph $k_{1,n}(n \geq 1)$ is a Root Mean Square graphs.

Proof:

Let G be a graph of $k_{1,n}$.
Let $v_1, v_2, \dots, v_n$ be the vertices of $K_{1,n}$ and c be the apex vertex.
Let $\{e_1, e_2, e_3, \dots\dots, e_n\}$ be the edges of $k_{1,n}$.
$E(G)=\{cv_i/1\leq i\leq n\}$
Let us set an arbitrary labeling for $k_{1,n}$.



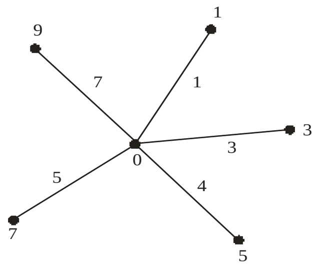
Let $k_{1,n}$ consists of n+1 vertices and n edges.
Define $f : V(G) \to \{0, 1, 2, 3, \dots 2p\}$ as follows

$f(c) = n-5$

$f(v_i) = 2i - 1 ; 1 \leq i \leq n$

The edges receive weight as a distinct value.
Hence the star graph is a Root Mean Square graph.

Example :2.2

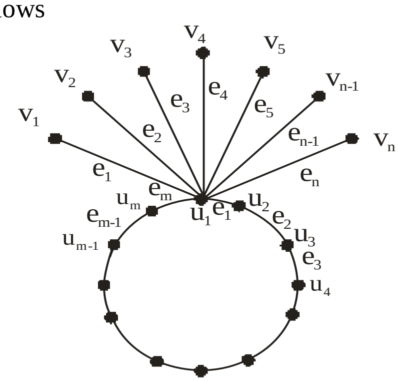


Theorem :2.3

The flower pot graph is a root mean square graph.

Proof :

Let G be a graph of flower pot.
Let $\{v_1, v_2, v_3, \dots\dots, v_n, u_1, u_2, u_3, \dots\dots u_m\}$ be the vertices of the flower pot.
Let us set an arbitrary labeling as follows



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Define f:V(G) —>{0,1,2,3,4,…….2p} as follows

$f(v_i)=2i-1$;$1\le i\le n$

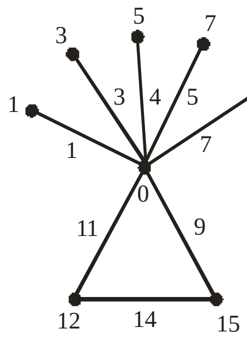
$f(u_1)=0$

$f(u_i)=5n-3i-4$; $2\le i\le m$

The edges receive weight as a distinct numbers.

Hence the flower pot graph is a Root Mean Square graph.

Example : 2.3



Theorem : 2.4

The Bistar graph < $K_{1,m}$ @ $K_{1,n}$ > is a Root Mean Square graph.

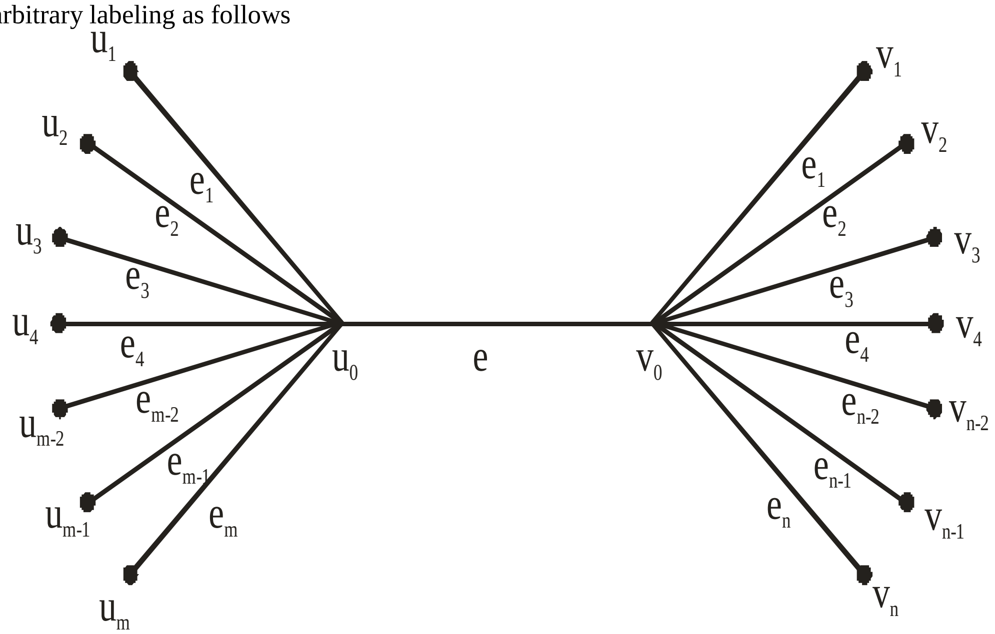
Proof :

Let G be a graph of $k_{1,m}@k_{1,n}$.

Let $v_1, v_2, \ldots, v_m$ be the vertices of $K_{1,m}$ and $v_1, v_2, \ldots, v_n$ be the vertices of $K_{1,n}$.

Let $e_1, e_2, \ldots, e_{m-1}, e'_1, e'_2, \ldots, e'_{n-1}$ be the edges of $K_{1,m}$ and $K_{1,n}$.

Let us set an arbitrary labeling as follows



Define f : V(G) → {0, 1, 2, 3, ...2p} as follows

$f(v_0) = 0$

$f(v_i) = 2i-1$; $1\le i\le n$

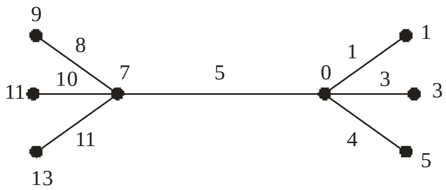
$f(u_0) = n+4$

$f(u_i) = 2n+2i+1$; $1\le i\le m$

The edges receive weight as a distinct values.

∴ The Bistar graph is a Root Mean Square graph.

Example: 2.4



Theorem : 2.5

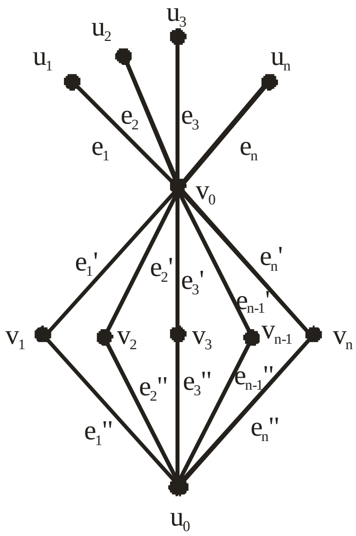
The Spliting graph spl($_{k1,n}$)is a Root Mean Square graph.

Proof :

Let G(V,E) be the splitting graph spl($k_{1,n}$).

Let $v_1,v_2,v_3,\ldots,v_n$ be the pendant vertices and v be the apex vertex of $K_{1,n}$ and $u,u_1,u_2,u_3,\ldots,u_n$ are added vertices corresponding to $v,v_1,v_2,v_3,\ldots,v_n$ to obtain spl($_{K1,n}$).

Let us set an arbitrary labeling as follows:



Define $f : V(G) \to \{0, 1, 2, 3, \ldots 2p\}$ as follows

$f(v) = 0$

$f(u_i) = 2i - 1$ for $1 \le i \le n$

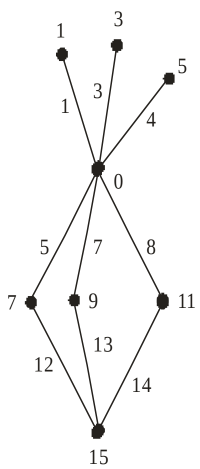
$f(v_i) = 4n + 1$

$f(u) = 2n+3$

The edges receive weight as distinct values.

$\therefore$ Split graph is a Root Mean Square graph.

Example : 2.5



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III. CONCLUSION

In this paper we introduce the definition for root mean square labeling. In this paper we investigated some families of graphs such as path, star, flower pot, Bistar; split graphs satisfy the condition of root mean square labeling. Our future research work is to apply graph labeling in various fields and to introduce different types of labeling for various families of graphs.

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