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Integer Programming Model for Fuzzy Time Cost and Quality Trade off Problem

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Abstract: Project Management is one of the most important fields in business and industry. One important aspect of the project management is to acquire the information related to an optimum balance between the project's objectives. The three interrelated and conflicting objectives of any project are time, cost and quality. These objectives are dependent on the related features of the activities of that project. The purpose of this paper is to develop mathematical model of cost, time and quality tradeoffs in conditions that time parameters of the project activities are estimated by triangular fuzzy number. The model is formulated in the form of Fuzzy Integer Linear Programming. This paper helps practicing project engineers to have realistic expectations of the method.

Key words: **Project Management - Fuzzy Time Cost and Quality Trade Off problem – Integer Programming model.**

I. INTRODUCTION

Project management is one of the most important fields in business and industry. Every task in an organization can be taken into account as a project. In scheduling a project, it is generally considered to expedite the duration of some activities through expending extra budget in order to compress the project completion time. This process can be considered under either some fixed available budget or a threshold of project completion time. This problem is known as Time Cost Trade off Problem (TCTP) in project management literature. The main objective of the TCTP is to determine the optimal amount of duration and cost assigned to the activities so that the overall cost is minimized. Hence this problem leads to a balance between the project completion time and the project total cost.

There are three main points that are the most important factors for a successful project: (1) a project must meet the customer requirements, (2) it has to be within project and (3) it has to be on time. These three criteria are often referred to as The Iron Triangle.

One important aspect of the project management is to acquire the information related to an optimum balance between the project's objectives. According to the Iron Triangle, time, cost and quality are important objectives of a project. Heretofore, extensive researchers have been conducted to develop cost-time trade off problems. Nowadays, the quality of a project is also added to the project time and cost. The aim of these problems (TCQTP) is to select a set of activities for crashing as well as an appropriate execution method for each activity such that the project cost and time is minimized while the project quality is maximized.

A. TIME COST AND QUALITY TRADE OFF PROBLEMS IN DIFFERENT NATURE

Babu and Suresh [5] presented the first paper considering the influence on project quality by project scheduling and developed three inter-related linear programming models to study the trade off among time, cost and quality in a deterministic CPM network. Each of the three proposed models, one of the three entries (ie) time, cost and quality by assigning desired levels to the other two entries. The linearity and deterministic assumptions led to simple solvable mathematical program which enabled the authors to investigate the idea of Time Cost and Quality Trade Off Problem (TCQTP). In Khang and Myint [14], the model proposed by Babu and Suresh [5] was applied to an actual cement factory construction project. The purpose was to evaluate the applicability of the method by highlighting the managerial insights gained, as well as pointing out key problems and difficulties faced. The problems investigated by Babu and Suresh [5], Khang and Myint [14] can be categorized in the class of continuous time, cost and quality trade off problem. Thereafter, many researchers have developed mathematical programming model for these kinds of problems.

In El-Rayes et al. [10] for the first time, the discrete Time Cost Quality Trade Off problem was investigated. They used a real world example and suggested new functions to enable the consideration of construction quality in the time, cost and quality optimization problem in construction industry. To estimate the project quality, they introduced some quality indicators, and used the weighed sum of the quality levels sassed by indicators as the project quality. In another work, a discrete model of time, cost and quality trade off was proposed by Tareghian

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[25] using three integer programming in which activities are performed in one of several available alternatives. The purpose of suggested model is to complete the project at a given deadline such that total accost is minimized and overall quality is maximized. Similar to Babu and Suresh [5] all the three entries are assumed to be deterministic parameters and consequently, the CPM framework were too applied to this research.

The authors employed a discrete multimode model using activity based resource utilization options to transform the time cost trade off to a time, cost and quality trade off model and solved it by genetic algorithm. Other works by Rahimi and Iranmanesh et al. [21], Johnson-Pollack and Liberatore [20] have been presented to optimize the discrete multi-mode model to TCQTP.

At this stage, meta heuristic methods were still regarded as an appropriate tool for solving these kinds of problems, for example in some works of Afsar et al. [2], Huang et al. [12], Yang [27], Tareghian and Taheri [26], Liberatore and Johnson-Pollack [20] and Lakshminayaranan et al. [15]

One of the most important issues in modeling this kind of problems is data uncertainty. This uncertainty is caused as the available information is often approximate or partial. The project managers require approximating the values of time, cost and quality of the activities and all these approximations deal with uncertainty. Many of the models, however, applied the crisp data as approximation of the parameters. These models neglect the inexact nature of such approximation. Some researchers include Cohen et al. [9], Abbasnia et al. [1], Ravi Shankar [19] and Zhang and Xing [28] considered the uncertainity problem of CTQT based on stochastic or fuzzy data. Meanwhile, Mokhtari et al. [17] developed a hybrid approach for the stochastic time-cost trade off problem in PERT networks. Amiri and Golozari [3] applied fuzzy multi arrtibute decision making techniques in project planning. Salmasnia et al. [4] regarded quality as an additional aspect in the traditional time cost trade off while the parameters are considered as stochastic. Seyed Hossein [24] developed a combination of fuzzy goal programming model and grey linear programming to solve the mathematical model.
Different heuristic approaches were presented by Siemens and Moselhi and Deb [18]. Meta heuristics such as genetic algorithm were exploited in solving this problem by Feng et al. [7] and Chau et al. [8]. Multi criteria techniques based on simulation model, stochastic dominance rules and a multi criteria aggregation procedure.
The research works in the field of time, cost and quality trade off, the subject of this research, can be categorized into two distinct categories:

- Continuous trade-off problems: in this category, the relation among time, cost and quality has been defined as continuous function. In these works, one of the three variables (usually time) is considered to vary independently and the two others are defined as functions of that variable. Research works of Babu and Suresh [5], Khang and Myint [14] are some examples.
- Discrete trade-off problems: in this class, the relation among time, cost and quality has been considered discrete. In other words, for each project activity, different modes of execution are defined, and for each mode, distinct time, cost, and quality are associated. So to trade-off among the objectives, one execution mode is selected for each activity. Works El-Rayes et al. [10], Tareghian and Taheri and Iranmanesh et al. [25] are a few to cite. When a project manager faces a project in which there are alternatives for executing activities, and each alternative have distinct time, cost and quality, discrete models are applicable. Project manager can select among these alternatives to optimize the trio of project objectives.

The purpose of this paper is to develop mathematical model with three different objectives representing cost, time and quality tradeoffs under the conditions that time and cost parameters of the project activities are estimated by triangular fuzzy number. The model is formulated in the form of Fuzzy Integer Linear Programming. The proposed model minimizes either the total cost or total fuzzy time or maximizes the total quality of the project and this model concerns both direct cost and indirect cost. This paper helps practicing project engineers to have realistic expectations of the method.

II. PRELIMINARIES

In this section, some basic definitions of fuzzy theory defined by Kaufmann, Gupta and Zimmermann [29], are presented.

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Definition 2.1

The characteristic function μ_A of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X. This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the universal set X fall within a specified range i.e. $\mu_{\tilde{A}} : X \to [0,1]$. The assigned values indicate the membership grade of the element in the set A. The function $\mu_{\tilde{A}}$ is called the membership function and the set $\tilde{A} = \{(A, \mu_A(x) : x \in X\}$ defined by $\mu_{\tilde{A}}(x)$ for each $x \in X$ is called a fuzzy set.

Definition 2.2

A fuzzy set $\tilde{A}$ defined on the set of real numbers R is said to be a fuzzy number if its membership function has the following characteristics:

1. $\mu_{\tilde{A}}(x) : R \to [0,1]$ is continuous.
2. $\mu_{\tilde{A}}(x) = 0$ for all $(-\infty, a] \cup [c, \infty)$.
3. $\mu_{\tilde{A}}(x)$ is strictly increasing on [a, b] and strictly decreasing on [b, c].
4. $\mu_{\tilde{A}}(x) = 1$ for all $x \in b$ where $a \le b \le c$.

Definition 2.3

Triangular fuzzy number is a fuzzy number represented with three points as follows:
$A = (a_1, a_2, a_3)$ this representation is interpreted as membership functions.

$$\mu_A(x) = \begin{cases} 0 & if \ \ x < a_1 \ and \ x > a_3 \\ \dfrac{x - a_1}{a_2 - a_1} & if \ \ a_1 \le x \le a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & if \ \ a_2 \le x \le a_3 \end{cases}$$

2.4 Fully Fuzzy Linear Programming problem (FFLP)

Linear programming is one of the most frequently applied operations research technique. We assume that all parameters and variables are real numbers. But in real world environment, do not have precise information. So, the fuzzy numbers and fuzzy variables should be used Linear Programming problem. The standard form FFLP problems with m fuzzy equality constraints and n fuzzy variables as follows:

Maximize (or Minimize) ($\tilde{C}^T \otimes \tilde{X}$)

Subject to $\tilde{A} \otimes \tilde{X} = \tilde{b}$

$\tilde{X}$ is a non-negative fuzzy number. Where $\tilde{C}^T = [\tilde{c}_j]_{1\times n}$, $\tilde{X} = [\tilde{x}_j]_{n\times 1}$, $\tilde{A} = [\tilde{a}_{ij}]_{m\times n}$, $\tilde{b} = [\tilde{b}_i]_{m\times 1}$ and $\tilde{c}_j, \tilde{x}_j, \tilde{a}_{ij}, \tilde{b}_i \in F(R)$ where i = 1, 2. . . m and j = 1, 2, . . . , n.

2.5 Integer Programming Model

Integer Programming models are optimization models in which some or all of the variables must be integer. The main difference between Linear Programming and Integer Programming models is that Linear Programming model allows fractional values for the decision variables, whereas Integer Programming models allow only integer values for integer constrained decision variables. The clever use of binary variables allows us to solve many interesting and difficult problems and our ability to model complex problems increases tremendously when we use binary variables. A 0-1 variable is a decision variable that must be equal to 0 or 1, (ie) an activity that either is or is not undertaken.

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If the variable is 1, the activity is undertaken; if it is equal to 0, the activity is not undertaken. A 0-1 variable is often called a binary variable. Our objective in this paper is to develop an Mixed Integer Programming model which enables us to minimize the cost spent in building construction meeting the desired deadline of the project.

III. MODEL FORMULATION

To model the Fuzzy Time cost trade off problem, we start with standard assumptions for modeling projects: that the project has no cycles, that the start activity is the only activity that is not an immediate successor of any activity, and that the finish activity is the only activity has no successors. Define the following parameters and variables

N_i - Number of options for activity i , d_{ij} - duration of activity i ,

C_{ij} - Cost of option j of activity i T_i - starting time of activity i

T - deadline of the project y_{ij} - 0-1 variable, for $i = 1, 2, \ldots n$ and $j = 1, 2, \ldots N_i$

n - Number of actual activities Q - average quality level

q_{ij} - Quality of activity i Q_{MIN} - assigned minimum average quality

We will now model of this problem, where we set upper limit on total project time and total and lower limit on average quality.

Precedence relationship constraint

The completion time of project could be constrained by one of the two methods. The first approach is to allow for a precedence constraint for each immediate preceding relationship in the project network. This approach was used in almost all existing optimization techniques. The second is to allow for one constraint for each path from the first activity to the last one in the project network. In the present model, the first approach will be adopted. The logical relationship between any two consecutive activities i and its immediate predecessor j, is expressed mathematically as $\tilde{T}_1 = 0$, $\tilde{T}_j - \tilde{T}_i - \sum_{j=1}^{N_i} \tilde{d}_{ij} y_{ij} \geq 0$ $\quad (a)$

- ***Project completion constraint***

Project completion is controlled by the latest finish time of ending activities. If the number of ending activities is denoted by n, the project completion constraint is given by the equation, in which T is the desired deadline of the project. $\tilde{T}_n \leq \tilde{T}$ $\quad (b)$

- ***Cost constraint***

The set of constraints mentioned in this section is used to calculate the cost of each activity based on the values of fuzzy duration and quality selected for that activity where the fuzzy duration is represented in the form of triangular fuzzy number. Instead of defining this set of constraint,

$$\tilde{C}_i = \sum_{j=1}^{N_i} \tilde{d}_{ij} y_{ij} \forall i, \quad and \quad \tilde{C} = \sum_{i,j} \tilde{C}_{ij} + \tilde{I} * \tilde{T}_n \qquad (c)$$

- ***Selection constraint***

Only one option should be selected from the number of options given for every task. Mathematically, it can be expressed as follows:

$$\sum_{j=1}^{N_i} y_{ij} = 1 \forall i \qquad (d)$$

where N_i is the number of options for activity i, and y_{ij} are binary variables.

- ***Quality constraint***

The following set of constraints (e) is used to calculate the quality of each activity. The constraints (f) and (g) take care of the maintenance of minimum level of quality Q_{MIN} that is prescribed and algorithm always aims to maximize the quality, the best possible.

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$$Q_i = \sum_{j=1}^{N_i} q_{ij} y_{ij} \forall i, \qquad (e)$$

$$Q = \frac{\sum_{i=1}^{n} Q_i}{n}, \quad Q_{MIN} \le Q \le 100 \qquad (f),(g)$$

where Q_{MIN} denote the assigned minimum level of quality.
The Complete Fuzzy Integer Mathematical Model for Fuzzy Time Cost and Quality Trade Off Problem is given below:

$$\begin{aligned}
&Min \quad \tilde{C} \\
&\tilde{T}_j \ge \tilde{T}_i + \tilde{D}_{ij}, \quad \tilde{T}_i = 0, \quad \tilde{T}_n \le \tilde{T} \\
&\tilde{C}_i = \sum_{j=1}^{N_i} \tilde{c}_{ij} y_{ij} \forall i, \quad \tilde{C} = \sum_{i,j} \tilde{C}_{ij} + (\tilde{I} * \tilde{T}) \\
&Q_i = \sum_{j=1}^{N_i} q_{ij} y_{ij} \forall i, \quad Q = \frac{\sum_{i=1}^{n} Q_i}{n}, \quad Q_{MIN} \le Q \le 100 \qquad (2) \\
&y_{ij} = 0(OR)1 \quad \forall i \,\&\, j, \quad \sum_{j=1}^{N_i} y_{ij} = 1 \forall i = 1...n
\end{aligned}$$

IV. ALGORITHM TO SOLVE FULLY FUZZY TIME COST TRADE OFF PROBLEM

The following is the algorithm to find an optimal solution of Fuzzy Time Cost Trade Off problem using ILP technique. The steps of proposed algorithm are given as below:
Step 1: Set up the mathematical formulation of the Fuzzy Time Cost Trade Off problem as given in (2).
Step 2: Problem (2) can be written in the following way:

$$\begin{aligned}
&Min \quad (C_1, C_2, C_3) \\
&Subject \quad to \\
&(T_{11}, T_{12}, T_{13}) = 0 \\
&(T_{j1}, T_{j2} T_{j3}) - (T_{i1}, T_{i2}, T_{i3}) - \sum_{j=1}^{N_i} (d_{ij1}, d_{ij2}, d_{ij3}) y_{ij} \ge 0 \\
&(T_{n1}, T_{n2}, T_{n3}) \le (T_1, T_2, T_3) \\
&(C_{i1}, C_{i2}, C_{i3}) = \sum_{j=1}^{N_i} (C_{ij1}, C_{ij2}, C_{ij3}) y_{ij} \quad \forall i = 1...n \qquad (3) \\
&(C_1, C_2, C_3) = \sum_{i=1}^{n} (C_{i1}, C_{i2}, C_{i3}) + (I_1, I_2, I_3) * (T_{n1}, T_{n2}, T_{n3}) \\
&Q_i = \sum_{j=1}^{N_i} q_{ij} y_{ij} \forall i, \quad Q = \frac{\sum_{i=1}^{n} Q_i}{n}, \quad Q_{MIN} \le Q \le 100 \\
&\sum_{j=1}^{N_i} y_{ij} = 1 \quad \forall (i,k) \in P, \quad y_{ij} = 0(or)1 \\
&T_{i3} - T_{i2} \ge 0, \quad T_{i2} - T_{i1} \ge 0 \quad \forall i
\end{aligned}$$

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Step 3: The above mixed integer linear programming model (3) may be written as follows:

$$
\begin{aligned}
& Min \quad C_1, \quad Min\ C_2, \quad Min\ C_3 \\
& Subject \quad to \\
& T_{11} = 0, \quad T_{12} = 0, \quad T_{13} = 0 \\
& T_{k1} - T_{i1} - \sum_{j=1}^{N_i} d_{ij1} y_{ij} \geq 0, \quad T_{k2} - T_{i2} - \sum_{j=1}^{N_i} d_{ij2} y_{ij} \geq 0, \\
& T_{k3} - T_{i3} - \sum_{j=1}^{N_i} d_{ij3} y_{ij} \geq 0 \ \forall (i,k) \in P \\
& T_{n1} \leq T_1, \quad T_{n2} \leq T_2, \quad T_{n3} \leq T_3 \\
& C_{i1} = \sum_{j=1}^{N_i} C_{ij1} y_{ij}, \quad C_{i1} = \sum_{j=1}^{N_i} C_{ij2} y_{ij}, \quad C_{i1} = \sum_{j=1}^{N_i} C_{ij3} y_{ij} \quad \forall i = 1 \dots n \\
& C_1 = \sum_{i=1}^{n} C_{i1} + I_1 * T_{n1} \quad C_2 = \sum_{i=1}^{n} C_{i2} + I_2 * T_{n2} \quad C_3 = \sum_{i=1}^{n} C_{i3} + I_3 * T_{n3} \qquad (4) \\
& Q_i = \sum_{j=1}^{N_i} q_{ij} y_{ij} \forall i, \quad Q = \frac{\sum_{i=1}^{n} Q_i}{n}, \quad Q_{MIN} \leq Q \leq 100 \\
& \sum_{j=1}^{N_i} y_{ij} = 1 \quad \forall (i,k) \in P, \quad y_{ij} = 0(or)1 \\
& T_{i3} - T_{i2} \geq 0, \quad T_{i2} - T_{i1} \geq 0 \quad \forall i
\end{aligned}
$$

Step 4: Model (4) can be decompose into three Crisp Integer Linear Programming models (5),(6) and (7) respectively. These three CILP models are given below:

$$
\begin{aligned}
& Min \quad C_1 \\
& subject \quad to: \ (a),(b),(c) \qquad (5) \\
& \qquad (d),(e),(f),(g)
\end{aligned}
\qquad
\begin{aligned}
& Min \quad C_2 \\
& subject \quad to: \ (a),(b),(c) \qquad (6) \\
& \qquad (d),(e),(f),(g)
\end{aligned}
$$

$$
\begin{aligned}
& Min \quad C_3 \\
& subject \quad to: \ (a),(b),(c) \qquad (7) \\
& \qquad (d),(e),(f),(g)
\end{aligned}
$$

Solve these decomposed models (5), (6) and (7) using existing procedure for solve Integer Programming problem.

Step (5): The optimal solution of the Fuzzy Time Cost Trade off Problem can be obtained by substituting the obtained results C_1, C_2 and C_3 from the models (5), (6) and (7) in (C_1, C_2, C_3).

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V. NUMERICAL ILLUSTRATION

To illustrate the developed Mathematical model, consider the simple example project used by Pollack-Johnson, B. and M. Liberatore [20]. Consider a general contractor planning to start construction of a new house. He has broken down the project into activities, with corresponding immediate successor activities, as given in Table 1. He has put the various activities individually up for bids, and has received bids for both duration and cost from different subcontractors. For example, one contractor might offer multiple time and cost pairs, as different possible scenario options for the same activity, different contractors could have submitted separate bid pairs for the same activity, or a combination of the two might occur. He wants to complete this project within (19, 21, 23) days. The possible triplets (time, cost and quality) are listed in Table 2. In this project, time parameter and costs of the project are considered in triangular fuzzy number form. Indirect cost of the project per day is Rs (2000, 2000, 2000).

Table 1 Construction Project

Activity	Description
$1 \to 2$ (A)	Excavate and Pour Footers
$1 \to 3$ (B)	Pour Concrete Foundation
$2 \to 3$ (C)	Erect Rough Wall and Roof
$2 \to 4$ (D)	Install Siding
$3 \to 4$ (E)	Install Plumbing
$3 \to 5$ (F)	Install Electrical
$4 \to 5$ (G)	Install Wallboard
$4 \to 6$ (H)	Lay Flooring
$5 \to 7$ (I)	Do Interior Painting
$5 \to 8$ (J)	Install Interior Fixtures
$6 \to 8$ (K)	Install Gutters & Downspouts
$7 \to 8$ (L)	Do Grading & Landscaping

Table 2 Options for Construction example (Time, Cost and Quality)

ACTIVITY i→j	TIME	COST	QUALITY
A(1, 2)	(2, 3, 4)	(21400, 21600, 21800)	70
B(1, 3)	(0,1,2)	(7000, 7200, 7400)	70
C(2, 3)	(4, 4, 4) (2, 3,4) (3, 4, 5)	(28800, 28800, 28800) (39300, 39600, 39900) (34500, 35000, 35500)	80, 70, 90
D(2, 4)	(5, 6, 7) (3, 3, 3) (3, 4, 5)	(28600, 28800, 29000) (43200, 43200, 43200) (35600, 36000, 36400)	70, 70, 60
E(3, 4)	(3, 3, 3) (1, 2, 3) (2, 3, 4)	(14000, 14000, 14000) (19000, 19200,19400) (11500, 12000, 12500)	70, 60, 60
F(3, 5)	(2, 4, 6) (2, 2, 2)	(9400, 9600, 9800) (16800, 16800, 16800)	70, 80
G(4, 5)	(5, 5, 5) (4, 4, 4)	(11500, 12000, 12500) (14400, 14400, 14400)	70, 80
H(4, 6)	(5, 6, 7) (1, 3, 5)	(28500, 28800, 29100) (57300, 57600, 57900)	70, 60
I(5, 7)	(2, 3, 4) (1, 2, 3) (3, ,3, 3)	(10500. 10800, 11100) (16400, 16800, 17200) (9000, 9000, 9000)	70, 70, 60
J(5, 8)	(3, 3, 3)	(7200, 7200, 7200)	70
K(6, 8)	(1, 2, 3) (1, 1, 1) (1, 2, 3)	(4400, 4800, 5200) (6000, 6000, 6000) (1700, 1800, 1900)	60, 70, 50
L(7, 8)	(2, 3, 4) (2, 2, 2) (2, 3, 4)	(14200, 14400, 14600) (19200, 19200, 19200) (15800, 16000, 16200)	70, 70, 80

Using step 1, the Fuzzy Integer Linear Mathematical Model of the sample project has been formulated.

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$$Min \quad (C_1, C_2, C_3)$$
$$Subject \quad to$$
$$(T_{11}, T_{12}, T_{13}) = 0$$
$$(T_{j1}, T_{j2} T_{j3}) - (T_{i1}, T_{i2}, T_{i3}) - \sum_{j=1}^{N_i} (d_{ij1}, d_{ij2}, d_{ij3}) y_{ij} \geq 0$$
$$(T_{n1}, T_{n2}, T_{n3}) \leq (T_1, T_2, T_3)$$
$$(C_{i1}, C_{i2}, C_{i3}) = \sum_{j=1}^{N_i} (C_{ij1}, C_{ij2}, C_{ij3}) y_{ij} \quad \forall i = 1 \ldots 12 \tag{8}$$
$$(C_1, C_2, C_3) = \sum_{i=1}^{n} (C_{i1}, C_{i2}, C_{i3}) + (I_1, I_2, I_3) * (T_{n1}, T_{n2}, T_{n3})$$
$$Q_i = \sum_{j=1}^{N_i} q_{ij} y_{ij} \forall i, \quad Q = \frac{\sum_{i=1}^{12} Q_i}{n}, \quad Q_{MIN} \leq Q \leq 100$$
$$\sum_{j=1}^{N_i} y_{ij} = 1 \quad \forall i \quad and \quad y_{ij} = 0 (or) 1$$
$$T_{i3} - T_{i2} \geq 0, \quad T_{i2} - T_{i1} \geq 0 \quad \forall i$$

According to step (4) in section IV, Fuzzy Mixed Integer Programming Model can be decomposing into three Crisp Integer Programming Models.

Solve these three Crisp Mixed Integer Programming Models using existing procedure. The values of minimum fuzzy total cost and planned fuzzy duration and maximum allowable quality of the project have been determined using LINGO solver. A computer package called LINGO (LINGO 2000) is used on a personal computer to solve the mathematical model of the example project. LINGO is a commercial package using the power of linear and non-linear optimization to formulate large problems concisely, solve them, and analyze the solution. In all tested runs, the linear mathematical model of the example project requires less than one second on LINGO to obtain the optimal solution.

The above Fuzzy Integer Programming Model can be easily solved by the proposed approach and the computation results are tabulated. The optimal time cost and quality for the Mixed Integer Programming Model (8) is presented in Table 3. The total cost of the project is Rs. (219300, 225200, 231100) at (19, 21, 23) days.

Table 3 Obtained result for the sample project

Activity	Time, Cost and Quality
$1 \rightarrow 2$ (A)	(2, 3, 4) (21400, 21600, 21800) 70
$1 \rightarrow 3$ (B)	(0, 1, 2) (7000, 7200, 7400) 70
$2 \rightarrow 3$ (C)	(4, 4, 4) (28800, 28800, 28800) 80
$2 \rightarrow 4$ (D)	(5, 6, 7) (28600, 28800, 29000) 70
$3 \rightarrow 4$ (E)	(3, 3, 3) (14000, 14000, 14000) 70
$3 \rightarrow 5$ (F)	(2, 4, 6) (9400, 9600, 9800) 70
$4 \rightarrow 5$ (G)	(5, 5, 5) (11500, 12000, 12500) 70
$4 \rightarrow 6$ (H)	(5, 6, 7) (28500, 28800, 29100) 70
$5 \rightarrow 7$ (I)	(3, 3, 3) (9000, 9000, 9000) 60
$5 \rightarrow 8$ (J)	(3, 3, 3) (7200, 7200, 7200) 70

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$6 \rightarrow 8$ **(K)**	**(1, 2, 3) (1700, 1800, 1900) 50**
$7 \rightarrow 8$ **(L)**	**(2, 3, 4) (14200, 14400, 14600) 70**

VI. CONCLUSION

This paper investigated the time cost and quality trade off problem in the project network with several fuzzy parameters which makes the problem complicated. The underlying idea is based on integer programming formulation. Fuzzy models are more effective in determining durations and cost in a real project network. The proposed model helps the decision maker to decide the best solution in fuzzy environment. The model is also applicable to more complicated project networks in real world. Clearly, the proposed approach is not confined to the fuzzy parameters of triangular type. Other types such as trapezoidal type and interval type are also applicable. This is illustrated by successfully solving an example with fuzzy parameters. As an extension of this study, interesting problem existing under Fuzzy Time Cost and Quality Trade off Problems by using Goal Programming technique.

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