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A Method for Solving Fuzzy Differential Equations Using Runge-Kutta Method with Harmonic Mean of Three Quantities

D.Paul Dhayabaran[1]
Associate Professor & Principal, PG and Research Department of Mathematics,
Bishop Heber College (Autonomous), Trichirappalli -620 017
J.Christy kingston[2]
Assistant Professor , PG and Research Department of Mathematics,
Bishop Heber College (Autonomous),Trichirappalli -620 017

Abstract*—In this paper a numerical solution for the first order fuzzy differential equations has been determined by runge- kutta method of order three with some parameters. The formula is involved with Harmonic mean of three quantitie k_i 's. The accuracy and efficiency of the proposed method is illustrated by solving a fuzzy initial value problem.

***Keywords* —Fuzzy Differential Equations, Runge-kutta method of order three, Trapezoidal Fuzzy Number.**

I. INTRODUCTION

Fuzzy differential equations are a natural way to model dynamical systems under uncertainty. First order linear fuzzy differential equations are one of the simplest fuzzy differential equations, which appear in many applications. The concept of fuzzy derivative was first introduced by S.L.Chang and L.A.Zadeh in [6].D.Dubois and Prade [7] discussed differentiation with fuzzy features.M.L.puri and D.A.Ralesec [17] and R.Goetschel and W.Voxman [10] contributed towards the differential of fuzzy functions. The fuzzy differential equation and initial value problems were extensively studied by O.Kaleva[11,12] and by S.Seikkala [18].Recently many research papers are focused on numerical solution of fuzzy initial value problems (FIVPS).Numerical Solution of fuzzy differential equations has been introduced by M.Ma, M. Friedman, A. Kandel [14] through Euler method and by S.Abbasbandy and T.Allahviranloo [1] by Taylor method.Runge – Kutta methods have also been studied by authors [2,16].Numerical solutions of fuzzy differential equations by Runge-kutta method of order three with some parameters has been studied by C.Duraisamy,and B.Usha[8].

This paper is organized as follows: In section 2 some basic results of fuzzy numbers and definitions of fuzzy derivative are given. In section 3 the fuzzy initial value problem is discussed .Section 4 contains the Runge-kutta method of order three. In section 5 the third order Runge-kutta method with some parameters based on harmonic mean is discussed. The method is illustrated by a solved numerical example in section 6 and the conclusion is in the last section.

II. PRELIMINARIES

A trapezoidal fuzzy number u is defined by four real numbers $k < l < m < n$, where the base of the trapezoidal is the interval $[k, n]$ and its vertices at $x = l$, $x = m$. Trapezoidal fuzzy number will be written as $u = (k, l, m, n)$. The membership function for the trapezoidal fuzzy number $u = (k, l, m, n)$ is defined as the following :

$$u(x) = \begin{cases} \frac{x-k}{l-k} & ; k \le x \le l \\ 1 & ; l \le x \le m \\ \frac{x-n}{m-n} & ; m \le x \le n \end{cases}$$

We have :

(1) $u > 0$ if $k > 0$;
(2) $u > 0$ if $\ell > 0$;
(3) $u > 0$ if $m > 0$;and

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(4) $u > 0$ if $n > 0$.

Let us denote R_F by the class of all fuzzy subsets of R (i.e. $u : R \to [0,1]$) satisfying the following properties:

(i) $\forall\, u \in R_F$, u is normal, i.e. $\exists\, x_0 \in R$ with $u(x_0) = 1$;

(ii) $\forall\, u \in R_F$, u is convex fuzzy set (i.e. $u(tx + (1 - t)\, y) \geq \min\{u(x), u(y)\}$, $\forall\, t \in [0,1]$, $x, y \in R$);

(iii) $\forall\, u \in R_F$, u is upper semi continuous on R;

(iv) $\{ x \in R;\, u(x) > 0 \}$ is compact, where A denotes the closure of A. Then R_F is called the space of fuzzy numbers(see eg:[12]).Obviously $R \subset R_F$. Here $R \subset R_F$ is understood as

$$R = \{\mathcal{X}_x \text{ ; x is usual real number}\}$$

We define the r-level set, $x \in R$;

$$[u]_r = \{ x \backslash u(x) \geq r \} \ ;\ 0 \leq r \leq 1$$

Clearly, $[u]_0 = \{ x \backslash u(x) > 0 \}$ *is compact*, which is a closed bounded interval and we denote by $[u]_r = [\underline{u}(r), \overline{u}(r)]$. It is clear that the following statements are true,

1. $\underline{u}(r)$ is a bounded left continuous non decreasing function over [0,1],
2. $\overline{u}(r)$ is a bounded right continuous non increasing function over [0,1],
3. $\underline{u}(r) \leq \overline{u}(r)$ for all $r \in (0,1]$,

for more details see [3],[4].

2.1 ∝ - Level Set

Let I be the real interval. A mapping $y: I \to E$ is called a fuzzy process and its ∝ - *level Set* is denoted by $[y(t)]_\propto = [\underline{y}(t;\propto), \overline{y}(t;\propto)]$, $t \in I$, $0 < \propto < 1$

2.2 Seikkala Derivative

The Seikkala Derivative $y'(t)$ of a fuzzy process is defined by $[y'(t)]_\propto = [\underline{y}'(t;\propto), \overline{y}'(t;\propto)]$, $t \in I$, $0 < \propto \leq 1$ provided that this equation defines a fuzzy number,as in [18]

Lemma 2.3

If the sequence of non-negative numbers $\{w_n\}_{n=0}^{N}$, satisfy

$$|Wn+1| \leq A\,|Wn| + B, \quad 0 \leq n \leq N-1$$

for the given positive constants A and B, then

$$|Wn| \leq A^n |W_0| + B\,\frac{A^n - 1}{A-1}, \quad 0 \leq n \leq N$$

Lemma 2.4

If the sequence of non-negative numbers $\{w_n\}_{n=0}^{N}$, $\{V_n\}_{n=0}^{N}$ satisfy

$$|Wn+1| \leq |Wn| + A \max\{|W_n|, |V_n|\} + B$$

$$|Vn+1| \leq |Vn| + A \max\{|W_n|, |V_n|\} + B$$

for the given positive constants A and B, then $U_n = |W_n| + |Vn|$, $0 \leq n \leq N$

we have $U_n \leq \overline{A}^n U_0 + \overline{B}\,\dfrac{\overline{A}^n - 1}{\overline{A} - 1}$, $0 \leq n \leq N$ where $\overline{A} = 1+2A$ and $\overline{B} = 2B$

Lemma 2.5

Let $F(t, u, v)$ and $G(t, u, v)$ belong to $C^1(R_F)$ and the partial derivatives of F and G be bounded over R_F. Then for arbitrarily fixed r, $0 \leq r \leq 1$,

$D(y(t_{n+1}), y^0(t_{n+1})) \leq h^2 L(1+2C)$,where L is a bound of partial derivatives of F and G, and

$$C = \max\left\{ \left| G[t_N, \underline{y}(t_N; r), \overline{y}(t_{N-1}; r)] \right|, r \in [0,1] \right\} < \infty$$

Theorem 2.6

Let $F(t, u, v)$ and $G(t, u, v)$ belong to $C^1(R_F)$ and the partial derivatives of F and G be bounded over R_F. Then for arbitrarily fixed r, $0 \leq r \leq 1$, the numerical solutions of $\underline{y}(t_{n+1}; r)$ and $\overline{y}(t_{N+1}; r)$ converge to the exact solutions $\underline{Y}(t;r)$ and $\overline{Y}(t; r)$ uniformly in t.

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Theorem 2.7
Let $F(t, u, v)$ and $G(t, u, v)$ belong to $C^1(R_F)$ and the partial derivatives of F and G be bounded over R_F and $2Lh < 1$. Then for arbitrarily fixed $0 \le r \le 1$,the iterative numerical solutions of $\underline{y}^{(j)}(t_n;r)$ and $\overline{y}^{(j)}(t_n;r)$ converge to the numerical solutions $\underline{y}(t_n; r)$ and $\overline{y}(t_n; r)$ in $t_0 \le t_n \le t_N$, when $j \to \infty$.

III. FUZZY INITIAL VALUE PROBLEM

Consider a first-order fuzzy initial value differential equation is given by

$$\begin{cases} y'(t) = f(t, y(t)), & t \in [t_0, T] \\ y(t_0) = y_0 \end{cases} \quad (3.1)$$

where y is a fuzzy function of t, $f(t, y)$ is a fuzzy function of the crisp variable t and the fuzzy variable y, y' is the fuzzy derivative of y and $y(t_0) = y_0$ is a trapezoidal or a trapezoidal shaped fuzzy number.
We denote the fuzzy function y by $y = [\underline{y}, \overline{y}]$. It means that the r-level set of $y(t)$ for $t \in [t_0, T]$ is

$$[y(t)]_r = [\underline{y}(t; r), \overline{y}(t; r)],$$

$$[y(t_0)]_r = [\underline{y}(t_0; r), \overline{y}(t_0; r)],\ r \in (0,1]$$

we write $f(t, y) = [\underline{f}(t, y), \overline{f}(t, y)]$ and

$$\underline{f}(t, y) = F[t, \underline{y}, \overline{y}],$$

$$\overline{f}(t, y) = G[t, \underline{y}, \overline{y}].$$

Because of $y' = f(t, y)$ we have

$$\underline{f}(t, y(t); r) = F[t, \underline{y}(t; r), \overline{y}(t; r)] \quad (3.2)$$

$$\overline{f}(t, y(t); r) = G[t, \underline{y}(t; r), \overline{y}(t; r)] \quad (3.3)$$

By using the extension principle, we have the membership function

$$f(t, y(t))(s) = sup\{y(t)(\tau) \backslash s = f(t,\tau)\},\ s \in R \quad (3.4)$$

so fuzzy number $f(t, y(t))$. From this it follows that

$$[f(t, y(t))]r = [\underline{f}(t, y(t); r), \overline{f}(t, y(t); r)],\ r \in (0,1], \quad (3.5)$$

where

$$\underline{f}(t, y(t); r) = min\{ f(t, u) | u \in [y(t)]_r\} \quad (3.6)$$

$$\overline{f}(t, y(t); r) = max\{ f(t, u) | u \in [y(t)]_r\}. \quad (3.7)$$

Definition 3.1 A function f: $R \to R_F$ is said to be fuzzy continuous function, if for an arbitrary fixed $t_0 \in R$ and $\varepsilon > 0$, $\delta > 0$ such that $|t - to| < \delta \Rightarrow D[f(t), f(t_0)] < \in$ exists.

Throughout this paper we also consider fuzzy functions which are continuous in metric D. Then the continuity of $f(t,y(t);r)$ guarantees the existence of the definition of $f(t, y(t); r)$ for $t \in [t_o, T]$ and $r \in [0,1]$ [10]. Therefore, the functions G and F can be definite too.

IV. THE THIRD ORDER RUNGE-KUTTA METHOD

The basis of all Runge-Kutta methods is to express the difference between the value of y at t_{n+1} and t_n as

$$y_{n+1} - y_n = \sum_{i=0}^{m} w_i k_i \quad (4.1)$$

Where w_i s are constant for all i and $k_i = hf(t_n + a_i h, y_n + \sum_{j=1}^{i-1} c_{ij} k_j)$ (4.2)

Most efforts to increase the order of accuracy of the Runge-Kutta methods have been accomplished by increasing the number of Taylor's series terms used and thus the number of functional evaluations required [5].The method proposed by Goeken .D and Johnson .O[9] introduces new terms involving higher order derivatives of 'f' in the Runge-Kutta k_i terms (i > 0)to obtain a higher order of accuracy without a corresponding increase in evaluations of 'f ', but with the addition of evaluations of f'.

The Third order Runge-Kutta method for with three slopes was proposed by C.Duraisamy, B.Usha [8]
Consider $y(t_{n+1}) = y(t_n) + W_1 k_1 + W_2 k_2 + W_3 k_3$ (4.3)

$$Where\ k_1 = h f(t_n, y(t_n)) \quad (4.4)$$

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$k_2 = h\, f(t_n + c_2 h, y(t_n) + a_{21} k_1)$

$k_3 = h\, f(t_n + c_3 h, y(t_n) + a_{31} k_1 + a_{32} k_2)$ (4.5)

and the parameters $W_1, W_2, W_3, a_{21}, a_{31}, a_{32}$ are chosen to make y_{n+1} closer to y(t_{n+1}).Using Taylor's series expansion the value of parameters is $a_{21} = \frac{2}{3}$, $a_{31} = 0$, $a_{32} = \frac{2}{3}$

The RK-method with harmonic mean of three quantities was discussed and used f' in the stage k_2 and k_3 by C.Duraisamy,B.Usha[8] .Then Runge- kutta method of order three with harmonic mean of three quantities is given by

$$y(t_{n+1}) = y(t_n) + \frac{3\, k_1 k_2 k_3}{k_2 k_3 + k_1 k_3 + k_1 k_2} \quad (4.6)$$

where $k_1 = h\, f(t_n, y(t_n))$ (4.7)

$k_2 = h\, f(t_n + c_2 h, y(t_n) + a_{21} k_1)$ (4.8)

$k_3 = h\, f(t_n + c_3 h, y(t_n) + a_{31} k_1 + a_{32} k_2)$ (4.9)

V. THE THIRD ORDER RUNGE-KUTTA METHOD WITH HARMONIC MEAN OF THREE QUANTITIES

Let the exact solution $[Y(t)]_r = [\underline{Y}(t;r), \overline{Y}(t;r)]$,is approximated by some $[y(t)]_r = [\underline{y}(t;r), \overline{y}(t;r)]$. The grid points at which the solutions is calculated are h=$\frac{T-t_0}{N}$, $t_i = t_0 + ih$; $0 \le i \le N$

From 4.6 to 4.9 we define

$$\underline{Y}(t_{n+1}, r) - \underline{Y}(t_n; r) = \frac{3\underline{k_1}(t_n, y(t_n, r))\underline{k_2}(t_n, y(t_n, r))\underline{k_3}(t_n, y(t_n, r))}{\underline{k_2}(t_n, y(t_n, r))\underline{k_3}(t_n, y(t_n, r)) + \underline{k_1}(t_n, y(t_n, r))\underline{k_3}(t_n, y(t_n, r)) + \underline{k_1}(t_n, y(t_n, r))\underline{k_2}(t_n, y(t_n, r))}$$

Where

$k_1 = h\, F(t_n, \underline{Y}(t_n; r), \overline{Y}(t_n; r))$

$k_2 = h\, F(t_n + \frac{2h}{3}, \underline{Y}(t_n; r) + \frac{2}{3}k_1, \overline{Y}(t_n; r) + \frac{2}{3}k_1)$ (5.1)

$k_3 = h\, F(t_n + \frac{2h}{3}, \underline{Y}(t_n; r) + \frac{2}{3}k_2, \overline{Y}(t_n; r) + \frac{2}{3}k_2)$

And

$$\overline{Y}(t_{n+1}, r) - \overline{Y}(t_n; r) = \frac{3\overline{k_1}(t_n, y(t_n, r))\overline{k_2}(t_n, y(t_n, r))\overline{k_3}(t_n, y(t_n, r))}{\overline{k_2}(t_n, y(t_n, r))\overline{k_3}(t_n, y(t_n, r)) + \overline{k_1}(t_n, y(t_n, r))\overline{k_3}(t_n, y(t_n, r)) + \overline{k_1}(t_n, y(t_n, r))\overline{k_2}(t_n, y(t_n, r))}$$

Where

$k_1 = h\, G(t_n, \underline{Y}(t_n; r), \overline{Y}(t_n; r))$

$k_2 = h\, G(t_n + \frac{2h}{3}, \underline{Y}(t_n; r) + \frac{2}{3}k_1, \overline{Y}(t_n; r) + \frac{2}{3}k_1)$ (5.2)

$k_3 = h\, G(t_n + \frac{2h}{3}, \underline{Y}(t_n; r) + \frac{2}{3}k_2, \overline{Y}(t_n; r) + \frac{2}{3}k_2)$

Also we have

$$\underline{y}(t_{n+1}, r) - \underline{y}(t_n; r) = \frac{3\underline{k_1}(t_n, y(t_n, r))k_2(t_n, y(t_n, r))\underline{k_3}(t_n, y(t_n, r))}{\underline{k_2}(t_n, y(t_n, r))\underline{k_3}(t_n, y(t_n, r)) + \underline{k_1}(t_n, y(t_n, r))\underline{k_3}(t_n, y(t_n, r)) + \underline{k_1}(t_n, y(t_n, r))\underline{k_2}(t_n, y(t_n, r))}$$

Where

$k_1 = h\, F(t_n, \underline{y}(t_n; r), \overline{y}(t_n; r))$

$k_2 = h\, F(t_n + \frac{2h}{3}, \underline{y}(t_n; r) + \frac{2}{3}k_1, \overline{y}(t_n; r) + \frac{2}{3}k_1)$ (5.3)

$k_3 = h\, F(t_n + \frac{2h}{3}, \underline{y}(t_n; r) + \frac{2}{3}k_2, \overline{y}(t_n; r) + \frac{2}{3}k_2)$

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and

$$\overline{y}(t_{n+1}, r) - \overline{y}(t_n; r) = \frac{3\overline{k_1}(t_n, y(t_n, r))\overline{k_2}(t_n, y(t_n, r))\overline{k_3}(t_n, y(t_n, r))}{\overline{k_2}(t_n, y(t_n, r))\overline{k_3}(t_n, y(t_n, r)) + \overline{k_1}(t_n, y(t_n, r))\overline{k_3}(t_n, y(t_n, r)) + \overline{k_1}(t_n, y(t_n, r))\overline{k_2}(t_n, y(t_n, r))}$$

Where

$$k_1 = h\,G\,(t_n, \underline{y}(t_n; r), \overline{y}(t_n; r))$$

$$k_2 = h\,G(t_n + \frac{2h}{3}, \underline{y}(t_n; r) + \frac{2}{3}k_1, \overline{y}(t_n; r) + \frac{2}{3}k_1) \quad (5.4)$$

$$k_3 = h\,G(t_n + \frac{2h}{3}, \underline{y}(t_n; r) + \frac{2}{3}k_2, \overline{y}(t_n; r) + \frac{2}{3}k_2)$$

We define

$$F[t, y(t, r)] = \frac{3\underline{k_1}(t, y(t, r))\underline{k_2}(t, y(t, r))\underline{k_3}(t, y(t, r))}{\underline{k_2}(t, y(t, r))\underline{k_3}(t, y(t, r)) + \underline{k_1}(t, y(t, r))\underline{k_3}(t, y(t, r)) + \underline{k_1}(t, y(t, r))\underline{k_2}(t, y(t, r))} \quad (5.5)$$

$$G[t, y(t, r)] = \frac{3\overline{k_1}(t, y(t, r))\overline{k_2}(t, y(t, r))\overline{k_3}(t, y(t, r))}{\overline{k_2}(t, y(t, r))\overline{k_3}(t, y(t, r)) + \overline{k_1}(t, y(t, r))\overline{k_3}(t, y(t, r)) + \overline{k_1}(t, y(t, r))\overline{k_2}(t, y(t, r))} \quad (5.6)$$

Therefore we have

$$\underline{Y}(t_{n+1}, r) = \underline{Y}(t_n; r) + F[t_n, Y(t_n; r)]$$

$$\overline{Y}(t_{n+1}, r) = \overline{Y}(t_n; r) + G[t_n, Y(t_n; r)]$$

And

$$\underline{y}(t_{n+1}, r) = \underline{y}(t_n; r) + F[t_n, y(t_n; r)]$$

$$\overline{y}(t_{n+1}, r) = \overline{y}(t_n; r) + G[t_n, y(t_n; r)]$$

Clearly $\underline{y}(t; r)$ and $\overline{y}(t; r)$ converge to $\underline{Y}(t; r)$ and $\overline{Y}(t; r)$ whenever $h \to 0$

VI. NUMERICAL EXAMPLE

Example 6.1 Consider fuzzy initial value problem

$$\begin{cases} y'(t) = y(t), & t \geq 0 \\ y(0) = (0.8 + 0.125r, 1.1 - 0.1r) \end{cases} \quad (6.1)$$

The exact solution is given by

$$Y(t;r) = [(0.8 + 0.125r)e^t,\ (1.1\text{-}0.1r)\,e^t]$$

At t=1we get

$$Y(1;r) = [(0.8 + 0.125r)e,\ (1.1\text{-}0.1r)\,e], \quad 0 \leq r \leq 1$$

The values of exact and approximate solution with h= 0.1 is given in Table: 1. The approximate solutions obtained by the proposed method is plotted in Fig:1

Table: 1

r	Exact Solution	Approximation of RK-Method with HM of Three quantities (h=0.1)
0	2.174625 , 2.990110	2.161249 , 2.971716
0.2	2.242583 , 2.935744	2.228695 , 2.917680
0.4	2.310540 , 2.881379	2.296326 , 2.863654
0.6	2.378497 , 2.827013	2.363865 , 2.809615
0.8	2.446454 , 2.772647	2.432121 , 2.755592
1	2.514411 , 2.718282	2.498943 , 2.701560

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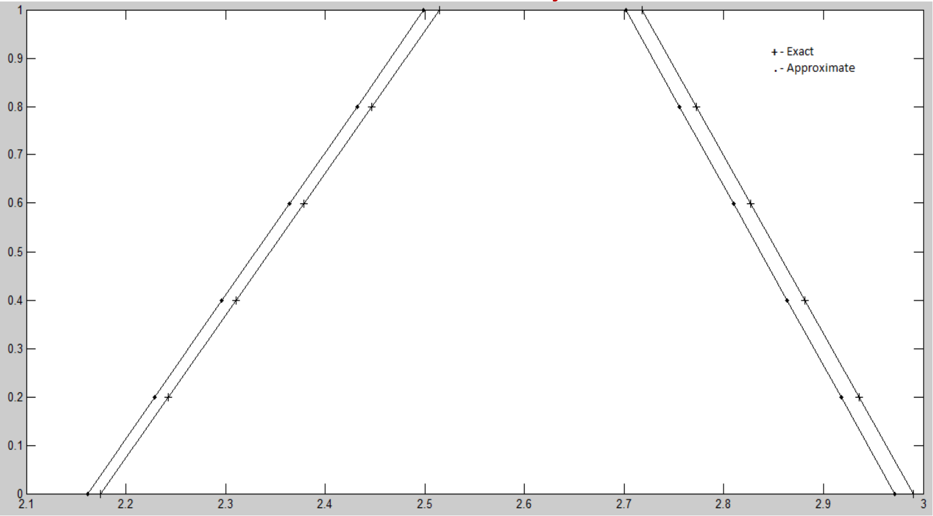

Fig : 1

VII. CONCLUSION

In this paper the Runge-Kutta method of order three with three quantities has been applied for finding the numerical solution of fuzzy differential equations. The higher order convergence O(h^3) is obtained by the proposed method. By minimizing the step size'h' the exact solution at 'h' and the solution obtained by the proposed method almost coincide.

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