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# An Algorithm to Solve Fully Fuzzy Time Cost Trade off Problems

M. Evangeline Jebaseeli<sup>1\*</sup>, D. Paul Dhayabaran<sup>2</sup>

*Abstract: Project management is one of the most important fields in business and industry. Every task in an organization can be taken into account as a project. In this paper, we have presented an algorithm for solving Fully Fuzzy Time Cost Trade Off problems through Multi Objective Linear Programming technique. The application of the presented method is easier when compared with the existing methods for solving time cost trade off problems occurring in real life situations. Using this technique, the project manager will be able to determine how much to crash each activity in order to minimize the total cost of meeting any specified deadline for the project. An illustration is provided to demonstrate the efficiency of the method.*

**Key words:** Fully Fuzzy Time Cost Trade Off problem – Multi Objective Linear Programming – Triangular fuzzy number.

## I. INTRODUCTION

Project Management is a very important field employed for scheduling activities and monitoring the progress, in competitive and fluctuating environments. The feasible duration required to perform a specific project is determined using critical path method. However, because of competitive priorities, time is important and the completion time of a project determined using critical path method should be reduced to meet a deadline requested.

In scheduling a project, it is generally considered to expedite the duration of some activities through expanding extra budget in order to compress the project completion time. This procedure can be considered under either some fixed available budget or a threshold of project completion time. This problem is known as time cost trade off problem or project crashing problem in the project management literature.

The main objective of these kinds of problem is to determine the optimum duration and cost should be assigned to the activities such that the overall cost is minimized. The project duration can be shortened by the acceleration of the critical activity times. The acceleration of the activity times can be achieved using more resources (using more productive equipment, material or hiring more workers) which means higher costs. Project crashing problem analyzes how to modify project activities so as to achieve the tradeoff between the project cost and the completion time.

### A. Time Cost Trade Off Problems in Different Nature

By reviewing the literature, it is observed that there are several studies investigated and analyzed the project management problems. Mathematical and heuristic methods are the two major approaches used to solve the time cost trade off problems in project scheduling. Mathematical methods convert the project time cost trade off problems to mathematical models and utilize linear programming, integer programming, dynamic programming, goal programming or multi-objective linear programming to solve the problems. However, formulating the objective function as well as the required constraints is time-consuming and prone to errors. Heuristic methods provide a way to obtain good solutions but do not guarantee optimality. However, they require less computational effort than mathematical methods.

The problem of project time cost trade off was first introduced by Kelly [11]. By assuming that direct cost of an activity changes with time, mathematical programming models were developed to minimize the project's direct cost [11]. Thereafter, many researchers have developed mathematical programming model for these kinds of problems. The time cost trades off problems have been extensively investigated. Many models have been proposed and they can be categorized into two types: Deterministic scheduling and non-deterministic scheduling. Recently, Yang [21] proposed a chance-constrained programming model to analyze the time cost trade off problem, where funding variability is considered. Yang [21] took budget uncertainty into account on project time cost trade off in a chance-constrained programming model. A hybrid intelligent algorithm integrating simulation and genetic algorithm was designed for solving the proposed models.



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The above mentioned time cost trade off models mainly based on probability theory. As generally known, it requires a prior predictable regularity or a posterior frequency distribution to construct the probability distribution of activity times. However, in real world applications some activity times must be forecasted subjectively; for example, we have to use human judgment instead of stochastic assumptions to determine activity times. An alternative way to deal with imprecise data is to employ the concept of fuzziness, whereby the vague activity times can be represented by fuzzy sets. The main advantages of methodologies based on fuzzy theory are that they do not require prior predictable regularities or posterior frequency distributions, and they can deal with imprecise input information containing feelings and emotions quantified based on the decision-makers subjective judgment.

In the literature, there are several studies that have investigated the project management problems with fuzzy parameters. Liu et al. [16] proposed a Fuzzy Optimal construction Time Cost Trade Off method. In their study, the activity duration is accepted as fuzzy number. An acceptable risk level is defined as the minimum concept of the fuzzy set theory; fuzzy durations are transformed into crisp sets. Then the genetic algorithm techniques are used to find the optimal or near optimal solutions. Arikani and Gungor [2] applied Fuzzy Goal Programming to the Time Cost Trade Off problem with two objectives which are minimum completion time and crashing costs. The aspiration levels of the objective are defined as fuzzy numbers. The goal programming is solved using max-min approach. Wang and Liang [17] solved project management decision problem with multiple fuzzy goals in their study. The goals of the problem are defined using linear membership functions, and the multiple Fuzzy Goal Programming problem is solved after transforming into its crisp equivalent using Bellman and Zadeh's fuzzy decision concept. Eshtehardian et al. [4] presented a new approach for the solution of Time Cost Trade Off problems with uncertain costs. An appropriate genetic algorithm is used to find the solution of the Multi Objective Fuzzy Time Cost Problem. Lin [14] proposed an approach to solve project crashing problems with uncertain activity times and crash costs. The confidence-interval estimates and the previous statistical data are used to solve the fuzzy project crashing problem. In this study, level  $(1 - \alpha)$  fuzzy numbers were derived from  $(1 - \alpha) \times 100\%$  confidence interval estimates of the statistical data with a distance ranking which is used to define the fuzzy ordering.

The activities execution times and costs, the daily costs are accepted as triangular fuzzy numbers. The proposed approach explicitly embeds the fuzzy set theory into the optimization procedure and then a multi objective genetic algorithm is used to solve the discontinuous and multi objective fuzzy time cost model. Maa and Ke [12] solved Time Cost Trade Off problem with fuzzy activity duration times. The fuzzy activity duration times defined as fuzzy variables based on self-dual credibility measure. Then, the obtained Time Cost Trade Off problem is solved with a hybrid intelligent algorithm integrating fuzzy simulation and genetic algorithm. Ghazanfari et al. [8, 9] proposed a mathematical model to deal with Fuzzy Time Cost Trade Off problem. The normal and crash durations of activities are considered as triangular fuzzy numbers. For the solution of the fuzzy problem, a ranking fuzzy numbers method used. Liang [18] proposed a possibilistic linear programming approach for the solution of Fuzzy Multi Objective project management decision problem. The fuzzy parameters are defined using the triangular possibility distribution. In the proposed possibilistic linear programming approach, the fuzzy objectives and the fuzzy constraints are transformed into their crisp equivalents. Then, the obtained multi objective linear programming problem transformed into an equivalent linear programming problem using Zimmerman's fuzzy decision concept and the minimum operator. Chen and Tsai [3] proposed a new approach to solve Time Cost Trade Off problems. This approach described the minimum total crash cost of a project network via a membership function which completely conserves all the fuzziness of parameters, and the corresponding optimal activity time for each activity under different possibility levels are obtained. Tolunay Gocken [20] proposed a solution process for the Fuzzy Multi Objective Project Crashing problem.

In this paper, we have presented an algorithm for solving Fully Fuzzy Time Cost Trade Off problems through Multi Objective Linear Programming technique. The proposed method minimizes the total cost of the project and this is one of the easiest methods to solve Fuzzy Time and Cost optimization problem occurring in real life situations. Using this technique, the project manager will be able to determine how much to crash each activity in order to minimize the total cost of meeting any specified deadline for the project. An illustration is provided to demonstrate the efficiency of the proposed method.



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**II. PRELIMINARIES**

In this section, some basic definitions of fuzzy theory defined by Kaufmann, Gupta and Zimmermann, are presented.

**Definition 2.1**

The characteristic function  $\mu_A$  of a crisp set  $A \subseteq X$  assigns a value either 0 or 1 to each member in  $X$ . This function can be generalized to a function  $\mu_{\tilde{A}}$  such that the value assigned to the element of the universal set  $X$  fall within a specified range i.e.  $\mu_{\tilde{A}} : X \rightarrow [0,1]$ . The assigned values indicate the membership grade of the element in the set  $A$ . The function  $\mu_{\tilde{A}}$  is called the membership function and the set  $\tilde{A} = \{(A, \mu_A(x) : x \in X)\}$

defined by  $\mu_{\tilde{A}}(x)$  for each  $x \in X$  is called a fuzzy set.

**Definition 2.2**

A fuzzy set  $\tilde{A}$  defined on the set of real numbers  $R$  is said to be a fuzzy number if its membership function has the following characteristics:

1.  $\mu_{\tilde{A}}(x) : R \rightarrow [0,1]$  is continuous.
2.  $\mu_{\tilde{A}}(x) = 0$  for all  $(-\infty, a] \cup [c, \infty)$ .
3.  $\mu_{\tilde{A}}(x)$  is strictly increasing on  $[a, b]$  and strictly decreasing on  $[b, c]$ .
4.  $\mu_{\tilde{A}}(x) = 1$  for all  $x \in b$  where  $a \leq b \leq c$ .

**Definition 2.3**

Triangular fuzzy number is a fuzzy number represented with three points as follows:  $\tilde{A} = (a_1, a_2, a_3)$  this representation is interpreted as membership functions.

$$\mu_A(x) = \begin{cases} 0 & \text{if } x < a_1 \text{ and } x > a_3 \\ \frac{x - a_1}{a_2 - a_1} & \text{if } a_1 \leq x \leq a_2 \\ \frac{a_3 - x}{a_3 - a_2} & \text{if } a_2 \leq x \leq a_3 \end{cases}$$

**Definition 2.4**

A fuzzy number  $\tilde{A}$  in parametric form is a pair  $(\underline{a}, \bar{a})$  of functions  $\underline{a}(r)$  and  $\bar{a}(r)$ ,  $0 \leq r \leq 1$ , which these functions satisfy the following requirements:

- i.  $\underline{a}(r)$  is a bounded monotonic increasing left continuous function,
- ii.  $\bar{a}(r)$  is a bounded monotonic decreasing left continuous function,
- iii.  $\underline{a}(r) \leq \bar{a}(r)$ ,  $0 \leq r \leq 1$ .

A popular fuzzy number is the trapezoidal fuzzy number  $\tilde{A} = (a, b, c, d)$  with interval defuzzifier  $[b, c]$  and left fuzziness  $(b - a)$  and right fuzziness  $(d - c)$ , where its membership function is given as follows:



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$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{b-a}{x-a} & \text{if } a \leq x \leq b, \\ 1 & \text{if } x \in [b, c] \\ \frac{d-c}{d-x} & \text{if } c \leq x \leq d, \\ 0 & \text{otherwise} \end{cases}$$

Its parametric form is  $\underline{a}(r) = a + (b-a)r$ ,  $\bar{a}(r) = d + (c-d)r$  where  $0 \leq r \leq 1$ . In particular,  $\tilde{A} = (a, b, c, d)$  can also signify a triangular fuzzy number  $\tilde{A} = (a, b, d) = (a_1, a_2, a_3)$  if  $b = c$ . The set of all these triangular fuzzy number is denoted by  $TF(R)$ .

**Definition 2.5**

A triangular fuzzy number  $\tilde{A} = (a_1, a_2, a_3)$  and  $\tilde{B} = (b_1, b_2, b_3)$  is said to be a non-negative triangular fuzzy number if and only if  $a_1 \geq 0$ . The set of all these triangular fuzzy numbers is denoted by  $TF(R)^+$ .

**Definition 2.6 Fully Fuzzy Linear Programming problem (FFLP)**

Linear programming is one of the most frequently applied operations research technique. We assume that all parameters and variables are real numbers. But in real world environment, do not have precise information. So, the fuzzy numbers and fuzzy variables should be used Linear Programming problem. The standard form FFLP problems with m fuzzy equality constraints and n fuzzy variables as follows:

$$\text{Maximize (or Minimize) } (\tilde{C}^T \otimes \tilde{X}) \tag{1}$$

$$\text{Subject to } \tilde{A} \otimes \tilde{X} = \tilde{b}$$

$\tilde{X}$  is a non-negative fuzzy number. Where  $\tilde{C}^T = [\tilde{c}_j]_{1 \times n}$ ,  $\tilde{X} = [\tilde{x}_j]_{n \times 1}$ ,  $\tilde{A} = [\tilde{a}_{ij}]_{m \times n}$ ,  $\tilde{b} = [\tilde{b}_i]_{m \times 1}$  and  $\tilde{c}_j, \tilde{x}_j, \tilde{a}_{ij}, \tilde{b}_i \in F(R)$  where  $i = 1, 2, \dots, m$  and  $j = 1, 2, \dots, n$ .

**Definition 2.7 [5]:** Let  $\tilde{u} = (x_1, x_2, x_3)$  and  $\tilde{v} = (y_1, y_2, y_3)$  be two arbitrary triangular fuzzy numbers.  $\tilde{u}$  is relatively less than  $\tilde{v}$ , which is denoted by  $\tilde{u} \prec \tilde{v}$ , if and only if:

- i.  $x_2 < y_2$  or
- ii.  $x_2 = y_2$  and  $(x_3 - x_1) > (y_3 - y_1)$
- iii.  $x_2 = y_2$ ,  $(x_3 - x_1) = (y_3 - y_1)$  and  $(x_3 + x_1) < (y_3 + y_1)$

**Remark 2.8 [5]:**  $\tilde{x}^* = (x_1^*, x_2^*, x_3^*)$  is said to be an exact optimal solution of problem (1) if it satisfies in the following statements:

- i.  $\tilde{x}^* = [\tilde{x}_j^*]_{n+1}$  where  $\tilde{x}^* \in TF(\mathfrak{R})^+$ ,  $j = 1, 2, \dots, n$
- ii.  $A\tilde{x}^* = b$
- iii.  $\forall \tilde{x} = (x_1, x_2, x_3) \in \tilde{S} = \{\tilde{x} / \tilde{A}\tilde{x} = \tilde{b}, \tilde{x} = [\tilde{x}_j^*]_{n+1} \text{ where } \tilde{x}_j \in TF(\mathfrak{R})^+\}$ , we have that  $\tilde{C}^T \tilde{x} \prec \tilde{C}^T \tilde{x}^*$  (in case minimization problem  $\tilde{C}^T \tilde{x} \succ \tilde{C}^T \tilde{x}^*$ )



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**Remark 2.9 [5]:** Let  $\tilde{x}^*$  be an optimal solution of problem (1) and there exist an  $\tilde{C}^T \tilde{x}' = \tilde{C}^T \tilde{x}^*$ , then  $\tilde{x}'$  is also an exact optimal solution of problem (1) is called an alternative exact optimal solution.

### III. PROBLEM DESCRIPTION

With the progress of the project, project managers always need to make tradeoff between the cost and the completion time. Sometimes managers may make decision in order to finish the project sooner with project cost augment by accelerating the project schedule, which is also named as project crashing in project management. In other cases, motivated by reducing the project cost, managers may be conscripted to sacrifice with prolonging the project completion time. Therefore, it is naturally desirable for managers to find a schedule to complete a project with the balance of the cost and completion time.

The total cost function of a project has two components: direct and indirect costs. Direct costs are incurred because of the performance of project activities, while indirect costs include those items that are not directly related to individual project activities and thus can be assessed for the entire project. In general, indirect cost increases almost linearly with the increase of project duration and usually assumed as a percentage of project direct cost. A possible way to solve time cost trade off problem is to use a mathematical programming model whose objective function is constructed so that project direct cost is minimized and the imposed constraints guarantee a desired project deadline, while the precedence requirements of the network are maintained.

A project can be represented by an activity-on-arc network  $G = (V, A)$ , where  $V = \{1, 2, \dots, n\}$  is the set of nodes representing the milestones and  $A$  is the set of arcs representing the activities. In the network, node 1 and  $n$  represent the start and end of the project respectively. The **Complete Fuzzy Mathematical Model** for Fully Fuzzy Time Cost Trade Off problem is presented as follows:

$$\begin{aligned}
 \text{Min } \tilde{Z} &= \sum_i \sum_j \tilde{C}_{ij} \\
 \text{Subject to} \\
 \tilde{T}_1 &= 0 \\
 \tilde{T}_j - \tilde{T}_i - \tilde{x}_{ij} &\geq 0 \\
 \tilde{T}_n &\leq \tilde{T} \\
 \tilde{C}_{ij} &= \tilde{s}_{ij} (N\tilde{T}_{ij} - \tilde{x}_{ij}) \\
 C\tilde{T}_{ij} &\leq \tilde{x}_{ij} \leq N\tilde{T}_{ij} \quad \forall (i, j) \in P \\
 \tilde{C}_{ij} &= \sum_i \sum_j \tilde{C}_{ij} + \tilde{I} * (\tilde{T}_n - \tilde{T}_1) + \sum_n \tilde{K}_n
 \end{aligned} \tag{2}$$

### IV. ALGORITHM TO SOLVE FULLY FUZZY TIME COST TRADE OFF PROBLEMS

The following is the algorithm to find an optimal solution of Fuzzy Time Cost Trade Off problems using MOLP technique. The steps of proposed algorithm are given as below:

Step 1: Set up the mathematical formulation of the Fuzzy Time Cost Trade Off problem as given in (2).

Step 2: With respect to definition 2.4 and 2.5, problem (2) can be shown as follows:



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$$\text{Min } \sum_i \sum_j (C_{ij1}, C_{ij2}, C_{ij3})$$

Subject to

$$(T_{11}, T_{12}, T_{13}) = 0$$

$$(T_{j1}, T_{j2}, T_{j3}) - (T_{i1}, T_{i2}, T_{i3}) - (D_{ij1}, D_{ij2}, D_{ij3}) \geq 0$$

$$(T_{n1}, T_{n2}, T_{n3}) \leq (T_1, T_2, T_3)$$

$$(C_{ij1}, C_{ij2}, C_{ij3}) = (s_{ij1}, s_{ij2}, s_{ij3}) \{ (NT_{ij1}, NT_{ij2}, NT_{ij3}) - (D_{ij1}, D_{ij2}, D_{ij3}) \} \quad (3)$$

$$(CT_{ij1}, CT_{ij2}, CT_{ij3}) \leq (D_{ij1}, D_{ij2}, D_{ij3}) \leq (NT_{ij1}, NT_{ij2}, NT_{ij3}) \quad \forall (i, j) \in P$$

$$(C_1, C_2, C_3) = \sum_i \sum_j (C_{ij1}, C_{ij2}, C_{ij3}) + I^* ((T_{n1}, T_{n2}, T_{n3}) - (T_{11}, T_{12}, T_{13})) + \sum_n (K_{n1}, K_{n2}, K_{n3})$$

Ste

p3: The above linear programming model (3) may be written as follows:

$$\text{Min } \sum_i \sum_j (C_{ij1}, C_{ij2}, C_{ij3})$$

Subject to

$$T_{11} = 0, \quad T_{12} = 0, \quad T_{13} = 0$$

$$T_{j1} - T_{i1} - D_{ij1} \geq 0, \quad T_{j2} - T_{i2} - D_{ij2} \geq 0, \quad T_{j3} - T_{i3} - D_{ij3} \geq 0$$

$$T_{n1} \leq T_1, \quad T_{n2} \leq T_2, \quad T_{n3} \leq T_3$$

$$C_{ij1} = NC_{ij1} + s_{ij1} \{ NT_{ij1} - D_{ij1} \},$$

$$C_{ij2} = NC_{ij2} + s_{ij2} \{ NT_{ij2} - D_{ij2} \},$$

$$C_{ij3} = NC_{ij3} + s_{ij3} \{ NT_{ij3} - D_{ij3} \} \quad (4)$$

$$CT_{ij1} \leq D_{ij1} \leq NT_{ij1}, \quad CT_{ij2} \leq D_{ij2} \leq NT_{ij2}, \quad CT_{ij3} \leq D_{ij3} \leq NT_{ij3}$$

$$C_1 = \sum_i \sum_j C_{ij1} + I_1^* (T_{n1} - T_{11}) + \sum_n K_{n1}$$

$$C_2 = \sum_i \sum_j C_{ij2} + I_2^* (T_{n2} - T_{12}) + \sum_n K_{n2}$$

$$C_3 = \sum_i \sum_j C_{ij3} + I_3^* (T_{n3} - T_{13}) + \sum_n K_{n3}$$

$$D_{ij2} - D_{ij1} \geq 0, \quad D_{ij3} - D_{ij2} \geq 0$$

$$C_{ij2} - C_{ij1} \geq 0, \quad C_{ij3} - C_{ij2} \geq 0, \quad \forall (i, j) \in P$$

Step 4: Regarding to definition 2.7, problem (4) is converted to the Multi Objective Linear Programming

problem with three crisp functions as follows:



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$$\text{Min } \sum_i \sum_j C_{ij2}, \quad \text{Max } \sum_i \sum_j (C_{ij3} - C_{ij1}), \quad \text{Min } \sum_i \sum_j (C_{ij3} + C_{ij1})$$

Subject to

$$T_{11} = 0, \quad T_{12} = 0, \quad T_{13} = 0$$

$$T_{j1} - T_{i1} - D_{ij1} \geq 0, \quad T_{j2} - T_{i2} - D_{ij2} \geq 0, \quad T_{j3} - T_{i3} - D_{ij3} \geq 0$$

$$T_{n1} \leq T_1, \quad T_{n2} \leq T_2, \quad T_{n3} \leq T_3$$

$$C_{ij1} = s_{ij1} \{NT_{ij1} - D_{ij1}\}, C_{ij2} = s_{ij2} \{NT_{ij2} - D_{ij2}\}, C_{ij3} = s_{ij3} \{NT_{ij3} - D_{ij3}\} \quad (5)$$

$$CT_{ij1} \leq D_{ij1} \leq NT_{ij1}, \quad CT_{ij2} \leq D_{ij2} \leq NT_{ij2}, \quad CT_{ij3} \leq D_{ij3} \leq NT_{ij3}$$

$$C_1 = \sum_i \sum_j C_{ij1} + I_1 * (T_{n1} - T_{11}) + \sum_n K_{n1}, \quad C_2 = \sum_i \sum_j C_{ij2} + I_2 * (T_{n2} - T_{12}) + \sum_n K_{n2}$$

$$C_3 = \sum_i \sum_j C_{ij3} + I_3 * (T_{n3} - T_{13}) + \sum_n K_{n3}$$

$$D_{ij2} - D_{ij1} \geq 0, \quad D_{ij3} - D_{ij2} \geq 0, \quad C_{ij2} - C_{ij1} \geq 0, \quad C_{ij3} - C_{ij2} \geq 0, \quad \forall (i, j) \in P$$

Step 5: In terms of the preference of objective function, the lexicographic method will be used to obtain a

lexicographically optimal solution of problem (5). So, we have

$$\text{Min } \sum_i \sum_j C_{ij2}$$

Subject to

$$T_{11} = 0, \quad T_{12} = 0, \quad T_{13} = 0$$

$$T_{j1} - T_{i1} - D_{ij1} \geq 0, \quad T_{j2} - T_{i2} - D_{ij2} \geq 0, \quad T_{j3} - T_{i3} - D_{ij3} \geq 0$$

$$T_{n1} \leq T_1, \quad T_{n2} \leq T_2, \quad T_{n3} \leq T_3$$

$$C_{ij1} = s_{ij1} \{NT_{ij1} - D_{ij1}\}, \quad C_{ij2} = s_{ij2} \{NT_{ij2} - D_{ij2}\}, \quad C_{ij3} = NC_{ij3} + s_{ij3} \{NT_{ij3} - D_{ij3}\}$$

$$CT_{ij1} \leq D_{ij1} \leq NT_{ij1}, \quad CT_{ij2} \leq D_{ij2} \leq NT_{ij2}, \quad CT_{ij3} \leq D_{ij3} \leq NT_{ij3} \quad \forall (i, j) \in P$$

$$C_1 = \sum_i \sum_j C_{ij1} + I_1 * (T_{n1} - T_{11}) + \sum_n K_{n1}$$

$$C_2 = \sum_i \sum_j C_{ij2} + I_2 * (T_{n2} - T_{12}) + \sum_n K_{n2}$$

$$C_3 = \sum_i \sum_j C_{ij3} + I_3 * (T_{n3} - T_{13}) + \sum_n K_{n3}$$

$$D_{ij2} - D_{ij1} \geq 0, \quad D_{ij3} - D_{ij2} \geq 0, \quad C_{ij2} - C_{ij1} \geq 0, \quad C_{ij3} - C_{ij2} \geq 0 \quad (6)$$

If problem (6) has a unique optimal solution, namely  $\tilde{D}_{ij}^* = (D_{ij1}^*, D_{ij2}^*, D_{ij3}^*)$  then it is an optimal solution of problem (6) and stop. Otherwise go to step 5.





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Step 6: Solve the following problem over the optimal solutions that are achieved in step 4 as

$$\text{Max } \sum_i \sum_j (C_{ij3} - C_{ij1})$$

Subject to

$$\sum_i \sum_j C_{ij2} = M$$

$$T_{11} = 0, \quad T_{12} = 0, \quad T_{13} = 0$$

$$T_{j1} - T_{i1} - D_{ij1} \geq 0, \quad T_{j2} - T_{i2} - D_{ij2} \geq 0, \quad T_{j3} - T_{i3} - D_{ij3} \geq 0$$

follows:  $T_{n1} \leq T_1, \quad T_{n2} \leq T_2, \quad T_{n3} \leq T_3$

$$C_{ij1} = s_{ij1} \{NT_{ij1} - D_{ij1}\}, \quad C_{ij2} = s_{ij2} \{NT_{ij2} - D_{ij2}\}, \quad C_{ij3} = s_{ij3} \{NT_{ij3} - D_{ij3}\} \quad (7)$$

$$CT_{ij1} \leq D_{ij1} \leq NT_{ij1}, \quad CT_{ij2} \leq D_{ij2} \leq NT_{ij2}, \quad CT_{ij3} \leq D_{ij3} \leq NT_{ij3} \quad \forall (i, j) \in P$$

$$C_1 = \sum_i \sum_j C_{ij1} + I_1 * (T_{n1} - T_{11}) + \sum_n K_{n1}, \quad C_2 = \sum_i \sum_j C_{ij2} + I_2 * (T_{n2} - T_{12}) + \sum_n K_{n2}$$

$$C_3 = \sum_i \sum_j C_{ij3} + I_3 * (T_{n3} - T_{13}) + \sum_n K_{n3}$$

$$D_{ij2} - D_{ij1} \geq 0, \quad D_{ij3} - D_{ij2} \geq 0, \quad C_{ij2} - C_{ij1} \geq 0, \quad C_{ij3} - C_{ij2} \geq 0$$

where M is the optimal value of problem (6). If problem (7) has a unique optimal solution, namely then it is also an optimal solution of problem (3) and stop. Otherwise go to Step 5.

Step 7: Solve the following problem over the optimal solutions that are achieved in Step 5 as follows:

$$\text{Min } \sum_i \sum_j (C_{ij3} + C_{ij1})$$

Subject to

$$\sum_i \sum_j C_{ij2} = M, \quad \sum_i \sum_j (C_{ij3} - C_{ij1}) = N$$

$$T_{11} = 0, \quad T_{12} = 0, \quad T_{13} = 0$$

$$T_{j1} - T_{i1} - D_{ij1} \geq 0, \quad T_{j2} - T_{i2} - D_{ij2} \geq 0, \quad T_{j3} - T_{i3} - D_{ij3} \geq 0$$

$$T_{n1} \leq T_1, \quad T_{n2} \leq T_2, \quad T_{n3} \leq T_3$$

$$C_{ij1} = s_{ij1} \{NT_{ij1} - D_{ij1}\}, \quad C_{ij2} = s_{ij2} \{NT_{ij2} - D_{ij2}\}, \quad C_{ij3} = NC_{ij3} + s_{ij3} \{NT_{ij3} - D_{ij3}\} \quad (8)$$

$$CT_{ij1} \leq D_{ij1} \leq NT_{ij1}, \quad CT_{ij2} \leq D_{ij2} \leq NT_{ij2}, \quad CT_{ij3} \leq D_{ij3} \leq NT_{ij3}$$

$$C_1 = \sum_i \sum_j C_{ij1} + I_1 * (T_{n1} - T_{11}) + \sum_n K_{n1}, \quad C_2 = \sum_i \sum_j C_{ij2} + I_2 * (T_{n2} - T_{12}) + \sum_n K_{n2}$$

$$C_3 = \sum_i \sum_j C_{ij3} + I_3 * (T_{n3} - T_{13}) + \sum_n K_{n3}$$

$$D_{ij2} - D_{ij1} \geq 0, \quad D_{ij3} - D_{ij2} \geq 0, \quad C_{ij2} - C_{ij1} \geq 0, \quad C_{ij3} - C_{ij2} \geq 0, \quad \forall (i, j) \in P$$

Where N is the optimal value of problem 7. So the optimal solution of (2) is obtained by solving problem (8).





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**Theorem 4.1 [5]:**

If  $\tilde{x}^* = (x_1^*, x_2^*, x_3^*)$  be an optimal solution of problems (6) - (8) (and hence is a lexicographic optimal solution of problem (5)), then it is also an exact optimal solution of problem (3).

**V. NUMERICAL ILLUSTRATION**

List of activities for creating a canteen in a factory is given below with other relevant details. Job A must precede all others while job G must follow and others, job can run concurrently. Table 1 represents the description of the project. In this project, time parameter and costs of the project are considered in triangular fuzzy number form. Indirect cost of the project per day is Rs.(3000, 3000, 3000). In this project, the project manager decides to complete within (51, 55, 59) days. Activities information is given in Table 2.

**Table 1 Description of the project**

Activity	Description
1 → 2 (A)	Plan approval and site preparation
1 → 3 (B)	Laying foundations
1 → 4 (C)	Raising building walls
2 → 4 (D)	Tile proofing
3 → 4 (E)	Install electricity
3 → 5 (F)	Install Plumbing
4 → 5 (G)	Connect services to finish

**Table 2 Fuzzy Data of the project**

Activity $i \rightarrow j$	Normal time $(NT_{ij})$	Crash time $(CT_{ij})$	Normal cost $(\tilde{C}_{ij})$	Slope cost $(\tilde{S}_{ij})$
A(1, 2)	(12, 14, 16)	(9,10, 11)	(9500, 10000, 10500)	(1200, 1500, 1800)
B(1, 3)	(17, 19, 21)	(15, 17, 19)	(9500, 10000, 10500)	(800, 1000, 1200)
C(1, 4)	(17, 18, 19)	(14, 15, 16)	(36000, 40000, 44000)	(1600, 1800, 2000)
D(2, 4)	(12, 15, 18)	(11, 13, 15)	(16000, 20000, 24000)	(800, 1200, 1600)
E(3, 4)	(18, 18, 18)	(15, 15, 15)	(150000, 160000, 170000)	(4700, 5000, 5300)
F(3, 5)	(17, 19, 21)	(15, 16, 17)	(20000, 21000, 22000)	(1000, 1300, 1600)
G(4, 5)	(20, 22, 24)	(18, 20, 22)	(38000, 40000, 44000)	(2000, 3000, 4000)

Using step 1 given in section 4, the Fully Fuzzy Linear Mathematical Model of the project has been formulated.



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$$\text{Min } (C_{A1} + C_{B1} + C_{C1} + C_{D1} + C_{E1} + C_{F1} + C_{G1}, C_{A2} + C_{B2} + C_{C2} + C_{D2} + C_{E2} + C_{F2} + C_{G2}, \\ C_{A3} + C_{B3} + C_{C3} + C_{D3} + C_{E3} + C_{F3} + C_{G3})$$

Subject to

$$(T_{11}, T_{12}, T_{13}) = 0$$

$$(T_{j1} - T_{i1} - D_{ij1}, T_{j2} - T_{i2} - D_{ij2}, T_{j3} - T_{i3} - D_{ij3}) \geq 0 \quad \forall i = 2, \dots, 5 \quad j = 1, 2, \dots, 4$$

$$(T_{n1}, T_{n2}, T_{n3}) \leq (T_1, T_2, T_3)$$

$$(C_{ij1}, C_{ij2}, C_{ij3}) = (s_{ij1}(NT_{ij1} - D_{ij1}), s_{ij2}(NT_{ij2} - D_{ij2}), s_{ij3}(NT_{ij3} - D_{ij3})) \quad (9)$$

$$(CT_{ij1}, CT_{ij2}, CT_{ij3}) \leq (D_{ij1}, D_{ij2}, D_{ij3}) \leq (NT_{ij1}, NT_{ij2}, NT_{ij3}) \quad \forall (i, j) \in P$$

$$(C_1, C_2, C_3) = \sum_i \sum_j ((C_{ij1} + I_1 * (T_{n1} - T_{11}) + \sum_n (K_{n1}), C_{ij2} + I_2 * (T_{n2} - T_{12}) + \sum_n (K_{n2}), \\ C_{ij3} + I_3 * (T_{n3} - T_{13}) + \sum_n (K_{n3}))$$

$$D_{ij2} - D_{ij1} \geq 0, \quad D_{ij3} - D_{ij2} \geq 0$$

$$C_{ij2} - C_{ij1} \geq 0, \quad C_{ij3} - C_{ij2} \geq 0$$

With respect to the definition 2.7 and step 4, problem (9) is converted to the Multi Objective Programming problem as follows:

$$\text{Min } C_{A2} + C_{B2} + C_{C2} + C_{D2} + C_{E2} + C_{F2} + C_{G2}$$

$$\text{Max } C_{A3} + C_{B3} + C_{C3} + C_{D3} + C_{E3} + C_{F3} + C_{G3} - (C_{A1} + C_{B1} + C_{C1} + C_{D1} + C_{E1} + C_{F1} + C_{G1})$$

$$\text{Min } C_{A3} + C_{B3} + C_{C3} + C_{D3} + C_{E3} + C_{F3} + C_{G3} + C_{A1} + C_{B1} + C_{C1} + C_{D1} + C_{E1} + C_{F1} + C_{G1}$$

Subject to

$$T_{11} = 0, \quad T_{12} = 0, \quad T_{13} = 0$$

$$T_{j1} - T_{i1} - D_{ij1} \geq 0, \quad T_{j2} - T_{i2} - D_{ij2} \geq 0, \quad T_{j3} - T_{i3} - D_{ij3} \geq 0$$

$$T_{n1} \leq T_1, \quad T_{n2} \leq T_2, \quad T_{n3} \leq T_3$$

$$C_{ij1} = s_{ij1}(NT_{ij1} - D_{ij1}), C_{ij2} = s_{ij2}(NT_{ij2} - D_{ij2}), C_{ij3} = s_{ij3}(NT_{ij3} - D_{ij3})$$

$$CT_{ij1} \leq D_{ij1} \leq NT_{ij1}, \quad CT_{ij2} \leq D_{ij2} \leq NT_{ij2}, \quad CT_{ij3} \leq D_{ij3} \leq NT_{ij3} \quad \forall (i, j) \in P \quad (10)$$

$$C_1 = \sum_i \sum_j C_{ij1} + I_1 * (T_{n1} - T_{11}) + \sum_{n=1}^5 K_{n1},$$

$$C_2 = \sum_i \sum_j C_{ij2} + I_2 * (T_{n2} - T_{12}) + \sum_{n=1}^5 K_{n2},$$

$$C_3 = \sum_i \sum_j C_{ij3} + I_3 * (T_{n3} - T_{13}) + \sum_{n=1}^5 K_{n3}$$

$$D_{ij2} - D_{ij1} \geq 0, \quad D_{ij3} - D_{ij2} \geq 0, \quad C_{ij2} - C_{ij1} \geq 0, \quad C_{ij3} - C_{ij2} \geq 0, \quad \forall i = 2, \dots, 5 \quad j = 1, 2, \dots, 4$$

Using step 5, 6 and 7, the optimal solution of problem (10) is achieved. The following table shows that the obtained crashing cost for each activity of the project.



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Table 3 Crashing cost for each activity

Activity	Project Duration	Crashing Cost
A	(12, 14, 16)	(0, 0, 0)
B	(15, 17, 19)	(1600, 2000, 2400)
C	(17, 18, 19)	(0, 0, 0)
D	(12, 15, 18)	(0, 0, 0)
E	(18, 18, 18)	(0,0,0)
F	(17, 19, 21)	(0, 0, 0)
G	(18, 20, 22)	(2400, 3000, 3600)

The values of minimum fuzzy total cost and planned fuzzy duration of the project have been determined using LINGO solver. A computer package called LINGO (LINGO 2000) is used on a personal computer to solve the mathematical model of the example project. LINGO is a commercial package using the power of linear and non-linear optimization to formulate large problems concisely, solve them, and analyze the solution. In all tested runs, the linear mathematical model of the example project requires less than one second on LINGO to obtain the optimal solution.

The optimum crash cost for each activity of the sample project is presented in Table 3. The optimal fuzzy total cost of the project is (436000, 471000, 506000) at (51, 55, 59) days.

## VI. CONCLUSION

In this paper, an algorithm has been presented to solve the Fully Fuzzy Time Cost Trade Off problem. Based on a lexicographic ordering on triangular fuzzy numbers, the Fully Fuzzy Linear Programming problem is converted to its equivalent Multi Objective Linear Programming Problem. By a simple example, the obtained result of presented algorithm has been compared with the method presented by Evangeline [6]. We conclude that the results coincide with each other. As an extension of this study, interesting problems existing under Fuzzy Time Cost Trade off Problems and Fuzzy Time Cost and Quality Trade off Problems by using Goal Programming technique.

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#### AUTHOR BIOGRAPHY



**M. Evangeline Jebaseeli** is working as an Assistant Professor in Mathematics, Bishop Heber College and currently pursuing Ph. D (Part-Time) in Mathematics under the supervision of Dr. D. Paul Dhayabaran. She has completed M. Phil. (Mathematics) from Bishop Heber College (Bharathidasan University), Tiruchirappalli, Tamil Nadu. Her area of research is Fuzzy Network Crashing.



**Dr. D. Paul Dhayabaran** is a Principal, Bishop Heber College, Tiruchirappalli, Tamil Nadu. He received the Ph. D. degree in Mathematics from Bharathidasan University. He served as an Associate Professor in Mathematics, Bishop Heber College for 24 years and served as an Assistant Professor in Mathematics at BITS PILANI, Dubai, UAE for seven semesters. He has published 20 papers in International Journals and has attended 8 workshops and conferences. His current area of research is Graph Theory, Fuzzy Network Crashing and Fuzzy Differential Equations.