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On Rg-Compact Spaces

A. M. Al-Shibani[1] and S. M. Al-Areefi[2]
Department of Mathematics, College of science
University of Dammam, Dammam, Saudi Arabia

Abstract: ***The purpose of this paper is to offer some more properties of rg-compact spaces such as the finite union of rg-compact sets, and then we studied its connection with rg-closed sets and rg-Hausdorff spaces.***

Key Words: **rg-compact spces, rg-Hausdorff spaces, rg-closed sets.**

I. INTRODUCTION

Regular generalized closed sets have been introduced and studied by Palaniappar and Chandra [3], Chawlit [2] introduced the concept of rg-Hausdroff spaces, Al-Shibani [1] introduced and investigated rg-compact spaces. Throughout this paper (X,τ)(or simply X) represents a topological space. For any subset $A$of X, cl $A$and int Adenote the closure of $A$and the interior of A, respectively.

Defnition 1 A subset A of X is said to be:

1. Regular open (briefly, r-open) (resp. regular closed (briefly, r-closed)) if A= int(cl A) (resp. A= cl(int A)) [4].
2. rg-closed if cl $A \subseteq U$ whenever $A \subseteq U$, where U is r-open.

It is said to be rg-open if $X \setminus A$ is rg-closed (equivalently $F \subseteq$ int Awhenever $F \subseteq A$ and F is r-closed) [3].

Remark 1 It is easy to see that
r-open$\rightarrow$ open$\rightarrow$ rg-open

Defnition 2 A topological space X is said to be rg-Hausdorff if whenever x and y are distinct points of$X$ there are disjoint rg-open sets U and V with $x \in U$ and $y \in V$ [2].

Remark 2 It is easy to see that
Hausdorff $\rightarrow$ rg-Hausdorff.

Defnition 3 A topological space X is called regular generalized compact (briefly, rg-compact) if every rg-open cover of$X$ has a finite subcover [1].

Remark 3 It is easy to see that
rg -compact $\rightarrow$ compact.

Example 1 Every finite subset of a topological space is rg-compact.
Proof. Let $A=\{a_1,\dots,a_n\}$be finite subset of a topological space X, let $\{U_\alpha\}$be rg-open cover of A which means $A \subseteq \bigcup_{\alpha\in I} U_\alpha$.

Then each element of A belongs to at least one of the sets $\{U_\alpha\}$, hence $a_1 \in U_{\alpha_1},\dots,a_n \in U_{\alpha_n}$

Thus $A \subseteq \bigcup_{i=1}^{n} U_{\alpha_i}$

Which is a finite subcover for A. So $A$is rg-compact.■

Theorem 1 Let A, B be rg-compact subsets of a topological space X.
Then $A \cup B$ is rg-compact.

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Proof. Let $\{U_\alpha\}_{\alpha\in I}$ be rg-open cover of $A\cup B$, i.e. $A\cup B\subseteq \bigcup_{\alpha\in I} U_\alpha$

So $A\subseteq \bigcup_{\alpha\in I} U_\alpha$, $B\subseteq \bigcup_{\alpha\in I} U_\alpha$.

Since A and B are rg-compact, so we can choose finite subcover, say $A\subseteq U_{\alpha_1}\cup,\dots,\cup U_{\alpha_m}, B\subseteq U_{\alpha_{m+1}}\cup\dots\cup U_{\alpha_{m+k}}$

Hence $A\bigcup B\subseteq \bigcup_{i=1}^{m+k} U_{\alpha_i}$.

Therefor $A\cup B$ is rg-compact. ■

Corollary 1 Any finite union of rg-compact sets are rg-compact.

Theorem 2 Each rg-compact subset of a rg-Hausdorff space is rg-closed.

Proof. Let A be rg-compact subset of a Hausdorff space X. We will prove that $X\backslash A$ is rg-open and hence A is rg-closed.

Let $x\in X\backslash A$, for each $y\in A$ their exists disjoint rg-open sets U_y and V_x with $y\in U_y$ and $x\in V_x$.

The collection $\{U_y: y\in A\}$ is rg-open cover of A.

Since A is rg-compact so it has a finite subcover $\{U_{y_i}\}_{i=1}^{n}$, $A\subseteq \bigcup_{i=1}^{n} U_{y_i}$.

Put $U=\bigcup_{i=1}^{n} U_{y_i}$, $V=\bigcup_{i=1}^{n} V_{x_i}$.

So U and V are dijoint, V is rg-open set, also $x\in V$ and $A\cap V=\emptyset$.
So $x\in V\subseteq X\backslash A$.
Therefore $X\backslash A$ is rg-open set, hence A is rg-closed set. ■

Theorem 3 Let X be rg-compact, rg-Hausdorff space, A subset A of X is rg-compact if and only if it is rg-closed.

Proof. Obvious from Theorem (2) and Theorem 4.2 in [3]. ■

Theorem 4 If A is rg-compact, F is rg-closed subset of a topological space X, then $A\cap F$ is rg-compact.
Proof. Let $\{U_\alpha\}$ be rg-open cover of $A\cap F$, i.e. $A\cap F\subseteq \bigcup_{\alpha\in I} U_\alpha$, then $A\subseteq \bigcup_{\alpha\in I} U_\alpha \bigcup (X\setminus F)$

Since F is rg-closed subset, so $X\backslash F$ is rg-open subset, therefore $\{\{U_\alpha\}, X\backslash F\}$ is rg-open cover of A. Since A is rg-compact, so it has a finite subcover for A, that is $A\subseteq U_{\alpha_1}\cup\dots\cup U_{\alpha_n}\cup X\backslash F$, Thus we have $A\cap F\subseteq U_{\alpha_1}\cup\dots\cup U_{\alpha_n}$, therefor $A\cap F$ is rg-compact. ■

Theorem 5 The following conditions are equivalent:
1. X is rg-compact
2. Every family $\{F_\alpha\}$ where I is index set of rg-closed subsets of X with $\bigcap_{\alpha\in I}^{\alpha\in I} F_\alpha=\phi$, contains a finite subsets $\{F_{\alpha_1},\dots,F_{\alpha_k}\}$ such that $\bigcap_{i=1}^{k} F_{\alpha_i}=\phi$.

Proof. 1. →2. Let $\bigcap_{\alpha\in I} F_\alpha=\phi$, then by taking complement

$$X\setminus \bigcap_{\alpha\in I} F_\alpha = X$$
$$\Rightarrow \bigcup_{\alpha\in I}(X\setminus F_\alpha)=X$$

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So $\{X \backslash F_\alpha\}_{\alpha \in I}$ is rg-open cover of X, since X is rg-compact, it has a finite subcover, thus

$$X = \bigcup_{i=1}^{k} \left(X \setminus F_{\alpha_i} \right)$$

Now taking complement

$$\phi = X / \bigcup_{i=1}^{k} \left(X \setminus F_{\alpha_i} \right)$$

$$= \bigcap_{i=1}^{k} F_{\alpha_i}$$

2. →1. Let $\{U_\alpha\}_{\alpha \in I}$be rg-open cover of X.

i.e. $X = \bigcup_{\alpha \in I} U_\alpha$, so $X \backslash U_\alpha,\ \alpha \in I$ are rg-closed subsets which have an emptyintersection.

By hypothes it contains a finite subset $\{X \backslash U_{\alpha_i}, \dots, X \backslash U_{\alpha_k}\}$such that $\bigcap_{i=1}^{k} \left(X \setminus U_{\alpha_i} \right) = \phi.$

By taking complement

$$X \setminus \left(\bigcap_{i=1}^{k} X U_{\alpha_i} \right) = X \setminus \phi$$

$$\Rightarrow \bigcup_{i=1}^{k} U_{\alpha_i} = X$$

Thus X is rg-compact. ■

Theorem 6 A topological space X is rg-compact if and only if every family of rg-closed subset of X which has the finite intersection property, has a nonempty intersection.

Proof. From the previous theorem we can take the negation of 2 i.e. for every rg-closed sets $F_\alpha,\ \alpha \in I$ such that $\bigcap_{i=1}^{k} F_{\alpha_i} \neq \phi$ implies $\bigcap_{\alpha \in I} F_\alpha \neq \phi.$

So the proof is completed. ■

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