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Optimal Design of Digital Differentiator Using Simulated Annealing

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Abstract: - In this paper, optimal design of digital differentiator is presented. Optimal coefficients of digital differentiators are calculated based on the globally optimized techniques. Many optimization techniques exist; we are using simulating annealing (SA) optimization to get the optimized differentiator coefficients. Simulated annealing (SA) is a random-search technique which exploits an analogy between the way in which a metal cools and freezes into a minimum energy crystalline structure and the search for a minimum in a more general system. The performance of the designed differentiator is evaluated based on different parameters such as magnitude response, phase response, absolute magnitude error and relative magnitude error. In this paper, we are using absolute magnitude error as the control parameter for the optimal design of digital differentiators. The results show that the response of the design differentiator will perform better than the other existing differentiator over the whole frequency range. Simulation result also shows that the designed differentiator is outperforms the all existing digital differentiator in terms of magnitude error but only in a particular frequency range. Result also reveals that the performance of the differentiator improves as the order of the differentiator increases.

***Keywords:-*Digital differentiator, absolute magnitude error (AME), Simulating Annealing (SA), optimization techniques, digital filter.**

I. INTRODUCTION

Digital differentiator finds a wide range of application in different area of engineering such as control, biomedical engineering and radar processing. It also finds application in the field o image processing with 2-D digital differentiator. The frequency response of an ideal digital differentiator is

$$H_{diff}(\omega) = j\omega \qquad (1)$$

Where $j = \sqrt{-1}$ and ω is the angular frequency in radians.

In analogue signal processing, differentiators are often obtained by inverting the transfer functions of analogue integrators [2]-[4]. The concept can be extended to digital differentiators. The new approach can be broken down into the following steps: (1) obtain or design an integrator that has the same range and accuracy as the desired differentiator; (2) invert the transfer function of the integrator; (3) stabilise the transfer function by reflecting the poles that lie outside the unit circle to inside the unit circle; (4) compensate the magnitude appropriately by noting that if a pole that lies at a radius *r* is replaced by a pole that lies at a radius of l/r, the magnitude of the resulting transfer function will be multiplied by r; thus to compensate for the resulting change in magnitude, the resulting transfer function should be multiplied by l / r .

Different approaches for the design of digital integrators are as follows simple linear interpolation between the magnitude responses of the classical rectangular, trapezoidal and Simpson digital integrators [5] [10], linear programming optimisation approach [6], optimising the pole-zero locations [7]-[8], Simulated Annealing [9], Genetic Algorithms [9], and Fletcher and Powell Optimization [9].

Simulated annealing is a popular local search meta-heuristic used to address discrete and, to a lesser extent, continuous optimization problems. The key feature of simulated annealing is that it provides a means to escape local optima by allowing hill-climbing moves (i.e., moves which worsen the objective function value) in hopes of finding a global optimum. At each iteration of a simulated annealing algorithm applied to a discrete optimization problem, the objective function generates values for two solutions (the current solution and a newly selected solution) are compared. Improving solutions are always accepted; while a fraction of non-improving (inferior) solutions are accepted in the hope of escaping local optima in search of global optima. The probability of accepting non-improving solutions depends on a temperature parameter, which is typically non-increasing with each iteration of the algorithm.

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The rest of the paper is organized as follows: In section II, explain the simulating annealing algorithm applied to a discrete optimization problem, the objective function generates values for two solutions and the flowchart of SA algorithm is given for filter approximation.. In Section III, implementation of simulating annealing algorithm is presented. Section IV explains designed digital differentiator based on the simulating annealing algorithm. In Section V, shows the experimental results to analysis the performance of the differentiator in terms of Absolute magnitude error (AME). Finally, a conclusion is made.

II. SIMULATED ANNEALING ALGORITHM

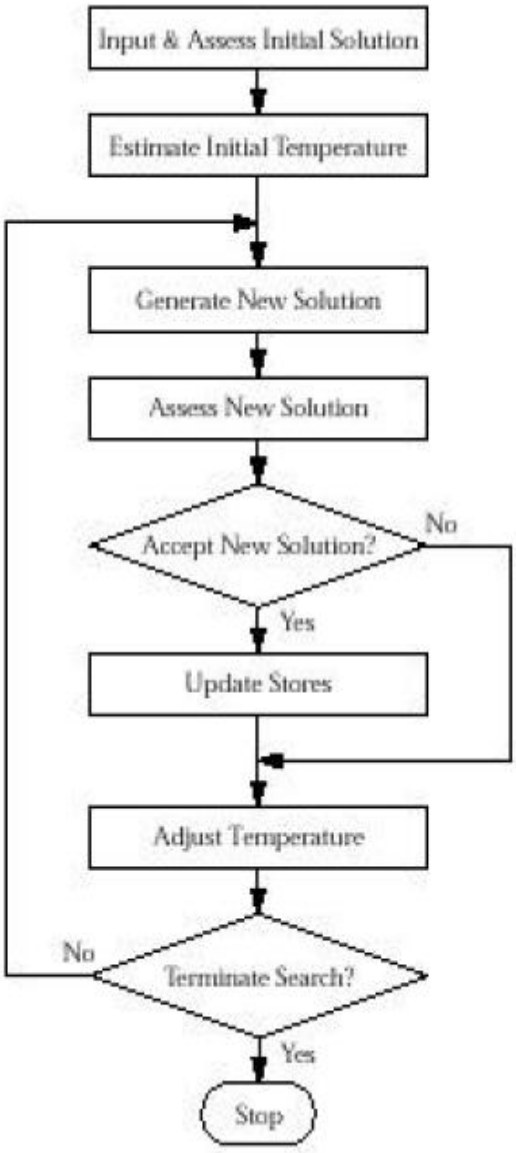

Fig. 1 Flow chart of simulated annealing algorithm

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Simulated annealing [11] is a method for solving unconstrained and bound-constrained optimization problems. The simulated annealing algorithm [12] was originally inspired from the process of annealing in metal work. Annealing involves heating and cooling a material to alter its physical properties due to the changes in its internal structure. As the metal cools its new structure becomes fixed, consequently causing the metal to retain its newly obtained properties. In simulated annealing we keep a temperature variable to simulate this heating process. We initially set it high and then allow it to slowly 'cool' as the algorithm runs. While this temperature variable is high the algorithm will be allowed, with more frequency, to accept solutions that are worse than our current solution. This gives the algorithm the ability to jump out of any local optimums it finds itself in early on in execution. As the temperature is reduced so is the chance of accepting worse solutions, therefore allowing the algorithm to gradually focus in on a area of the search space in which hopefully, a close to optimum solution can be found. This gradual 'cooling' process is what makes the simulated annealing algorithm remarkably effective at finding a close to optimum solution when dealing with large problems which contain numerous local optimums [13]. SA's major advantage over other methods is an ability to avoid becoming trapped in local minima. The algorithm employs a random search which not only accepts changes that decrease the objective function f (assuming a minimisation problem), but also some changes that increase it. The latter are accepted with a probability

$$p = \exp (-\delta f / T) \quad (2)$$

where df is the increase in f and T is a control parameter, which by analogy with the original application is known as the system "temperature" irrespective of the objective function involved. The implementation of the basic SA algorithm is straightforward.

The following elements must be provided:

1. a representation of possible solutions
2. a generator of random changes in solutions
3. a means of evaluating the problem functions and
4. an annealing schedule - an initial temperature and rules for lowering it as the search progresses.

III. IMPLEMENTATION OF SA ALGORITHM

Solution Representation and Generation: When attempting to solve an optimisation problem using the SA algorithm, the most obvious representation of the control variables is usually appropriate. However, the way in which new solutions are generated may need some thought. The solution generator should introduce small random changes, and allow all possible solutions to be reached.

Solution Evaluation: The SA algorithm does not require or deduce derivative information; it merely needs to be supplied with an objective function for each trial solution it generates. Thus, the evaluation of the problem functions is essentially a "black box" operation as far as the optimisation algorithm is concerned. Obviously, in the interests of overall computational efficiency, it is important that the problem function evaluations should be performed efficiently; especially as in many applications these function evaluations are by far the most computationally intensive activity. Some thought needs to be given to the handling of *constraints* when using the SA algorithm. In many cases the routine can simply be programmed to reject any proposed changes which result in constraint violation, so that a search of feasible space only is executed.

Annealing schedule: Annealing schedule determines the degree of uphill movement permitted during the search and is thus critical to the algorithm's performance. The principle underlying the choice of a suitable annealing schedule is easily stated: the initial temperature should be high enough to "melt" the system completely and should be reduced towards its "freezing point" as the search progresses. Choosing an annealing schedule for practical purposes is something of an art.

Control parameters: In SA algorithm homogeneous Markov chains of finite length are generated at decreasing temperatures. The following parameters should therefore be specified:

1. an initial temperature T_0
2. a final temperature T_f or a stopping criterion
3. a length for the Markov chains and
4. a rule for decrementing the temperature.

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Initial Temperature: A suitable initial temperature T0 is one that results in an average increase of acceptance probability p0 of about 0.8. In other words, there is an 80% chance that a change which increases the objective function will be accepted. The value of T0 will clearly depend on the scaling of f and, hence, be problem-specific. It can be estimated by conducting an initial search in which all increases are accepted and calculating the average objective increase observed df^+.

Final Temperature: In some simple implementations of the SA algorithm the final temperature is determined by fixing

1. The number of temperature values to be used, or
2. The total number of solutions to be generated.

Length of Markov Chains: An obvious choice for L_k, the length of the k-th Markov chain, is a value that depends on the size of the problem, so L_k is independent of k. Alternatively it can be argued that a minimum number of transitions Nmin should be accepted at each temperature. However, as T_k approaches 0, transitions are accepted with decreasing probability so the number of trials required to achieve N_{min} acceptances approaches 1 . Thus, in practice, an algorithm in which each Markov chain is terminated after

1. L_k transitions or
2. N_{min} acceptances,

IV. PROPOSED DIFFERENTIATOR

Several methods have been used for their design. Proposed second order differentiator is obtained as

$$H_{Diff\_SA\_2}(z)=\frac{1.1538-0.5408z^{-1}-0.613z^{-2}}{1+0.7121z^{-1}+0.067z^{-2}} \quad (3)$$

For the third order differentiator, the zeros and poles of the second order differentiator are used as starting points. Thus, the differentiator becomes

$$H_{Diff\_SA\_3}(z)=\frac{1.1555-0.3582z^{-1}-0.714z^{-2}-0.0833z^{-3}}{1+0.8662z^{-1}+0.1612z^{-2}+0.0028z^{-3}} \quad (4)$$

The fourth order differentiator was built upon using the third order

$$H_{Diff\_SA\_4}(z)=\frac{1.1540+0.2290z^{-1}-0.8794z^{-2}-0.4486z^{-3}-0.0549z^{-4}}{1+1.3788z^{-1}+0.623z^{-2}+0.1059z^{-3}+0.0059z^{-4}} \quad (5)$$

V. SIMULATION RESULTS

The optimal filter coefficient of IIR differentiators is obtained using simulated annealing (SA) algorithm. Simulation has been done in MATLAB and optimization toolbox of matlab. In this section, we are taking the absolute magnitude error for the analysis of the designed differentiator. Fig. 2 shows the Absolute magnitude error for optimized Al-Alaoui first order differentiator. Fig. 3 depicts the comparative analysis of Absolute magnitude error for optimized Al-Alaoui first order and Simulating Annealing Algorithm optimized second order differentiator. Fig. 4 demonstrate the comparative analysis of Absolute magnitude error for optimized Al-Alaoui first order, Simulating Annealing Algorithm optimized second order and Simulating Annealing Algorithm optimized third order differentiator. Fig.5 shows the comparative analysis of Absolute magnitude error for optimized Al-Alaoui first order, Simulating Annealing Algorithm optimized second order, Simulating Annealing Algorithm optimized third order and Simulating Annealing Algorithm optimized fourth order differentiator. Fig.6 shows the comparative analysis of Absolute magnitude error for optimized Al-Alaoui first order, Simulating Annealing Algorithm optimized second order, Simulating Annealing Algorithm optimized third order, Simulating Annealing Algorithm optimized fourth order differentiator and existing differentiator.

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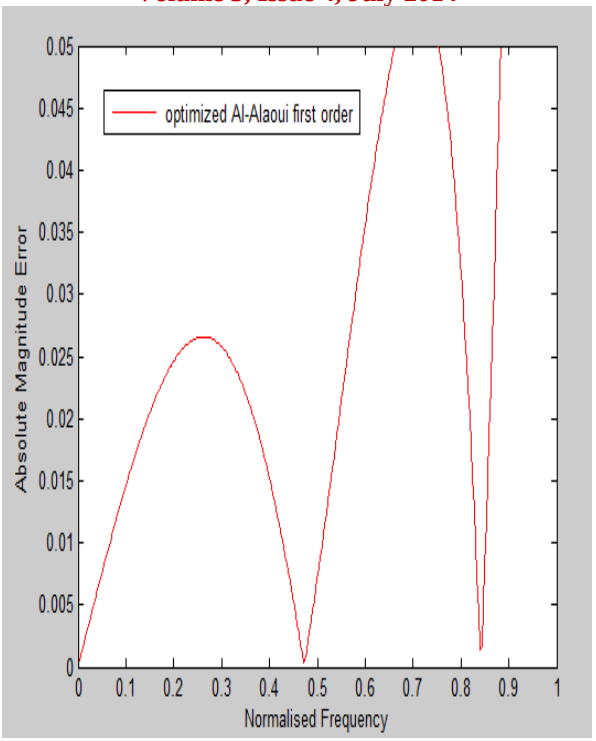

Fig.2 Absolute magnitude error for optimized Al-Alaoui first order differentiator

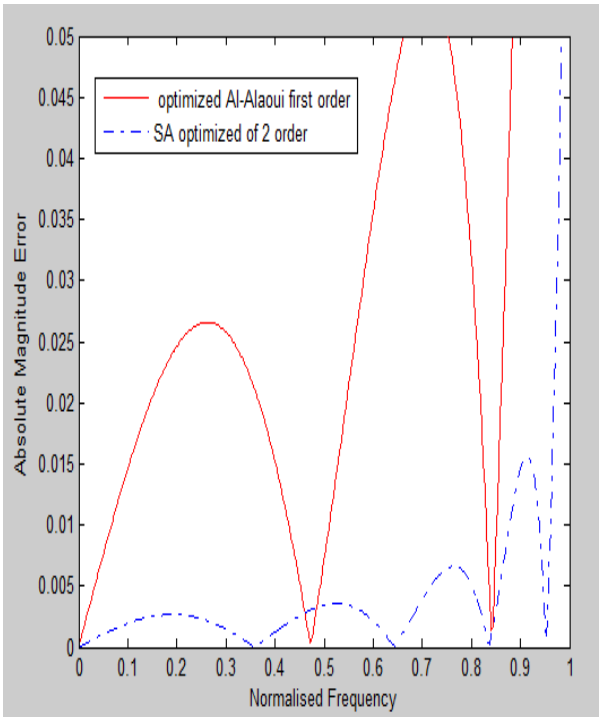

Fig.3 comparative analysis of Absolute magnitude error for optimized Al-Alaoui first order and Simulating Annealing Algorithm optimized second order differentiator

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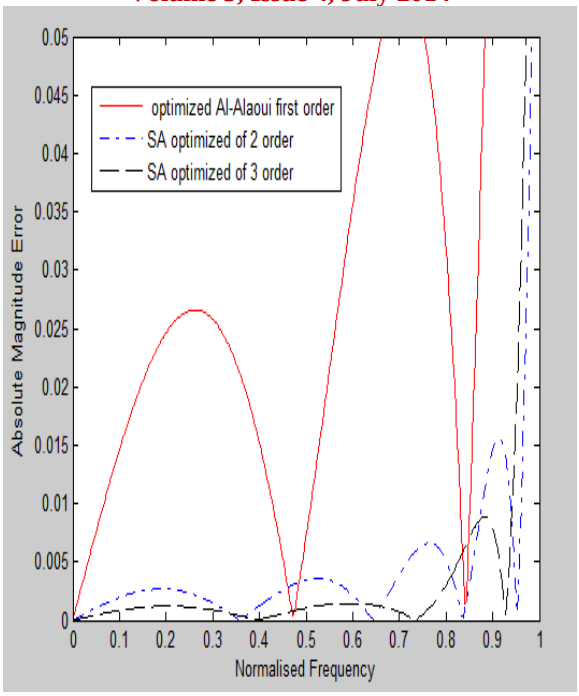

Fig.4 comparative analysis of Absolute magnitude error for optimized Al-Alaoui first order, Simulating Annealing Algorithm optimized second order and Simulating Annealing Algorithm optimized third order differentiator

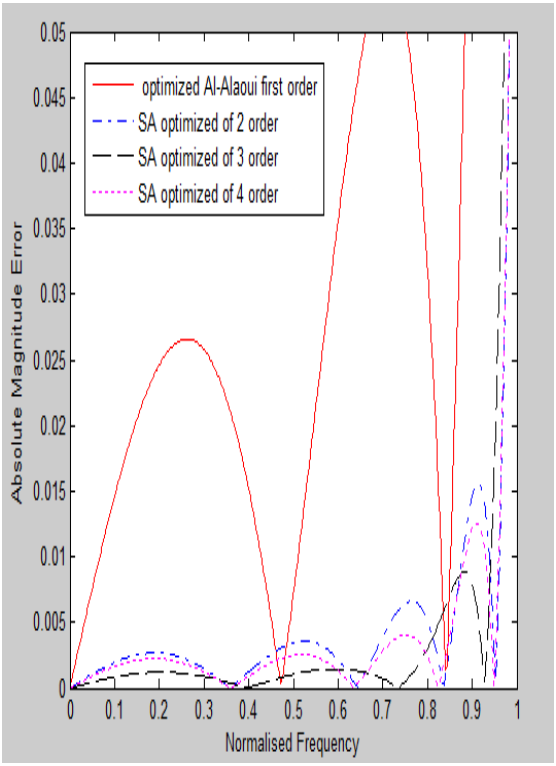

Fig.5 comparative analysis of Absolute magnitude error for optimized Al-Alaoui first order, Simulating Annealing Algorithm optimized second order, Simulating Annealing Algorithm optimized third order and Simulating Annealing Algorithm optimized fourth order differentiator

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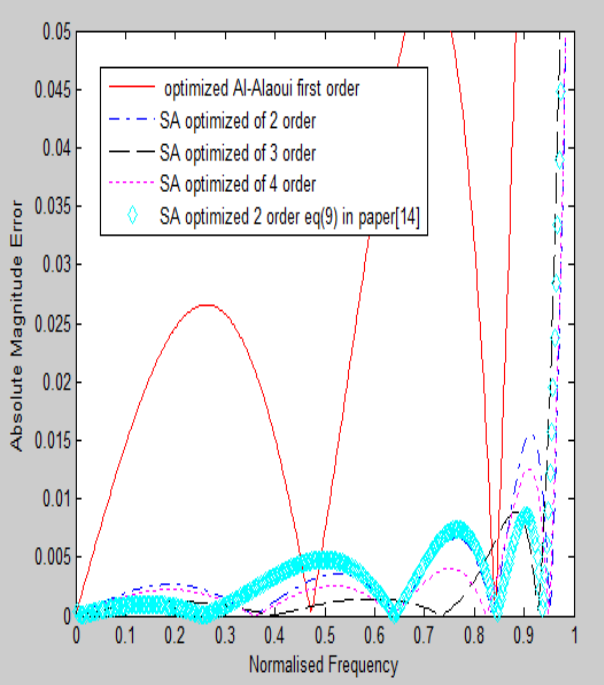

Fig.6 comparative analysis of Absolute magnitude error for optimized Al-Alaoui first order, Simulating Annealing Algorithm optimized second order, Simulating Annealing Algorithm optimized third order, Simulating Annealing Algorithm optimized fourth order differentiator and existing differentiator

VI. CONCLUSIONS

In this paper, optimal design of digital differentiator is presented. Optimal coefficients of digital differentiators are calculated based on the globally optimized techniques. Many optimization techniques exist; we are using simulating annealing (SA) optimization to get the optimized differentiator coefficients. In this paper, we are using absolute magnitude error as the control parameter for the optimal design of digital differentiators. Simulation result also shows that the designed differentiator is outperforms the all existing digital differentiator in terms of magnitude error but only in a particular frequency range. Result also reveals that the performance of the differentiator improves as the order of the differentiator increases. Results show the effectiveness of the proposed differentiator.

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