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High shear capillary viscometer with continuously varying pressure drop: Fast rheological test of coal-water slurry (CWS)

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Abstract*— *The pressure-driven capillary viscometer, recently designed by one of the authors has been modified to test the rheological behavior of CWS. Operation principles involve the detection of the continuous varying pressure drop across a capillary tube to provide a wide shear rate range during a single run. The flow curve is determined from the non-linear fitting of the derived analytical expression for the pressure drop as a function of time to measured data. It is found that for coal-loading up to 40 wt. %, CWS behaves as a Newtonian fluid, while above 50 wt. % of coal the CWS exhibits a non-Newtonian pseudo plastic behavior with yield stress, obeying the Bingham-plastic model.

***Index Terms*—Capillary viscometer, Coal water slurry, Viscosity function, Bingham-Plastic model.**

I. INTRODUCTION

There has been considerable activity in recent years, concerning the production and research of coal-water slurries (CWS) as an alternative fuel due to its lower coast and similarity to oil in several industrial applications [1, 2]. The rheological properties of CWS play a critical role in slurry processing and flow equipment design. Depending on the coal content, type and size of coal particles, chemical additives, CWS exhibits the regions of Newtonian, dilatation, pseudo-plastic behavior and most often, yield stress [3]. Typical CWS containing $50-75$ wt. % of coal and ≤ 1 wt. % chemical additives (surfactants) have an apparent viscosity $\eta_a = 0.8 \div 1.2\ \mathrm{Pa \cdot s}$ [4]. It is often required to prepare CWS [5] with a high coal content and low viscosity to improve its heating values and atomization.

Although there are many methods and instruments to test the rheological behavior of fuels, most current techniques can only operate at a single or in a limited range of shear rate, while rheological characterization of CWS requires a wide range of shear rate [5, 6]. One of the most popular techniques is capillary viscometer [7], in which the viscosity η is calculated from experimental data involving the pressure drop, Δp and flow rate Q in capillary tube of radius R and length $L >> R$. The shear stress σ at a distance r from the axis of capillary is evaluated from the equation:

$$\sigma(r) = \Delta p \frac{r}{2L} = \sigma_w \frac{r}{R} \tag{1}$$

where $\sigma_w = (R/2L)\Delta p$ is the wall shear stress. The shear rate $\dot{\gamma}(r) = -\partial \upsilon / \partial r$, where $\upsilon(r)$ is the velocity profile of the fluid, is related to the flow rate Q by the expression:

$$Q = \pi \int_R^0 r^2 d\upsilon = \pi \int_0^R \dot{\gamma}(r) r^2 dr \tag{2}$$

which involves the non-slip boundary condition, $\partial \upsilon / \partial r \big|_{r=R} = 0$ for the laminar steady state flow. Using (1), we can change the variable r to σ in (2) to obtain the relationship for the apparent shear rate:

$$\dot{\gamma}_a = \frac{4Q}{\pi R^3} = \frac{4}{\sigma_w^3} \int_0^{\sigma_w} \dot{\gamma}(\sigma) \sigma^2 d\sigma \tag{3}$$

This equation may be differentiated with respect to σ_w and rearranged to obtain the Weissenberg-Rabinowitsch-Mooney equation [8]:

$$\dot{\gamma}_w = \frac{\dot{\gamma}_a}{4}\left(3+\frac{d\ln\dot{\gamma}_a}{d\ln\sigma_w}\right) \tag{4}$$

Where $d\ln\dot{\gamma}_a / d\ln\sigma_w$ is the shear thinning index, which is unit for Newtonian fluids and may be constant for several non-Newtonian fluids over wide range of shear rate.

Thus, if σ_w and Q are defined, the viscosity function, $\eta = \sigma_w / \dot{\gamma}_w$ is calculated as:

$$\eta = \pi R^3 \frac{\sigma_w}{Q}\left(3+\frac{d\ln Q}{d\ln\sigma_w}\right)^{-1} \tag{5}$$

Effective supply of CWS to burner chamber requires understanding of its behavior in cases with high flow rate and consequently high shear rate (more than in usually used **HAAKE** rotational rheometer). The scarcity of data concerning the behavior of CWS with high flow rate supply, calls for the conduction and analysis of related experiments. In this context, the behavior of CWS has not yet been clearly demonstrated. For example, this concerns to coal concentrations role in the mentioned systems.

In the present study, within the above theoretical framework, we modify the operation principles of the pressure-driven capillary viscometer, recently designed by one of the authors [9], to test the rheological behavior of CWS.

II. THEORETICAL BACKGROUND

Consider two vessels coupled at the bottom via a valve by horizontal capillary of radius R and length $L >> R$ as is shown in Fig. 1. One vessel with the test liquid, open to air is under the atmospheric pressure p_{atm}, the other, a vacuum vessel of volume V_0 is under pressure $p_0 < p_{atm}$. If at time zero we open the valve, the test liquid from the sample vessel starts to flow through the capillary into a vacuum vessel. For this flow situation to determine the volumetric flow rate we'll consider an incompressible fluid with a viscosity independent of pressure. The flow is assumed a fully developed, steady, isothermal and laminar with no slip at the walls and no velocity in the radial direction.

During the fluid flows, the pressure difference, $\Delta p = p_{atm} - p(t)$, where $p(t)$ is the pressure in the vacuum vessel at time t, which gradually increases from the initial value p_0 to end equilibrium value $p(\infty)$, at which the flow stops. Let us denote by $V(t)$ the volume of air in the vacuum vessel at time t. Assuming that air is an ideal gas, for which the pressure-volume product, at any time t is constant, one uses the relation $p_0V_0 = p(t)V(t)$. Since the change of air volume in a vacuum vessel at any time t, $\delta V = V_0 - V(t)$ is equal to the fluid volume passed through the capillary tube into the vacuum vessel, the volumetric flow rate $Q(t)$ can be calculated as:

$$Q(t) = \frac{d}{dt}\delta V(t) = -\frac{dV(t)}{dt} = -p_0V_0\frac{d}{dt}\left(\frac{1}{p(t)}\right) = \frac{p_0V_0}{p^2}\frac{dp(t)}{dt} \tag{6}$$

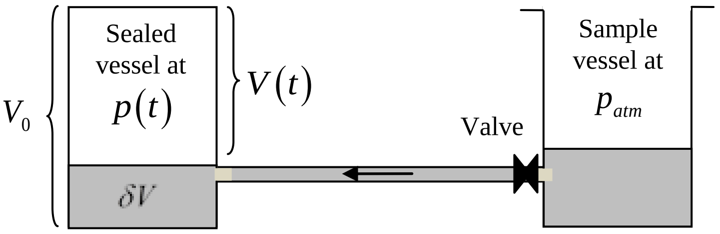


Fig. 1. Schematic diagram of the transient discharge device

Numerous experimental studies [10] suggest a description of slurry's rheology by the Bingham-plastic constitutive equation:

$$\sigma = \sigma_c + \eta_\infty \dot{\gamma} \tag{7}$$

where σ is the applied shear stress, $\dot{\gamma}$ is the maintained shear rate, and the yield stress σ_c and the plastic viscosity η_∞ are two empirical constants independent on the shear rate. The yield stress σ_c can be determined at low (zero) shear rates, from the final pressure difference Δp_∞ as:

$$\sigma_c = \frac{R}{2L}\Delta p_\infty \tag{8}$$

The Bingham-plastic viscosity η_∞ characterizes the fluid viscosity at high shear rates and can be calculated as follows. Using (7), we can express $\dot{\gamma}$ through σ to substitute in (3) and obtain after integration the expression for the volumetric flow rate:

$$Q = \frac{\pi R^4}{8\eta_\infty L}\Delta p\left(1 - \frac{4}{3}\frac{\Delta p_\infty}{\Delta p} + \frac{1}{3}\left(\frac{\Delta p_\infty}{\Delta p}\right)^4\right) \tag{9}$$

At high shear, or more specifically, at $\Delta p_\infty / \Delta p < 0.5$, the effect of the last term in (9) becomes negligible in which case, we can omit it and combine with (6) to obtain the differential equation for $p(t)$:

$$\frac{dp(t)}{dt} = \frac{\pi R^4}{8\eta_\infty L p_0 V_0} p^2(t)\left[p_{atm} - \frac{4}{3}\Delta p_\infty - p(t)\right] \tag{10}$$

Introducing the dimensionless variables:

$$\tilde{p}(t) = p(t)/p_\infty \quad \text{and} \quad \tilde{t} = \beta t \quad \text{with} \quad p_\infty = p_{atm} - \frac{4}{3}\Delta p_\infty \quad \text{and} \quad \beta = \frac{\pi R^4 p_0}{8\eta_\infty L V_0}(1+\alpha)^2 \tag{11}$$

We obtain (10) in the dimensionless form:

$$\frac{d\tilde{p}}{d\tilde{t}} = -\tilde{p}^2(\tilde{p} - 1) \tag{12}$$

Which can be integrated with the initial condition $\tilde{p}(0) = \tilde{p}_0$ to give:

$$\frac{1}{\tilde{p}} + \ln\left(\frac{1}{\tilde{p}} - 1\right) = -\tilde{t} + \frac{1}{\tilde{p}_0} + \ln\left(\frac{1}{\tilde{p}_0} - 1\right) \tag{13}$$

The solution of (13) explicitly for $\tilde{p}(\tilde{t})$ can be written in the terms of the Lambert-W function [11]:

$$\tilde{p}(\tilde{t}) = \frac{1}{1 + W\left(\alpha e^{\alpha - \tilde{t}}\right)} \tag{14}$$

where $\alpha = 1/\tilde{p}_0 - 1$, and $\mathrm{W}(z)$ is the Lambert-W function defined as the inverse function associated with the solution of the functional equation $\mathrm{W}(z)e^{\mathrm{W}(z)} = z$ implemented in many computer-algebra systems such as Mathematical, Maple or Macsyma. Since $0 < \tilde{p}_0 \le \tilde{p}(t) < 1$, the principal branch of the W-function should be used [11]. In rescale variables, $\alpha = p_\infty / p_0 - 1$ and (14) reads:

$$p(t) = \frac{p_\infty}{1 + \mathrm{W}\left(\alpha e^{\alpha - \beta t}\right)} \tag{15}$$

which links the Bingham Plastic model with the temporal variation of pressure in the vacuum vessel. Equation (15) satisfies the initial condition $p(0)=p_0$ and provides the limit, $p(t\to\infty)=p_\infty$ as per the Lambert W-function definition. Parameters α and β can be determined from the nonlinear curve fitting of experimental data $p(t)$ to (15). Then, knowing instrumental parameters of the system, the Bingham-Plastic viscosity can be calculated from (11) as:

$$\eta_\infty=\frac{\pi R^4 p_0}{8\beta L V_0}(1+\alpha)^2 \qquad (16)$$

Taking into account (16) the apparent shear rate can be calculated from (9) as:

$$\dot{\gamma}_a=\frac{4Q}{\pi R^3}=\frac{4\beta V_0\Delta p(t)}{\pi R^3 p_0(1+\alpha)^2}\left[1-\frac{4}{3}\frac{\Delta p_\infty}{\Delta p(t)}+\frac{1}{3}\left(\frac{\Delta p_\infty}{\Delta p(t)}\right)^4\right] \qquad (17)$$

Substituting (17) in (4) yieds the expression for the wall shear rate $\dot{\gamma}_w(t)$

$$\dot{\gamma}_w=\frac{4\beta V_0\Delta p(t)}{\pi R^3 p_0(1+\alpha)^2}\left[1-\frac{\Delta p_\infty}{\Delta p(t)}\right] \qquad (18)$$

The viscosity function is calculated from its definition, $\eta(\dot{\gamma}_w)=\sigma_w/\dot{\gamma}_w$ as:

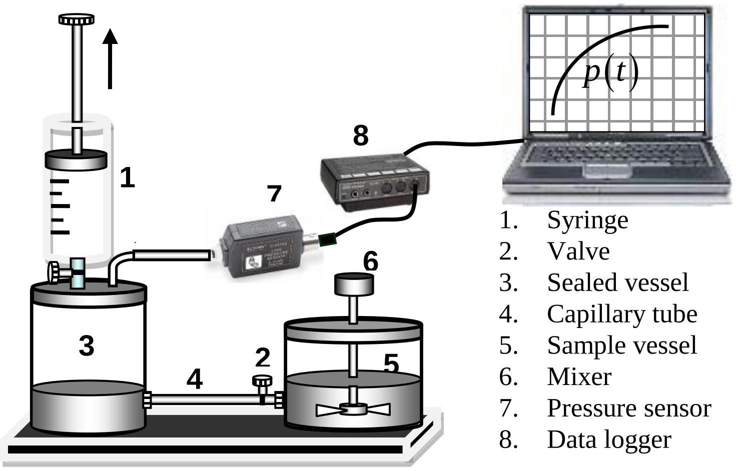
$$\eta=\frac{\pi R^4 p_0}{8\beta L V_0}(1+\alpha)^2\left[1-\frac{\Delta p_\infty}{\Delta p(t)}\right]^{-1} \qquad (19)$$

Thus, the rheological behavior of CWS may be determined if the $p(t)$ data are available and value $\Delta p_\infty=p_{atm}-p(\infty)$ is given. At $\Delta p_\infty=0$, CWS behaves as Newtonian liquid with the constant viscosity, $\eta=\eta_\infty$ independent on the shear rate.

Below, we describe the experimental set up and operation procedure for the fast determination of $p(t)$. As an example, we present the rheological test of CWS samples with 40 and 50 wt. % of coal.

III. EXPERIMENTAL

The high shear capillary rheometer schematically shown in Fig. 2 consists of two vessels. The sample vessel of volume 300 mL, open to air, contains the test sample of CWS rotated on a mill pot rotator to keep a uniform dispersion. The other is the pressurized vacuum vessel of volume $V_0=280\text{ mL}$, which is equipped by a 50 mL syringe and the pressure sensor PASCO CI-6532A with a range 0-700kPa connected through the data logger PASCO CI-6400 to a computer. Two vessels are connected via a valve by glass capillary tube. The principles of measurement are as follows. First, the data acquisition system is activated, all valves are opened and the ambient pressure, p_{atm} is fixed. Then, the valve of the sample vessel is closed and about 100 mL of the test CWS is introduced into vessel. The mixer is turned on to keep a uniform dispersion. The syringe piston moves up to set the pressure in the vacuum vessel, $p_0<p_{atm}$. Then, the syringe valve is closed through test and the valve of the sample vessel is opened allowing the liquid to flow through the capillary tube under pressure difference $\Delta p(t)=p_{atm}-p(t)$ between two vessels, which decreases during the flow because the free air volume in the vacuum vessel decreases. The pressure data versus time, $p(t)$ recorded at the sample rate 50 s^{-1}, read out to a PC and fed further for processing. When the pressure difference $\Delta p(t)$ reaches its equilibrium value, the flow is ceased and Δp_∞ is fixed.

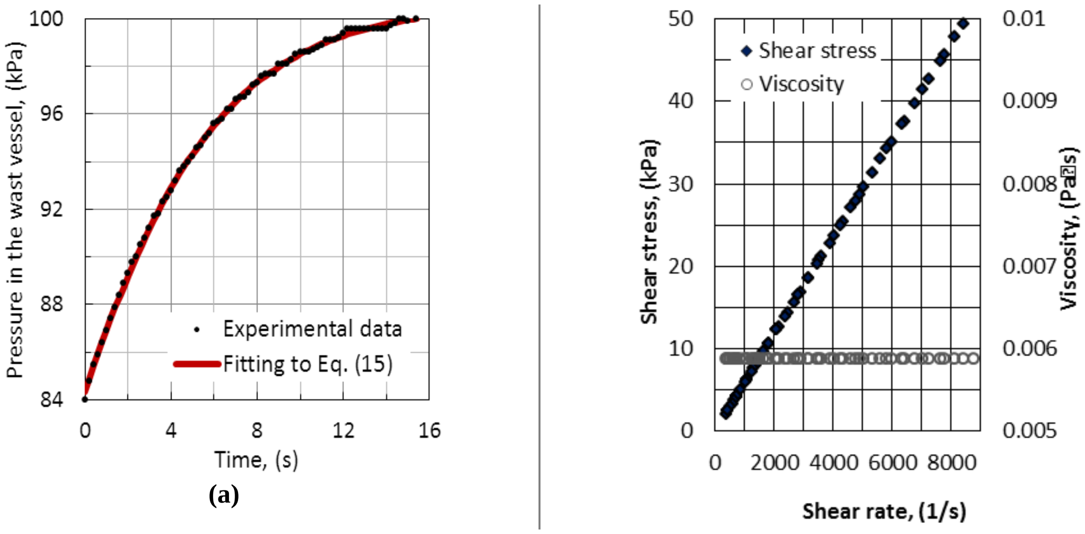


1. Syringe
2. Valve
3. Sealed vessel
4. Capillary tube
5. Sample vessel
6. Mixer
7. Pressure sensor
8. Data logger

Fig. 2. Schematic diagram of the experimental setup

IV. RESULTS

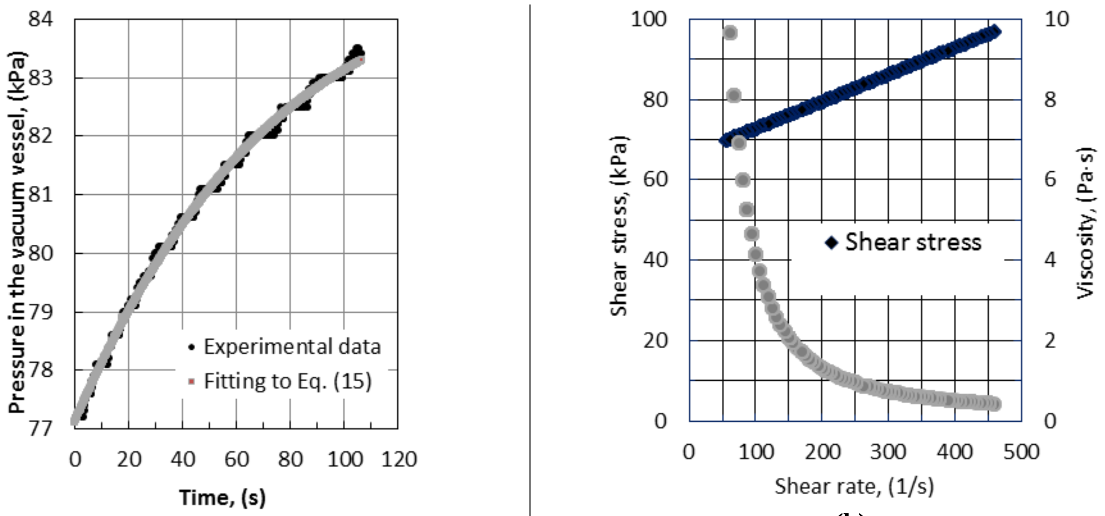
The experimental set up and operation procedure described in Section III were used for determination of the rheological behavior of CWS. The test samples were prepared from the South African (Billiton-Prime) coal with particle size ≤ 20 microns and coal content 40 – 50 wt. % without chemical additives to prevent the rheological modification of CWS. We used a glass capillary tube of radius $R = 1\ \text{mm}$ and length $L = 11.84\ \text{cm}$ so that the flow rate through the tube was significantly greater than the terminated settling velocity of coal particles, and the sedimentation effect was negligible. The recorded plot of the temporal variation of the pressure in the vacuum vessel during the flow of CWS with coal content 40 wt. % at room temperature 25 °C, atmospheric pressure $p_{atm} = 100\ \text{kPa}$ and initial air pressure in the vacuum vessel $p_0 = 84\ \text{kPa}$, is shown in Fig. 3a. As is expected when the fluid run through the tube, the pressure in the vacuum vessel, $p(t)$ gradually increased from the initial value to the equilibrium value $p(\infty) = 100\ \text{kPa}$, at which the flow was ceased. Typically it takes about 20 seconds. As the final pressure difference between the vessels $\Delta p_\infty = 0$, the yield stress $\sigma_c = 0$, indicating that the test CWS with coal content of 40 % behaves as a Newtonian liquid.



(a)

Fig. 3. The test results for CWS with the coal loading 40 wt %: (a) The temporal variation of air pressure in the vacuum vessel; (b) Newtonian flow curve

The fit of the experimental data $p(t)$ to (15) with $\alpha = 0.1930 \pm 0.0005$ and $\beta = 0.2379 \pm 0.0017$ shown in Fig. 3a is excellent, demonstrating the validity of Eq. (15) and above theoretical approach. The flow curve in Fig. 3b is built using the experimental data $p(t)$ shown in Fig. 3a. The viscosity calculated by (16) $\eta = 5.87 \pm 0.7 \text{ mPa}$ is constant independent of the flow rate. The Newtonian behavior of CWS for the coal concentration 40 wt. % agrees with finding that slurries tend to be Newtonian for low solid loading.



(a) (b)

Fig. 4. The test results for CWS with the coal concentration 50 wt %: (a) The temporal variation of air pressure in the vacuum vessel; (b) The Bingham plastic flow curves calculated from experimental data $p(t)$

In Fig. 4a is shown a variation of the pressure in the vacuum vessel during the flow of CWS with coal content 50 wt. %. At the room temperature 25 °C, the flow through the same capillary tube of radius $R = 1 \text{ mm}$ and length, $L = 11.84 \text{ cm}$ typically takes about 110 sec, in which case the shear rate decreased from 500 to 50 s^{-1}. It is reasonable to assume that the 110 s period is long enough to cause the sedimentation of coal particles if the sedimentation going to take place. To exclude this assumption, a large-diameter capillary (3 mm inside diameter) was used to decrease the test duration. Obtained results were in the well agreement with those obtained using a capillary of 2 mm diameter. The details of the additional experiment will be reported elsewhere in the future.

At atmospheric pressure $p_{atm} = 100 \text{ kPa}$, air pressure in the vacuum vessel $p(t)$ is slowly increased from an initial value $p_0 = 77.1 \text{ kPa}$ to a final value $p(\infty) = 83.4 \text{ kPa}$ with the final equilibrium pressure difference between the vessels, $\Delta p_\infty = 15.6 \text{ kPa}$, indicating the existence of the yield stress, value of which $\sigma_c = 65.88 \text{ Pa}$ as calculations by (8) show. The fit of experimental data $p(t)$ to Eq. (15) in Fig. 4a with $\alpha = 0.1006 \pm 0.0006$ and $\beta = 0.2379 \pm 0.0017$ is excellent. The Bingham-Plastic viscosity calculated by (16) using the values of α, β and instrumental parameters of the system, is: $\eta_\infty = 67.1 \pm 4.3 \text{ mPa}$, which is one order of magnitude greater than the viscosity of the first sample. The plot of the wall shear stress versus the wall shear rate and the viscosity function calculated by (1), (18) and (19), are shown in Fig. 4b. As can be seen from Fig. 4b, the viscosity of CWS with 50 wt. % of coal is a decreasing function of shear rate, approaching an asymptote $\eta_\infty = 67.1 \pm 4.3 \text{ mPa}$ as the rate of shear is increased above 400 s^{-1}, clearly indicating the non-Newtonian, shear thinning behavior over the whole interval of shear rate.

V. DISCUSSION AND CONCLUSION

We described theoretical and operation principles of a new shear-controlled capillary viscometer with continuously changing pressure head. The feature of the method is simplicity, ease in operation, quick measurement with a small volume of test sample and accurate results over a relatively broad shear rate range in a single measurement without any changes in the geometry. The validity of the theoretical treatment was illustrated by the rheological testing of CWS at room temperature. The main results can be summarized as follows:

- The viscometer is capable of giving excellent results for Newtonian and non-Newtonian behavior of CWS depending on the coal-loading. Explanation of this CWS behavior was represented by Boger in his review [12].
- It is found that at low coal concentrations, CWS tends to be Newtonian while by increasing the coal-loading above 40 wt %, CWS becomes more non-Newtonian (same us mentioned in studies [12, 13]) with shear thinning properties.
- The flow curves under a shear rate $\dot{\gamma} > 50\ \mathrm{s}^{-1}$ reveal a linear relation between the shear stress and rate of shear, often with yield stress, describing by the Bingham-Plastic constitutive equation.

It is difficult to quantitatively compare of the obtained results with "common" literature data [4,5, 12,13], since CWS samples under studies are usually prepared from different types of coal and different coal particles size distribution, strongly affected its rheological properties.

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REFERENCES

[1] C. B. Henderson, S R. Scheffee, and E.T McHale, "Coal-water slurries - a low-cost liquid fuel for boilers," Energy Progress, (United States), vol. 3, no 2, pp. 69–75, 1983.

[2] Y. Liang, Y. Tu and R. Zhang, "Coal Water Mixture Preparation Technology and Application in Replacing Oil to Generate Electricity, 2010.

[3] S. K. Mishra, S. B. Kanungo, "Factors affecting the preparation of highly concentrated Coal–Water slurry (HCCWS)," Journal of Scientific & Industrial Research, 59, pp. 765–790, 2000.

[4] A. Slaczka, A. Wasilczyk, "The effect of chemicals on the rheology of highly loaded coal water slurries (CWS)," Physicochem. Probl. Miner. Process. Vol. 48, no 1, pp. 141–148, 2012.

[5] F. Boylu, H. Dincer, and G. Atesok, "Effect of coal particle size distribution, volume fraction and rank on the rheology of coal-water slurries," Fuel Processing Technology 85, pp. 241–250, 2004.

[6] P. Buranasrisak and M. H. Narasingha, "Effects of Particle Size Distribution and Packing Characteristics on the Preparation of Highly-Loaded Coal-Water Slurry," International Journal of Chemical Engineering and Applications, vol. 3, no 1, pp. 31-35, 2012.

[7] S. H. Maron, I. M. Krieger, and A. W. Sidko, A capillary viscometer with continuously varying pressure head," J. Appl. Phys. 25, pp. 971-976, 1954.

[8] R. B. Bird, R. Armstrong, O. Hassager, Dynamics of polymeric liquids, vol. 1. Wiley- Interscience, New York, 1987.

[9] R. M. Digilov, " Pressure-driven capillary viscometer: Fundamental challenges in transient flow viscometry," Rev. Sci. Instrum. 82, 125111, 2011.

[10] P. K. Senapati, R. Mohapatra, G. K. Pani, B. K. Mishra, "Studies on rheological and leaching characteristics of heavy metals through selective additive in high concentration ash slurry," Journal of Hazardous Materials, 229-230, pp. 390–397, 2012.

[11] M. Corless, G.H. Gonnet, D.E. G. Hare, D. J. Jeffrey and D. E. Knuth, "On the Lambert W function," Adv. Comput. Math. 5, pp. 329-359, 1996.

[12] D. V. Boger "Rheology of Slurries and Environmental Impacts in the Mining Industry" Annual Review of Chemical and Bimolecular Engineering, 4, pp. 239-257, 2013.

[13] R. M Turian, J.F Attal, D.-J Sung, L. E Wedgewood “Properties and rheology of coal–water mixtures using different coals” Fuel , vol. 81, no 16, pp. 2019-2033, 2002.

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