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Consumers' Activities for Brand Selection and an Expansion to the Second Order Lag Model

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Abstract*—Focusing that consumers' are apt to buy superior brand when they are accustomed or bored to use current brand, new analysis method is introduced. Before buying data and after buying data is stated using liner model. When above stated events occur, transition matrix becomes upper triangular matrix. In this paper, equation using transition matrix is extended to the second order lag and the method is newly re-built. These are applied to the jewelry purchasing case and are confirmed by numerical examples. Some interesting results are obtained. This approach makes it possible to identify brand position in the market and it can be utilized for building useful and effective marketing plan.

***Key Words*—brand selection, matrix structure, brand position, jewelry, accessory.**

I. INTRODUCTION

It is often observed that consumers select upper class brand when they buy next time. Suppose that former buying data and current buying data are gathered. Also suppose that upper brand is located upper in the variable array. Then transition matrix becomes upper triangular matrix under the supposition that former buying variables are set input and current buying variables are set output. If the top brand were selected from lower brand in jumping way, corresponding part in upper triangular matrix would be 0. There may be also the case that customers select lower brand to seek suitable price when they have chosen higher brand. Then it may compose items of lower triangular matrix.

If transition matrix is identified, S-step forecasting can be executed. Unless planner for products does not notice its brand position whether it is upper or lower than other products, matrix structure makes it possible to identify those by calculating consumers' activities for brand selection. Thus, this proposed approach enables to make effective marketing plan and/or establishing new brand. Quantitative analysis concerning brand selection has been executed by Yamanaka (1982)[1], Takahashi et al.(2002)[2]. Yamanaka (1982)[1] examined purchasing process by Markov Transition Probability with the input of advertising expense. Takahashi et al.(2002)[2] made analysis by the Brand Selection Probability model using logistics distribution. In Takeyasu et al. (2007) [3], matrix structure was analyzed for the case brand selection was executed toward upper class. In this paper, equation using transition matrix is extended to the second order lag in order to improve model accuracy and thereby forecasting accuracy, and confirm them by the questionnaire investigation for jewelry purchasing case. Such research is quite a new one.

In this paper, matrix structure is analyzed for the case brand selection is executed for upper class and for lower class, utilizing jewelry/accessory purchasing history record of on-line shopping over three years. Hereinafter, matrix structure is clarified for the selection of brand in section 2. Extension of the model to the second order lag is executed in section 3. Forecasting is formulated in section 4. Purchase history investigation of jewelry/accessory on-line shopping is examined and its numerical calculation is executed in section 5. Remarks are described in 6. Section 7 is a conclusion.

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II. BRAND SELECTION AND ITS MATRIX STRUCTURE[3]

A. Upper Shift of Brand Selection

Now, suppose that x is the most upper class brand, y is the second upper class brand, and z is the lowest class brand. Consumer's behavior of selecting brand might be $z \to y$, $y \to x$, $z \to x$ etc. $x \to z$ might be few.

Suppose that x is current buying variable, and x_b is previous buying variable. Shift to x is executed from x_b, y_b, or z_b. Therefore, x is stated in the following equation. a_{ij} represents transition probability from j-th to i-th brand.

$$x = a_{11}x_b + a_{12}y_b + a_{13}z_b$$

Similarly,

$$y = a_{22}y_b + a_{23}z_b$$

And

$$z = a_{33}z_b$$

These are re-written as follows.

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{pmatrix} \begin{pmatrix} x_b \\ y_b \\ z_b \end{pmatrix} \quad (1)$$

Set

$$\mathbf{X} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad \mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{pmatrix}, \quad \mathbf{X_b} = \begin{pmatrix} x_b \\ y_b \\ z_b \end{pmatrix}$$

then, $\mathbf{X}$ is represented as follows.

$$\mathbf{X} = \mathbf{AX_b} \quad (2)$$

Here,

$$\mathbf{X} \in \mathbf{R}^3, \mathbf{A} \in \mathbf{R}^{3\times3}, \mathbf{X_b} \in \mathbf{R}^3$$

$\mathbf{A}$ is an upper triangular matrix. To examine this, generating following data, which are all consisted by the data in which transition is made from lower brand to upper brand,

$$\mathbf{X^i} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad \cdots \quad \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad (3)$$

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$$\mathbf{X_b^i} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad \cdots \quad \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{4}$$

$$i = 1, \quad 2 \quad \cdots \quad N$$

parameter can be estimated using least square method. Suppose

$$\mathbf{X}^i = \mathbf{A}\mathbf{X}_\mathbf{b}^i + \boldsymbol{\varepsilon}^i \tag{5}$$

Where

$$\varepsilon^i = \begin{pmatrix} \varepsilon_1^i \\ \varepsilon_2^i \\ \varepsilon_3^i \end{pmatrix} \qquad i = 1, 2, \cdots, N$$

and minimize following J

$$J = \sum_{i=1}^{N} \boldsymbol{\varepsilon}^{iT} \boldsymbol{\varepsilon}^i \rightarrow Min \tag{6}$$

$\hat{\mathbf{A}}$ which is an estimated value of $\mathbf{A}$ is obtained as follows.

$$\hat{\mathbf{A}} = \left(\sum_{i=1}^{N} \mathbf{X}^i \mathbf{X}_\mathbf{b}^{iT} \right) \left(\sum_{i=1}^{N} \mathbf{X}_\mathbf{b}^i \mathbf{X}_\mathbf{b}^{iT} \right)^{-1} \tag{7}$$

In the data group which are all consisted by the data in which transition is made from lower brand to upper brand, estimated value $\hat{\mathbf{A}}$ should be upper triangular matrix. If following data which shift to lower brand are added only a few in equation (3) and (4),

$$\mathbf{X}^i = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \qquad \mathbf{X}_\mathbf{b}^i = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$\hat{\mathbf{A}}$ would contain minute items in the lower part triangle.

B. Sorting Brand Ranking by Re-arranging Row

In a general data, variables may not be in order as x, y, z. In that case, large and small value lie scattered in $\hat{\mathbf{A}}$. But re-arranging this, we can set in order by shifting row. The large value parts are gathered in upper triangular matrix, and the small value parts are gathered in lower triangular matrix.

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \overset{\hat{\mathbf{A}}}{\begin{pmatrix} \bigcirc & \bigcirc & \bigcirc \\ \varepsilon & \bigcirc & \bigcirc \\ \varepsilon & \varepsilon & \bigcirc \end{pmatrix}} \xleftarrow{\text{Shifting row}} \begin{pmatrix} z \\ x \\ y \end{pmatrix} \overset{\hat{\mathbf{A}}}{\begin{pmatrix} \varepsilon & \varepsilon & \bigcirc \\ \bigcirc & \bigcirc & \bigcirc \\ \varepsilon & \bigcirc & \bigcirc \end{pmatrix}} \tag{8}$$

C. *Matrix Structure under the Case Skipping Intermediate Class Brand is Skipped*

It is often observed that some consumers select the most upper class brand from the most lower class brand and skip selecting the intermediate class brand.
We suppose v, w, x, y, z brands (suppose they are laid from upper position to lower position as $v > w > x > y > z$).
In the above case, selection shifts would be

$$v \leftarrow z$$
$$v \leftarrow y$$

Suppose they do not shift to y, x, w from z, to x, w from y, and to w from x, then Matrix structure would be as follows.

$$\begin{pmatrix} v \\ w \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ 0 & a_{22} & 0 & 0 & 0 \\ 0 & 0 & a_{33} & 0 & 0 \\ 0 & 0 & 0 & a_{44} & 0 \\ 0 & 0 & 0 & 0 & a_{55} \end{pmatrix} \begin{pmatrix} v_b \\ w_b \\ x_b \\ y_b \\ z_b \end{pmatrix} \tag{9}$$

III. EXTENSION OF THE MODEL TO THE SECOND ORDER LAG

We extend Eq.(2) to the second order lag in this section. We have analyzed the automobile purchasing case (Takeyasu et al. (2007) [3]). In that case, we could obtain the data (current buying data, former buying data, before former buying data). We have analyzed them by dividing the data (current buying data, former buying data) and (former buying data before former buying data), and put them to Eq.(5) to apply the model.
But this is a kind of simplified method to apply to the model. If we have a further time lag model and we can utilized the data as it is, the estimation accuracy of parameter would be more accurate and the forecasting would be more precise. Therefore we introduce a new model which extends Eq.(2) to the second order lag model as follows.

$$\mathbf{X}_t = \mathbf{A}_1 \mathbf{X}_{t-1} + \mathbf{A}_2 \mathbf{X}_{t-2} \tag{10}$$

Where

$$\mathbf{X}_t = \begin{pmatrix} x_1^t \\ x_2^t \\ \vdots \\ x_p^t \end{pmatrix} \qquad t = 1, 2 \cdots$$

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$$\mathbf{A}_1 = \begin{pmatrix} a_{11}^{(1)}, & a_{12}^{(1)}, & \cdots, & a_{1p}^{(1)} \\ a_{21}^{(1)}, & a_{22}^{(1)}, & \cdots, & a_{2p}^{(1)} \\ \vdots & \vdots & & \vdots \\ a_{p1}^{(1)}, & a_{p1}^{(1)}, & \ldots, & a_{pp}^{(1)} \end{pmatrix}$$

$$\mathbf{A}_2 = \begin{pmatrix} a_{11}^{(2)}, & a_{12}^{(2)}, & \cdots, & a_{1p}^{(2)} \\ a_{21}^{(2)}, & a_{22}^{(2)}, & \cdots, & a_{2p}^{(2)} \\ \vdots & \vdots & & \vdots \\ a_{p1}^{(2)}, & a_{p1}^{(2)}, & \ldots, & a_{pp}^{(2)} \end{pmatrix}$$

$$\mathbf{X}_t \in \mathbf{R}^p \ (t = 1,2,\cdots) \quad \mathbf{A}_1 \in \mathbf{R}^{p\times p}, \mathbf{A}_2 \in \mathbf{R}^{p\times p}$$

In order to estimate $\mathbf{A_1}, \mathbf{A_2}$, we set the following equation in the same way as before.

$$\mathbf{X}_t^i = \mathbf{A}_1\mathbf{X}_{t-1}^i + \mathbf{A}_2\mathbf{X}_{t-2}^i + \boldsymbol{\varepsilon}_t^i \qquad (t = 1,2,\cdots,N) \tag{11}$$

$$J = \sum_{i=1}^{N} \boldsymbol{\varepsilon}_t^{iT} \boldsymbol{\varepsilon}_t^{i} \rightarrow Min \tag{12}$$

Eq.(11) is expressed as follows.

$$\mathbf{X}_t^i = (\mathbf{A}_1, \quad \mathbf{A}_2)\begin{pmatrix} \mathbf{X}_{t-1}^i \\ \mathbf{X}_{t-2}^i \end{pmatrix} + \boldsymbol{\varepsilon}_t^i \tag{13}$$

$(\hat{\mathbf{A}}_1, \quad \hat{\mathbf{A}}_2)$ which is an estimated value of $(\mathbf{A}_1, \quad \mathbf{A}_2)$ is obtained as follows in the same way as Eq.(7).

$$\begin{aligned}(\hat{\mathbf{A}}_1, \quad \hat{\mathbf{A}}_2) &= \left(\sum_{i=1}^{N} \mathbf{X}_t^i (\mathbf{X}_{t-1}^{iT}, \quad \mathbf{X}_{t-2}^{iT})\right)\left(\sum_{i=1}^{N}\begin{pmatrix} \mathbf{X}_{t-1}^i \\ \mathbf{X}_{t-2}^i \end{pmatrix}(\mathbf{X}_{t-1}^{iT}, \quad \mathbf{X}_{t-2}^{iT})\right)^{-1} \\ &= \left(\sum_{i=1}^{N} \mathbf{X}_t^i \mathbf{X}_{t-1}^{iT}, \quad \sum_{i=1}^{N} \mathbf{X}_t^i \mathbf{X}_{t-2}^{iT}\right)\begin{pmatrix} \sum_{i=1}^{N} \mathbf{X}_{t-1}^i \mathbf{X}_{t-1}^{iT}, & \sum_{i=1}^{N} \mathbf{X}_{t-1}^i \mathbf{X}_{t-2}^{iT} \\ \sum_{i=1}^{N} \mathbf{X}_{t-2}^i \mathbf{X}_{t-1}^{iT}, & \sum_{i=1}^{N} \mathbf{X}_{t-2}^i \mathbf{X}_{t-2}^{iT} \end{pmatrix}^{-1}\end{aligned} \tag{14}$$

This is re-written as :

$$(\hat{\mathbf{A}}_1, \quad \hat{\mathbf{A}}_2) = \left(\sum_{i=1}^{N} \mathbf{X}_t^i \mathbf{X}_{t-1}^{iT}, \quad \sum_{i=1}^{N} \mathbf{X}_t^i \mathbf{X}_{t-2}^{iT}\right)\begin{pmatrix} \sum_{i=1}^{N} \mathbf{X}_{t-1}^i \mathbf{X}_{t-1}^{iT}, & \sum_{i=1}^{N} \mathbf{X}_{t-1}^i \mathbf{X}_{t-2}^{iT} \\ \sum_{i=1}^{N} \mathbf{X}_{t-2}^i \mathbf{X}_{t-1}^{iT}, & \sum_{i=1}^{N} \mathbf{X}_{t-2}^i \mathbf{X}_{t-2}^{iT} \end{pmatrix}^{-1} \tag{15}$$

We set this as :

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Volume 3, Issue 2, March 2014

$$\left(\hat{\mathbf{A}}_1, \quad \hat{\mathbf{A}}_2\right)=\left(\hat{\mathbf{B}}, \quad \hat{\mathbf{C}}\right)\begin{pmatrix}\hat{\mathbf{D}} & \hat{\mathbf{E}} \\ \hat{\mathbf{E}}^T & \hat{\mathbf{F}}\end{pmatrix}^{-1} \tag{16}$$

In the data group of upper shift brand, $\hat{\mathbf{B}}$, $\hat{\mathbf{C}}$ and $\hat{\mathbf{E}}$ becomes an upper triangular matrix. While $\hat{\mathbf{D}}$ and $\hat{\mathbf{F}}$ are diagonal matrix in any case. This will be made clear in the numerical calculation later.

IV. FORECASTING

After transition matrix is estimated, we can make forecasting. We show some of them in the following equations.

$$\hat{\mathbf{X}}_{t+1}=\hat{\mathbf{A}}_1\mathbf{X}_t+\hat{\mathbf{A}}_2\mathbf{X}_{t-1} \tag{17}$$

$$\hat{\mathbf{X}}_{t+2}=\left(\hat{\mathbf{A}}_1{}^2+\hat{\mathbf{A}}_2\right)\mathbf{X}_t+\hat{\mathbf{A}}_1\hat{\mathbf{A}}_2\mathbf{X}_{t-1} \tag{18}$$

$$\hat{\mathbf{X}}_{t+3}=\left(\hat{\mathbf{A}}_1{}^3+\hat{\mathbf{A}}_1\hat{\mathbf{A}}_2+\hat{\mathbf{A}}_2\hat{\mathbf{A}}_1\right)\mathbf{X}_t+\left(\hat{\mathbf{A}}_1{}^2+\hat{\mathbf{A}}_2\right)\hat{\mathbf{A}}_2\mathbf{X}_{t-1} \tag{19}$$

$$\hat{\mathbf{X}}_{t+4}=\left(\hat{\mathbf{A}}_1{}^4+\hat{\mathbf{A}}_1{}^2\hat{\mathbf{A}}_2+\hat{\mathbf{A}}_1\hat{\mathbf{A}}_2\hat{\mathbf{A}}_1+\hat{\mathbf{A}}_2\hat{\mathbf{A}}_1{}^2+\hat{\mathbf{A}}_2{}^2\right)\mathbf{X}_t+\left\{\hat{\mathbf{A}}_1\left(\hat{\mathbf{A}}_1{}^2+\hat{\mathbf{A}}_2\right)+\hat{\mathbf{A}}_2\hat{\mathbf{A}}_1\right\}\hat{\mathbf{A}}_2\mathbf{X}_{t-1} \tag{20}$$

V. PURCHASE HISTORY INVESTIGATION AND NUMERICAL CALCULATION

Jewelry/Accessory purchase history investigation is executed.
First of all, the framework of jewelry/accessory purchasing via on-line shopping is as follows.

- On-line shop: Ciao! / Happy gift
  Host site: http://www.happy-gift.jp/
  Branch site: http://www.rakuten.co.jp/ciao/
  http://store.shopping.yahoo.co.jp/b-ciao/index.html
  Managed by Cherish Co.Ltd.

- Customers: all over Japan (Every Prefecture)

- Data gathering period: April 2008 – May 2011

- Order number: 4411 (limited to the order number which has repeated order)

- Main residents of customers
  Tokyo 11.9%
  Kanagawa 8.7%
  Osaka 6.0%
  Aichi 5.8%
  Chiba 5.7%
  Saitama 5.4%
  The share of Tokyo capital area consists of 31.7%.

- Sales goods:
  Necklace / Pendant
  Pierced earrings
  Ring
  Bracelet / Bangle

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Volume 3, Issue 2, March 2014

Brooch
Necktie Pin
Miscellaneous （Package/Ribbon etc.）

- Classification of goods by price

Table Ⅰ. Classification of Goods by Price

Rank	Price(Yen)	Rank	Price(Yen)
Necklace / Pendant		Pierced earrings	
N6	40001～	P6	24001～
N5	～40000	P5	～24000
N4	～30000	P4	～16000
N3	～20000	P3	～10000
N2	～15000	P2	～6000
N1	～10000	P1	～2000
Ring		Bracelet / Bungle	
R6	40001～	B6	40001～
R5	～40000	B5	～40000
R4	～30000	B4	～35000
R3	～20000	B3	～30000
R2	～15000	B2	～15000
R1	～10000	B1	～10000

The purchase history data was the most for Necklace/ Pendant. Therefore we make focus on them.

Table Ⅱ. Shifting Results of Goods

$< \mathbf{X}_{t-2}$ to $\mathbf{X}_{t-1} >$		
1.	Shift from N1 to N1	36
2.	Shift from N1 to N1	8

$< \mathbf{X}_{t-1}$ to $\mathbf{X}_{t} >$		
1.	Shift from N1 to N1	36
2.	Shift from N1 to N2	8

ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 3, Issue 2, March 2014

3.	Shift from N1 to N1	3
4.	Shift from N1 to N1	1
5.	Shift from N1 to N2	2
6.	Shift from N1 to N2	1
7.	Shift from N1 to N2	1
8.	Shift from N1 to N3	1
9.	Shift from N1 to N3	1
10.	Shift from N2 to N1	7
11.	Shift from N2 to N1	3
12.	Shift from N2 to N2	4
13.	Shift from N2 to N2	52
14.	Shift from N2 to N2	1
15.	Shift from N2 to N2	1
16.	Shift from N2 to N3	1
17.	Shift from N2 to N3	1
18.	Shift from N2 to N4	1
19.	Shift from N3 to N1	4
20.	Shift from N3 to N1	2
21.	Shift from N3 to N2	6
22.	Shift from N3 to N2	1
23.	Shift from N3 to N3	4
24.	Shift from N3 to N3	2
25.	Shift from N3 to N3	25

3.	Shift from N1 to N3	3
4.	Shift from N1 to N4	1
5.	Shift from N2 to N1	2
6.	Shift from N2 to N2	1
7.	Shift from N2 to N3	1
8.	Shift from N3 to N1	1
9.	Shift from N3 to N2	1
10.	Shift from N1 to N1	7
11.	Shift from N1 to N2	3
12.	Shift from N2 to N1	4
13.	Shift from N2 to N2	52
14.	Shift from N2 to N3	1
15.	Shift from N2 to N5	1
16.	Shift from N3 to N2	1
17.	Shift from N3 to N3	1
18.	Shift from N4 to N2	1
19.	Shift from N1 to N1	4
20.	Shift from N1 to N3	2
21.	Shift from N2 to N2	6
22.	Shift from N2 to N3	1
23.	Shift from N3 to N1	4
24.	Shift from N3 to N2	2
25.	Shift from N3 to N3	25

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International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 3, Issue 2, March 2014

26.	Shift from N3 to N3	1
27.	Shift from N4 to N1	1
28.	Shift from N4 to N1	1
29.	Shift from N4 to N2	1
30.	Shift from N4 to N2	1
31.	Shift from N4 to N3	1
32.	Shift from N4 to N4	1
33.	Shift from N4 to N4	11
34.	Shift from N5 to N4	1
35.	Shift from N5 to N5	1
36.	Shift from N6 to N1	1
37.	Shift from N6 to N3	1
38.	Shift from N6 to N6	1

26.	Shift from N3 to N4	1
27.	Shift from N1 to N1	1
28.	Shift from N1 to N2	1
29.	Shift from N2 to N1	1
30.	Shift from N2 to N2	1
31.	Shift from N3 to N3	1
32.	Shift from N4 to N3	1
33.	Shift from N4 to N4	11
34.	Shift from N4 to N4	1
35.	Shift from N5 to N5	1
36.	Shift from N1 to N1	1
37.	Shift from N3 to N6	1
38.	Shift from N6 to N6	2

Vector $\mathbf{X}_t, \mathbf{X}_{t-1}, \mathbf{X}_{t-2}$ in these cases are expressed as follows.

1. $$\mathbf{X}_t = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad \mathbf{X}_{t-2} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

2. $$\mathbf{X}_t = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad \mathbf{X}_{t-2} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

3. $$\mathbf{X}_t = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad \mathbf{X}_{t-2} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

20. $$\mathbf{X}_t = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad \mathbf{X}_{t-2} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

21. $$\mathbf{X}_t = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad \mathbf{X}_{t-2} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

22. $$\mathbf{X}_t = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad \mathbf{X}_{t-2} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

4. $\mathbf{X}_t = \begin{pmatrix}0\\0\\1\\0\\0\\0\end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix}0\\0\\0\\0\\0\\1\end{pmatrix}, \quad \mathbf{X}_{t-2} = \begin{pmatrix}0\\0\\0\\0\\0\\1\end{pmatrix}$

5. $\mathbf{X}_t = \begin{pmatrix}0\\0\\0\\0\\0\\1\end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix}0\\0\\0\\0\\1\\0\end{pmatrix}, \quad \mathbf{X}_{t-2} = \begin{pmatrix}0\\0\\0\\0\\0\\1\end{pmatrix}$

6. $\mathbf{X}_t = \begin{pmatrix}0\\0\\0\\0\\1\\0\end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix}0\\0\\0\\0\\1\\0\end{pmatrix}, \quad \mathbf{X}_{t-2} = \begin{pmatrix}0\\0\\0\\0\\0\\1\end{pmatrix}$

7. $\mathbf{X}_t = \begin{pmatrix}0\\0\\0\\1\\0\\0\end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix}0\\0\\0\\0\\1\\0\end{pmatrix}, \quad \mathbf{X}_{t-2} = \begin{pmatrix}0\\0\\0\\0\\0\\1\end{pmatrix}$

8. $\mathbf{X}_t = \begin{pmatrix}0\\0\\0\\0\\0\\1\end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix}0\\0\\0\\1\\0\\0\end{pmatrix}, \quad \mathbf{X}_{t-2} = \begin{pmatrix}0\\0\\0\\0\\0\\1\end{pmatrix}$

9. $\mathbf{X}_t = \begin{pmatrix}0\\0\\0\\0\\1\\0\end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix}0\\0\\0\\1\\0\\0\end{pmatrix}, \quad \mathbf{X}_{t-2} = \begin{pmatrix}0\\0\\0\\0\\0\\1\end{pmatrix}$

10. $\mathbf{X}_t = \begin{pmatrix}0\\0\\0\\0\\0\\1\end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix}0\\0\\0\\0\\0\\1\end{pmatrix}, \quad \mathbf{X}_{t-2} = \begin{pmatrix}0\\0\\0\\0\\1\\0\end{pmatrix}$

23. $\mathbf{X}_t = \begin{pmatrix}0\\0\\0\\0\\0\\1\end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix}0\\0\\0\\1\\0\\0\end{pmatrix}, \quad \mathbf{X}_{t-2} = \begin{pmatrix}0\\0\\0\\1\\0\\0\end{pmatrix}$

24. $\mathbf{X}_t = \begin{pmatrix}0\\0\\0\\0\\1\\0\end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix}0\\0\\0\\1\\0\\0\end{pmatrix}, \quad \mathbf{X}_{t-2} = \begin{pmatrix}0\\0\\0\\1\\0\\0\end{pmatrix}$

25. $\mathbf{X}_t = \begin{pmatrix}0\\0\\0\\1\\0\\0\end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix}0\\0\\0\\1\\0\\0\end{pmatrix}, \quad \mathbf{X}_{t-2} = \begin{pmatrix}0\\0\\0\\1\\0\\0\end{pmatrix}$

26. $\mathbf{X}_t = \begin{pmatrix}0\\0\\1\\0\\0\\0\end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix}0\\0\\0\\1\\0\\0\end{pmatrix}, \quad \mathbf{X}_{t-2} = \begin{pmatrix}0\\0\\0\\1\\0\\0\end{pmatrix}$

27. $\mathbf{X}_t = \begin{pmatrix}0\\0\\0\\0\\0\\1\end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix}0\\0\\0\\0\\0\\1\end{pmatrix}, \quad \mathbf{X}_{t-2} = \begin{pmatrix}0\\0\\1\\0\\0\\0\end{pmatrix}$

28. $\mathbf{X}_t = \begin{pmatrix}0\\0\\0\\0\\1\\0\end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix}0\\0\\0\\0\\0\\1\end{pmatrix}, \quad \mathbf{X}_{t-2} = \begin{pmatrix}0\\0\\1\\0\\0\\0\end{pmatrix}$

29. $\mathbf{X}_t = \begin{pmatrix}0\\0\\0\\0\\0\\1\end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix}0\\0\\0\\0\\1\\0\end{pmatrix}, \quad \mathbf{X}_{t-2} = \begin{pmatrix}0\\0\\1\\0\\0\\0\end{pmatrix}$

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ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 3, Issue 2, March 2014

11. $\mathbf{X}_t = \begin{pmatrix} 0\\0\\0\\0\\1\\0 \end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix} 0\\0\\0\\0\\0\\1 \end{pmatrix}, \quad \mathbf{X}_{t-2} = \begin{pmatrix} 0\\0\\0\\0\\1\\0 \end{pmatrix}$

12. $\mathbf{X}_t = \begin{pmatrix} 0\\0\\0\\0\\0\\1 \end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix} 0\\0\\0\\0\\1\\0 \end{pmatrix}, \quad \mathbf{X}_{t-2} = \begin{pmatrix} 0\\0\\0\\0\\1\\0 \end{pmatrix}$

13. $\mathbf{X}_t = \begin{pmatrix} 0\\0\\0\\0\\1\\0 \end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix} 0\\0\\0\\0\\1\\0 \end{pmatrix}, \quad \mathbf{X}_{t-2} = \begin{pmatrix} 0\\0\\0\\0\\1\\0 \end{pmatrix}$

14. $\mathbf{X}_t = \begin{pmatrix} 0\\0\\0\\1\\0\\0 \end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix} 0\\0\\0\\0\\1\\0 \end{pmatrix}, \quad \mathbf{X}_{t-2} = \begin{pmatrix} 0\\0\\0\\0\\1\\0 \end{pmatrix}$

15. $\mathbf{X}_t = \begin{pmatrix} 0\\1\\0\\0\\0\\0 \end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix} 0\\0\\0\\0\\1\\0 \end{pmatrix}, \quad \mathbf{X}_{t-2} = \begin{pmatrix} 0\\0\\0\\0\\1\\0 \end{pmatrix}$

16. $\mathbf{X}_t = \begin{pmatrix} 0\\0\\0\\0\\1\\0 \end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix} 0\\0\\0\\1\\0\\0 \end{pmatrix}, \quad \mathbf{X}_{t-2} = \begin{pmatrix} 0\\0\\0\\0\\1\\0 \end{pmatrix}$

17. $\mathbf{X}_t = \begin{pmatrix} 0\\0\\0\\1\\0\\0 \end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix} 0\\0\\0\\1\\0\\0 \end{pmatrix}, \quad \mathbf{X}_{t-2} = \begin{pmatrix} 0\\0\\0\\0\\1\\0 \end{pmatrix}$

30. $\mathbf{X}_t = \begin{pmatrix} 0\\0\\0\\0\\1\\0 \end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix} 0\\0\\0\\0\\1\\0 \end{pmatrix}, \quad \mathbf{X}_{t-2} = \begin{pmatrix} 0\\0\\1\\0\\0\\0 \end{pmatrix}$

31. $\mathbf{X}_t = \begin{pmatrix} 0\\0\\0\\1\\0\\0 \end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix} 0\\0\\0\\1\\0\\0 \end{pmatrix}, \quad \mathbf{X}_{t-2} = \begin{pmatrix} 0\\0\\1\\0\\0\\0 \end{pmatrix}$

32. $\mathbf{X}_t = \begin{pmatrix} 0\\0\\0\\1\\0\\0 \end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix} 0\\0\\1\\0\\0\\0 \end{pmatrix}, \quad \mathbf{X}_{t-2} = \begin{pmatrix} 0\\0\\1\\0\\0\\0 \end{pmatrix}$

33. $\mathbf{X}_t = \begin{pmatrix} 0\\0\\1\\0\\0\\0 \end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix} 0\\0\\1\\0\\0\\0 \end{pmatrix}, \quad \mathbf{X}_{t-2} = \begin{pmatrix} 0\\0\\1\\0\\0\\0 \end{pmatrix}$

34. $\mathbf{X}_t = \begin{pmatrix} 0\\1\\0\\0\\0\\0 \end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix} 0\\0\\1\\0\\0\\0 \end{pmatrix}, \quad \mathbf{X}_{t-2} = \begin{pmatrix} 0\\0\\1\\0\\0\\0 \end{pmatrix}$

35. $\mathbf{X}_t = \begin{pmatrix} 0\\1\\0\\0\\0\\0 \end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix} 0\\1\\0\\0\\0\\0 \end{pmatrix}, \quad \mathbf{X}_{t-2} = \begin{pmatrix} 0\\1\\0\\0\\0\\0 \end{pmatrix}$

36. $\mathbf{X}_t = \begin{pmatrix} 0\\0\\0\\0\\0\\1 \end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix} 0\\0\\0\\0\\0\\1 \end{pmatrix}, \quad \mathbf{X}_{t-2} = \begin{pmatrix} 1\\0\\0\\0\\0\\0 \end{pmatrix}$

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International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 3, Issue 2, March 2014

18. $\mathbf{X}_t = \begin{pmatrix} 0\\0\\0\\0\\1\\0 \end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix} 0\\0\\1\\0\\0\\0 \end{pmatrix}, \quad \mathbf{X}_{t-2} = \begin{pmatrix} 0\\0\\0\\0\\1\\0 \end{pmatrix}$ 37. $\mathbf{X}_t = \begin{pmatrix} 1\\0\\0\\0\\0\\0 \end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix} 0\\0\\0\\1\\0\\0 \end{pmatrix}, \quad \mathbf{X}_{t-2} = \begin{pmatrix} 1\\0\\0\\0\\0\\0 \end{pmatrix}$

19. $\mathbf{X}_t = \begin{pmatrix} 0\\0\\0\\0\\0\\1 \end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix} 0\\0\\0\\0\\0\\1 \end{pmatrix}, \quad \mathbf{X}_{t-2} = \begin{pmatrix} 0\\0\\0\\1\\0\\0 \end{pmatrix}$ 38. $\mathbf{X}_t = \begin{pmatrix} 1\\0\\0\\0\\0\\0 \end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix} 1\\0\\0\\0\\0\\0 \end{pmatrix}, \quad \mathbf{X}_{t-2} = \begin{pmatrix} 1\\0\\0\\0\\0\\0 \end{pmatrix}$

Substituting these to Eq.(14), we obtain the following equation.

$$\left(\hat{\mathbf{A}}_1,\ \hat{\mathbf{A}}_2\right) = \left(\begin{array}{cccccc:cccccc} 1&0&0&1&0&0&2&0&0&0&0&0\\ 0&1&0&0&1&0&0&1&0&0&1&0\\ 0&0&12&1&0&1&0&1&11&1&0&1\\ 0&0&1&27&3&5&0&0&2&28&2&4\\ 0&0&1&4&60&12&0&0&2&8&57&10\\ 0&0&0&5&7&49&1&0&2&8&11&39 \end{array}\right) \left(\begin{array}{cccccc:cccccc} 2&0&0&0&0&0&1&0&0&0&0&0\\ 0&1&0&0&0&0&0&1&0&0&0&0\\ 0&0&14&0&0&0&0&1&12&0&1&0\\ 0&0&0&38&0&0&1&0&1&32&2&2\\ 0&0&0&0&71&0&0&0&2&7&58&4\\ 0&0&0&0&0&67&1&0&2&6&10&48\\ \hdashline 1&0&0&1&0&1&3&0&0&0&0&0\\ 0&1&1&0&0&0&0&2&0&0&0&0\\ 0&0&12&1&2&2&0&0&17&0&0&0\\ 0&0&0&32&7&6&0&0&0&45&0&0\\ 0&0&1&2&58&10&0&0&0&0&71&0\\ 0&0&0&2&4&48&0&0&0&0&0&54 \end{array}\right)^{-1} \quad (21)$$

As we have seen before, we can confirm that $\hat{\mathbf{B}}$ part in Eq.(16) is an upper triangular matrix and $\hat{\mathbf{D}}, \hat{\mathbf{F}}$ part in Eq.(16) are diagonal matrices.

While $\hat{\mathbf{C}}$ and $\hat{\mathbf{E}}$ parts are lower triangular matrices. This means that there was a rather big lower shift in $\mathbf{X}_{t-2}$ to $\mathbf{X}_{t-1}$.

We can find that if $\hat{\mathbf{E}}$ part becomes an upper triangular matrix, then the items compose upper shift or the same level shift.

Calculation results of $\left(\hat{\mathbf{A}}_1,\ \hat{\mathbf{A}}_2\right)$ become as follows.

$$\left(\hat{\mathbf{A}}_1,\ \hat{\mathbf{A}}_2\right) = \begin{pmatrix} 0.000 & -0.506 & -0.506 & -0.479 & -0.506 & -0.520 & 1.000 & 0.506 & 0.506 & 0.488 & 0.507 & 0.518\\ 0.000 & 1.000 & 0.000 & 0.001 & 0.008 & -0.001 & 0.000 & -0.000 & -0.001 & -0.001 & 0.007 & 0.000\\ 0.000 & -0.283 & 0.716 & 0.006 & 0.001 & -0.006 & 0.000 & 0.283 & 0.141 & 0.018 & -0.010 & 0.023 \end{pmatrix} \quad (22)$$

VI. REMARKS

Looking over the results, purchasing in N1, N2 and N3 are dominant and we can observe the same level

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shift from N1 to N1 and N2 to N2. It is quite natural for the consumers to buy the same level goods to purchase in the next time after they are satisfied with the first time purchasing. It may be required that N1, N2 and N3 classes should be separated into the more fine classes in the near future. Considerable lower shifts could be seen in $\mathbf{X}_{t-2}$ to $\mathbf{X}_{t-1}$ and on the contrary considerable upper shifts could be seen in $\mathbf{X}_{t-1}$ to $\mathbf{X}_t$. Shop owner thought that consumers would buy the upper class brand goods after they are satisfied with the purchasing. It may be true, but the results show that the consumers would buy upper shift brand goods after the second time purchasing, i.e., third time purchasing. They will have reliance to the shop after the second purchasing, and then they will become the repeater of the shop.

VII. CONCLUSION

Consumers often buy higher grade brand products as they are accustomed or bored to use current brand products they have. Formerly we have presented the paper and matrix structure was clarified when brand selection was executed toward higher grade brand. Matrix structure was analyzed for the case brand selection was executed for upper class by our research. In this paper, equation using transition matrix was extended to the second order lag and the method was newly re-built. In the numerical example, matrix structure's hypothesis was verified. We could utilize the data as it is for the data in which time lag exist by this new model and estimation accuracy of parameter became more accurate and forecasting became more precise. This method can be utilized for building useful and effective marketing plan. It is our future research to investigate other cases such as automobile purchasing case, brand bag purchasing case etc. Various cases should be examined hereafter in order to verify obtained result.

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