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# Pure Bending Analysis of SCSS and CSSS Platforms using New Approach

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***Abstract:*** ***This paper presents new approach to pure bending analysis of platforms with one clamped and three simply supported edges (SCSS and CSSS). Taylor-Mclaurin's polynomial shape function was derived and substituted into the Galerkin's functional that were minimized to obtain the coefficient of the maximum deflection of the platform. The aspect ratios from 0.1 to 2.0 with 0.1 increments were considered. The coefficient of the deflection and the deflection function profile were used to determine the centre of platform deflections for different aspect ratios. These centre of platform deflection were compared with those from previous studies. For aspect ratio of 1.0, the centre of platform deflection parameter is 0.0028 $qb^4/D$.Comparison of the centre of platform deflections from this study and previous studies showed that significant difference does not exist.***

*Key words:* **SCSS and CSSS, Galerkin's functional, Taylor-Mclaurin's polynomial shape function, centre of platform deflection.**

## I. INTRODUCTION

In this era applications of platform plays a major role in engineering structures like in architectural structures, bridges, oil platforms, helipad, hydraulic structures, pavements, containers, airplanes, ships and other structural components steel materials. It's very important to study their maximum deflection and slopes under uniformly distributed loads in order to understand their behavior and possible conditions of failure. The important factors on which the bending platforms depend are the load conditions and the support conditions.

The problem of some rectangular platform carrying a uniformly distributed load is one of the great challenge and many papers have been written based on some platforms. Many authors have calculated the deflections of uniformly loaded rectangular platforms with SCSS and CSSS. Some of them are Timoshenko and Krieger (1959), Variddhi & Nuttawit (2006) and Shuang (2007). This paper analyzed the maximum deflections of SCSS and CSSS rectangular platform under uniformly distributed loads. In this paper, Galerkin's functional and Taylor polynomial shape function presented here is approximate solution of SCSS and CSSS rectangular platform problem under uniformly distributed loads and boundary conditions. This new approach is found to be easier and more effective.

## II. FORMULATION OF GALERKIN'S FUNCTIONAL EQUATION

From the principle of the theory of elasticity, the governing differential equation for pure bending of rectangular platform is given as equation (2.1)

$$D\left(\frac{\partial^4 w}{\partial x^4}+\frac{2\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)-q \qquad (2.1)$$

Equation (2.1) can be of the form (Ventsel and Krauthammer, 2001)

$$\nabla^4 w(x,y) = \frac{q(x,y)}{D} \qquad (2.2)$$

$$\text{Where } \nabla - \text{is the biharmonic Operator}, (x,y) - \text{ is the co-ordinates of the platform},$$

$$q - \text{ is the applied load}, \quad D - \text{ is flexural rigidity of the platform}$$

Zhan and Ma (1945) gave the weighted residual function

$$R(x) = D[u(x)] = L[U(x)] + F(x) \qquad (2.3)$$

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Equation 2.2, after some mathematical operations, becomes

$$R(x,y) = \int_{x1}^{x2}\int_{y1}^{y2} W(x,y)(D[u(x,y)]\partial x\partial y = 0 \qquad (2.4)$$

Ventsel and Krauthammer (2001) gave the general Galerkin expression as

$$\iint(L(W_N) - q]f_i(x,y)dx\partial y = 0 \qquad (2.5)$$

Where $L(W_N)$ is equal to $D\nabla^4W$; $f_i(x,y)$ is the trial function; q is the force.

However,

$$D[U(x,y)]) = [L(W_N) - q] = \left[\frac{\partial^4 W}{\partial x^4} + \frac{2\partial^4 W}{\partial x^2\partial y^2} + \frac{\partial^4 w}{\partial y^4} - \frac{q}{D}\right] \qquad (2.6)$$

Substituting equation (2.6) into equation (2.4) gave;

$$\int_{x1}^{x2}\int_{y1}^{y2}\left[\frac{\partial^4 W}{\partial x^4} + \frac{2\partial^4 W}{\partial x^2\partial y^2} + \frac{\partial^4 w}{\partial y^4} - \frac{q}{D}\right] f_i(x,y) = G \qquad (2.7)$$

Hence, equation (2.7) is the Galerkin expression for rectangular platform in bending problem under uniform distributed transverse loads.

Expressing equation (2.7) in dimensionless co-ordinate of R and Q axes for aspect ratio, p = b/a gave:

$$\frac{1}{a^2}\int_0^1\int_0^1[\alpha\left(\frac{\partial^4(f_i)}{\partial R^4} + \frac{2\,\partial^4(f_i)}{p^2\,\partial R^2\,\partial Q^2} + \frac{\partial^4(f_i)}{p^4\,\partial Q^4} - \frac{qa^2}{D}\right)]\, f_i\,\partial R\,\partial Q = G \qquad (2.8)$$

$$where: X = aR, Y = BQ \text{ and aspect ratio } P = ab(b\frac{a}{p})$$

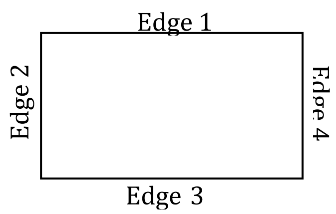
Equation (2.8) is the general Galerkin's functional for pure bending platforms in non-dimensional form.

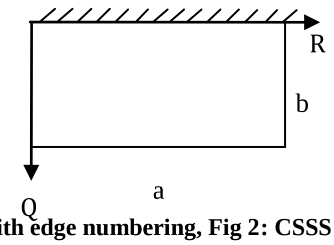
## III. DEFLECTION EQUATION AND BOUNDARY CONDITIONS

Figures 1, 2 and 3 show the SCSC and cscs platfoms in R and Q coordinates and the support numbering. Ibearugbulem (2012) gave the polynomial shape function as:

$$W= (a_0+a_1R+a_2R^2+a_3R^3+a_4R^4)\,(b_0+b_1Q+b_2Q^2+b_3Q^3+b_4Q^4) \quad = \alpha f_i \qquad (3.1)$$

Where: W denotes deflection function; $\alpha$ is constant, $f_i$ is derived shape function



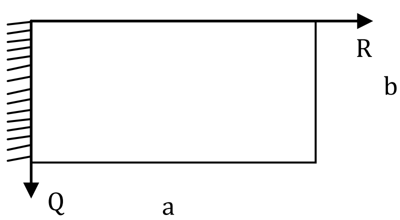




**Fig 1: rectangular platform with edge numbering, Fig 2: CSSS platform Fig 3 : SCSS platform**

FIGURE 1 FIGURE 2 FIGURE 3

**BOUNDARY CONDITIONS**

The boundary conditions of CSSS and SCSS platforms are:

FOR CSSS

$W(R,0) = W'(R,0) = 0$

$W(R,1) = W''(R,1) = 0$

$W(Q,0) = W''(Q,0) = 0$

$W(Q,1) = W''(Q,1) = 0$

FOR SCSS

$W(Q,0) = W'(Q,0) = 0$

$W(Q,1) = W''(Q,1) = 0$

$W(R,0) = W''(R,0) = 0$

$W(R,1) = W''(R,1) = 0$

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Substituting the boundary conditions above in the equation (3.1) gave;

For CSSS, $w = \alpha\,(1.5R^2 - 2.5R^3 + R^4)\,(Q-2Q^3+Q^4)$ (3.2)

For SCSS, $w = \alpha\,(1.5Q^2 - 2.5Q^3 + Q^4)\,(R-2R^3+R^4)$ (3.3)

Where $\alpha = a_4 b_4$

The equation (3.2) and (3.3) is the shape function for CSCS and SCSC platforms.

Substituting the shape functions into the Galerkin's functional one after other and also integrate them with the respect to $\partial R$ and $\partial Q$

$$\int_0^1\int_0^1 \frac{\partial^4 f_i}{\partial R^4}\cdot f_i \partial R \partial Q = 24\left(\frac{1}{3}-\frac{4}{5}+\frac{2}{6}+\frac{4}{7}-\frac{4}{8}+\frac{1}{9}\right)\left(\frac{1.5}{3}-\frac{2.5}{4}+\frac{1}{5}\right)$$

$$= 0.0885708 \qquad (3.4)$$

$$\int_0^1\int_0^1 \frac{\partial^4 f_i}{\partial R^2 \partial Q^2}\cdot f_i\, \partial R \partial Q = \left(\frac{4.5}{3}-\frac{30}{4}+\frac{58.5}{5}-\frac{45}{6}+\frac{12}{7}\right)$$

$$\left(\frac{12}{3}-\frac{12}{4}+\frac{24}{5}-\frac{24}{6}+\frac{1}{7}\right) = 0.0416321 \qquad (3.5)$$

$$\int_0^1\int_0^1 \frac{\partial^4 f_i}{\partial Q^4}\cdot f_i\, \partial R \partial Q = 24\left(\frac{2.25}{5}-\frac{7.5}{6}+\frac{9.25}{7}-\frac{5}{8}+\frac{1}{9}\right)\left(\frac{1}{2}-\frac{2}{4}+\frac{1}{5}\right)$$

$$=0.03619056 \qquad (3.6)$$

$$\int_0^1\int_0^1 f_i \partial R \partial Q = \left(\frac{1.5}{3}-\frac{2.5}{4}+\frac{1}{5}\right)\left(\frac{1}{2}-\frac{2}{4}+\frac{1}{5}\right)$$

$$(0.075)\,(0.2) = 0.015 \qquad (3.7)$$

Substituting equation (3.4), (3.5), (3.6) and (3.7) into equation function.

For CSSS platform the equation becomes:

$$\alpha\left[[0.08857080 + 2\frac{0.00293877}{P^2} + \frac{0.03619036}{P^4}]\right] - \frac{0015\,a^4 q}{PD} \qquad (3.8)$$

For SCSS platform the equation becomes:

$$\left[\alpha\left[0.03619036 + 2\frac{0.00293877}{P^2} + \frac{0.0885708}{P^4}\right]\right] - \frac{0015 a^4 q}{D} \qquad (3.9)$$

## IV. RESULTS OF CSSS and SCSS RECTANGULAR PLATFORM

The table 4.1 shows the maximum deflection parameters for CSSS and SCSS platforms from the present study. Tables 4.2 and 4.3 show the comparison of the solutions from this present study and the previous studies for different aspect ratios. The result of the table 4.2 in the present study was compared with the result from Timoshenko and Krieger (1959) and Shuang (2007). The average percentage difference between the solution from previous works in this exact solution and the present study based are 2.43% and 4.19% respectively. The closeness of the three solutions increased as the aspect ratio increases from 0.1 to 2. With this we can say the solution of this present study was a very close exact solution. the table 4.3 result of the present study were also compared with SCSS platform, the average percentage difference between the solution from Timoshenko and Krieger (1959) , Variddhi & Nuttawit (2006) and the present study based are 0.9561% and 2.414% respectively.

Table 4.1: Values for maximum deflection parameter for Different Aspect Ratio for CSSS and SCSS thin Platforms under Pure Bending from the present study

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(a) For CSSS

| Aspect ratio (P) | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 |
|---|---|---|---|---|---|---|---|---|---|---|
| Maximum Deflection parameters | 0 | 0.0000 | 0.0001 | 0.0003 | 0.0006 | 0.00010 | 0.0014 | 0.0019 | 0.0024 | 0.0028 |

| Aspect ratio (P) | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 | 1.7 | 1.8 | 1.9 | 2 |
|---|---|---|---|---|---|---|---|---|---|---|
| Maximum Deflection parameters | 0.0032 | 0.0036 | 0.0039 | 0.0042 | 0.0044 | 0.0046 | 0.0048 | 0.0050 | 0.0051 | 0.0052 |

(b) For SCSS

| Aspect ratio (P) | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 |
|---|---|---|---|---|---|---|---|---|---|---|
| Maximum Deflection parameters | 0 | 0.0000 | 0.0000 | 0.0001 | 0.0003 | 0.0006 | 0.0010 | 0.0015 | 0.0021 | 0.0035 |

| Aspect ratio (P) | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 | 1.7 | 1.8 | 1.9 | 2 |
|---|---|---|---|---|---|---|---|---|---|---|
| Maximum Deflection parameters | 0.0035 | 0.0043 | 0.0050 | 0.0058 | 0.0065 | 0.0071 | 0.0077 | 0.0083 | 0.0089 | 0.0094 |

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**Table 1: Values for Different Aspect Ratio for CSSS Thin Platform under Pure Bending**

| Aspect ratio (P) | 1 | 2 | 3 | % diff 1 and 2 | % diff 1 and 3 |
|---|---|---|---|---|---|
| 1 | 0.0028 | 0.0028 | 0.00279 | 0.00 | 1.119549 |
| 1.1 | 0.0032 | 0.0032 | | 0.55 | |
| 1.2 | 0.0036 | 0.0035 | | 2.18 | |
| 1.3 | 0.0039 | 0.0038 | | 2.45 | |
| 1.4 | 0.0042 | 0.004 | | 4.28 | |
| 1.5 | 0.0044 | 0.0042 | 0.00425 | 5.11 | 3.888627 |
| 2 | 0.0052 | | 0.00488 | | 7.57603 |

**(1) = present studies ,(2) = Timoshenko and Krieger and (3) = Shuang**

**Table 2: Values for Different Aspect Ratio for SCSS Thin Platform under Pure Bending**

| Aspect ratio (P) | 1 | 2 | 3 | % diff 1 and 2 | % diff 1 and 3 |
|---|---|---|---|---|---|
| 0.9 | 0.0021 | | | | |
| 1 | 0.0028 | 0.0028 | 0.0028 | 0.59 | 0.594246 |
| 1.1 | 0.0035 | 0.0035 | | 1.15 | |
| 1.2 | 0.0043 | 0.0043 | | -0.34 | |
| 1.3 | 0.0050 | 0.005 | | 0.61 | |
| 1.4 | 0.0058 | 0.0058 | | -0.69 | |
| 1.5 | 0.0065 | 0.0064 | 0.0061 | 0.95 | 5.911661 |
| 2 | 0.0094 | 0.0093 | 0.0093 | 0.74 | 0.735957 |

**(1) = present studies, (2) = Timoshenko and Krieger and (3) = Variddhi and Nuttawit**

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