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A Conservative Scheme Model of an Inclined Pad Thrust Bearing

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***Abstract:* Inclined pad thrust bearings are designed for high axial loads at high speeds and to reduce costs. In this paper, formulation of Reynolds equation governing the pressure distribution for the bearing is done in two dimensions and reduced into a conservative form. A finite difference scheme in conservative form is developed and used to convert the terms of the Reynolds equation into a set of simultaneous linear algebraic equations. A solution procedure for finding the value of the pressure in the oil film is described. R software programming language is used to fit a parametric function on the pressure values that on integration gives the axial load. The thrust bearing behaviour in a realistic configuration allowing sufficient parameter variation to check validity of the numerical model is developed supporting the validity and accuracy of the method. The results obtained here is a useful input for rotor dynamics especially in designing fluid dynamic bearings.**

Index Terms—**Axial load, Reynolds equation, pressure, thrust bearing.**

I. INTRODUCTION

Hydrodynamic Thrust bearings are used widely for high load carrying capacity at high speeds with reduced costs. Reynolds equation is solved to determine the bearing performance. To solve this equation, one requires knowledge about lubricant properties such as viscosity and density. In this model viscosity is treated as a function of pressure. A number of models of hydrodynamic thrust bearing performance have been proposed over the years. Kim *et al* [1] studied a three dimensional Thermo hydrodynamic bearing model taking into account radial tilt neglecting the elastic and thermal distortion of the bearing surfaces. In their study viscosity and density were treated as functions of the temperature of the pad and assumed the oil inlet temperature to be uniform. Rodkiewiez and Yang [2] studied an infinitely long centrally pivoted thrust bearing with both pressure and temperature dependent viscosity and density along with elastic and thermal deformation of bearing components by introducing a comprehensive term in the heat equation was included. Brockett *et al* [3] proposed a model for a fixed geometry of a fixed geometry thrust bearing that included temperatures and deformations in both runner and pad where viscosity as a function of both temperature and pressure. Almqvist *et al* [4] studied the Thermo hydrodynamic Lubrication analysis of tilting pad step bearing where a comparison was made between theory and experiments where viscosity and density are treated as functions of both temperature and pressure. Youngson [5] developed a mixed lubrication model to investigate the lubrication of coupled journal-thrust beaming systems. A conformal mapping method was implemented in the model formulation to facilitate a universal flow description of the governing equations into a computational domain. Neminath and Gudadappagouda [6] studied the dynamic Reynolds equation for micro polar fluid lubrication of porous slider bearing. In the study the rheological effects of micro polar fluid lubricants on the steady state and dynamic behavior of porous slider bearings by considering the squeezing action was made. Srikanth *et al*. [7] modeled a large Tilting pad Thrust bearing stiffness and damping coefficients where a formulation of Reynolds' equation for the bearing is done in two dimensions. A finite difference method is used to convert the terms of the Reynolds' equation in to a set of simultaneous linear algebraic equations. Recently, Gultekin [8] studied the effects of the total bearing deformation on the performance of hydrodynamic thrust bearings. In the study, the pressure distribution in the thrust bearing was obtained using the Reynolds equation for the case of stable lubricant viscosity and isothermal conditions. Then, the deformation is found out by applying the constitutive equations for the linear elastic materials to both pad and runner. Although many investigations have been made in the past two decades, only a few published papers take into account the true geometry of the pad and the incline. The pad geometry is represented as a polar rectangle in the $xy-$plane and mapped onto an ordinary rectangle in the $r,\theta-$ plane with the pad inclination taken as a function of the radial component θ. The conservative scheme developed here addresses the issue of the pad geometry.

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II. MATHEMATICAL FORMULATION

The two dimensional Reynolds equation in cylindrical form is given by Srikanth [7] as;

$$\frac{\partial}{\partial r}\left[\frac{rH^3}{\mu}\frac{\partial P}{\partial r}\right]+\frac{1}{r}\frac{\partial}{\partial\theta}\left[\frac{H^3}{\mu}\frac{\partial P}{\partial\theta}\right]=6u_\theta r\frac{\partial H}{\partial\theta} \quad (1)$$

On making usual assumptions in the analysis done herein, the Reynolds equation in non - dimensional form is:

$$r\frac{\partial}{\partial r}\left[\frac{rH^3}{\mu}\frac{\partial P}{\partial r}\right]+\frac{\partial}{\partial\theta}\left[\frac{H^3}{\mu}\frac{\partial P}{\partial\theta}\right]=6St\left[\frac{L_0}{H_0}\right]^2 r^2\frac{\partial H}{\partial\theta} \quad (2)$$

To obtain the solution to the non - dimensional equation (2), over one pad to obtain the pressure profiles, the pad geometry which represents a polar rectangle in the $x, y-$plane is transformed to

an ordinary rectangle in the $r,\theta-$plane using the transformation equations $x = r\cos\theta$ and $y = r\sin\theta$ whose Jacobian of transformation is;

$$\frac{\partial(x,y)}{\partial(r,\theta)}=\begin{vmatrix}\frac{\partial x}{\partial r} & \frac{\partial y}{\partial r}\\ \frac{\partial x}{\partial\theta} & \frac{\partial y}{\partial\theta}\end{vmatrix}=\begin{vmatrix}\cos\theta & -r\sin\theta\\ \sin\theta & r\cos\theta\end{vmatrix}=r \quad (3)$$

The transformed plane of reference is as shown in Figure 1 below.

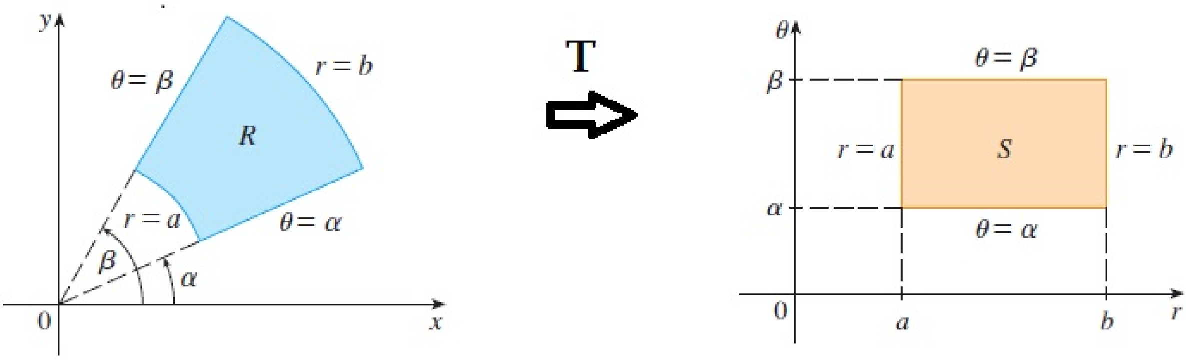

Fig 1: Transformed coordinate system.

On obtaining a uniform grid, we adopt the numerical procedure as proposed by Morinishi [9] by letting $r = r(\eta^r)$, $\theta = \theta(\eta^\theta)$ $\eta^r = \eta^r(r)$, $\eta^\theta = \eta^\theta(\theta)$, $h_r = \frac{1}{h_r}\frac{dr}{d\eta^r}$, $h_\theta = \frac{1}{h_\theta}\frac{d\theta}{d\eta^\theta}$ and $J = h_r h_\theta$.

Such that;

$$r\frac{\partial}{\partial r}=\frac{1}{h_r}\frac{\partial}{\partial\eta^r}, \frac{\partial}{\partial\theta}=\frac{1}{h_\theta}\frac{\partial}{\partial\eta^\theta} \quad (4)$$

Using (4) in (2) and doing the necessary expansions yields;

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$$\frac{1}{h_r}\left[\frac{1}{h_r\mu}\frac{\partial P}{\partial \eta^r}\frac{\partial H^3}{\partial \eta^r}+\frac{H^2}{\mu h_r}\frac{\partial^2 P}{\partial \eta^{r2}}+H^3\frac{\partial P}{\partial \eta^r}\frac{\partial(1/h_r\mu)}{\partial \eta^r}\right]+$$
$$\frac{1}{h_\theta}\left[\frac{1}{h_\theta\mu}\frac{\partial P}{\partial \eta^\theta}\frac{\partial H^3}{\partial \eta^\theta}+\frac{H^3}{\mu h_\theta}\frac{\partial^2 P}{\partial \eta^{\theta 2}}+\frac{H^3}{h_\theta}\frac{\partial P}{\partial \eta^\theta}\frac{\partial(1/\mu)}{\partial \eta^\theta}\right]=\frac{M_1}{h_\theta}\frac{\partial(r^2 H)}{\partial \eta^\theta} \quad (5)$$

This is simplified to;

$$\frac{1}{h_r}\left[\frac{H^3}{\mu h_r}\frac{\partial^2 P}{\partial \eta^{r2}}+H^3\frac{\partial P}{\partial \eta^r}\frac{\partial(1/h_r\mu)}{\partial \eta^r}\right]+\frac{1}{h_\theta}\left[\frac{1}{h_\theta\mu}\frac{\partial P}{\partial \eta^\theta}\frac{\partial(H^3/\mu)}{\partial \eta^\theta}+\frac{H^3}{\mu h_\theta}\frac{\partial^2 P}{\partial \eta^{\theta 2}}\right]=\frac{M_1}{h_\theta}\frac{\partial(r^2 H)}{\partial \eta^\theta} \quad (6)$$

III. COMPUTATIONAL PROCEDURE

The approximation the derivatives in the governing equations is based on the interconnection of five points in a five – point stencil discretization difference method where the grid point (i, j) and its four neighbors are used. The five points are displayed in the figure below. A total of 441 nodes in the form of the grid shown in Figure 2 below are used.

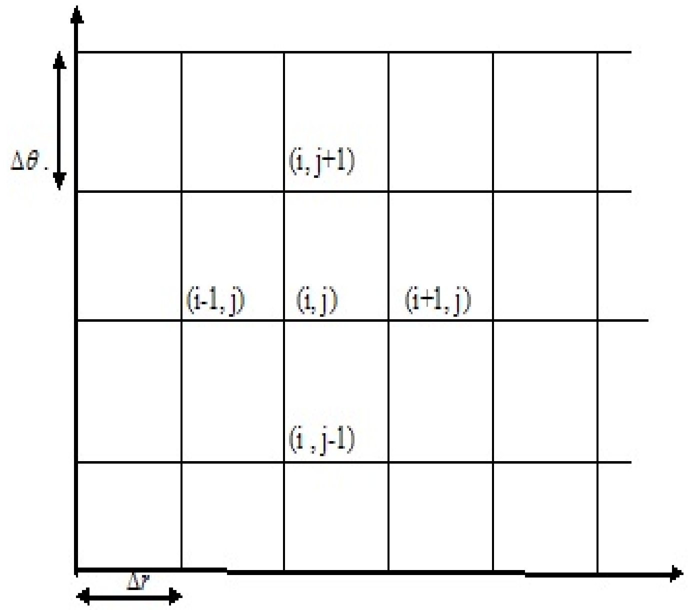

Fig 2: Discretization of Pad for Reynolds equation [7].

Equation (6) is now solved to obtain the pressure distribution over one pad. Letting $h_r = \frac{\Delta r}{r}$ and $h_\theta = \Delta\theta$ where Δr and $\Delta\theta$ are the grid spacing at a defined point (r,θ) in the physical domain. We apply the approximations for the first and second order partial derivatives in the equation as proposed by Morinishi [9] to have;

$$\frac{H^3}{\mu h_r^2}\left[\frac{p_{i+1,j}-2p_{i,j}+p_{i-1,j}}{(\Delta\eta^r)^2}\right]+\frac{H^3}{h_r}\left[\frac{p_{i+1,j}-p_{i-1,j}}{2\Delta\eta^r}\right]\frac{\partial}{\partial\eta^r}\left[\frac{1}{h_r\mu}\right]+$$

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$$\frac{1}{h_\theta^2}\frac{\partial\left(H^3/\mu\right)}{\partial\eta^\theta}\left[\frac{p_{i,j+1}-p_{i,j-1}}{2\Delta\eta^\theta}\right]+\frac{H^3}{h_\theta^2\mu}\left[\frac{p_{i,j+1}-2p_{i,j}+p_{i,j-1}}{\left(\Delta\eta^\theta\right)^2}\right]=\frac{M_1}{h_\theta}\frac{\partial\left(r^2H\right)}{\partial\eta^\theta} \quad (7)$$

Making $p_{i,j}$ the subject yields;

$$p_{i,j}=\left\{\frac{-H^3}{\mu h_r^2}\left[\frac{p_{i+1,j}+p_{i-1,j}}{\left(\Delta\eta^r\right)^2}\right]-\frac{H^3}{h_r}\left[\frac{p_{i+1,j}-p_{i-1,j}}{2\Delta\eta^r}\right]\frac{\partial}{\partial\eta^r}\left[\frac{1}{h_r\mu}\right]-\frac{1}{h_\theta^2}\frac{\partial\left(H^3/\mu\right)}{\partial\eta^\theta}\left[\frac{p_{i,j+1}-p_{i,j-1}}{2\Delta\eta^\theta}\right]\right\}$$

$$-\frac{H^3}{\mu h_\theta^2}\left[\frac{p_{i,j+1}+p_{i,j-1}}{\left(\Delta\eta^\theta\right)^2}\right]+\frac{M_1}{h_\theta}\frac{\partial\left[r^2H\right]}{\partial\eta^\theta}-\left[\frac{2H^2}{h_r^2\mu\left(\Delta\eta^r\right)^2}+\frac{2H^2}{h_\theta^2\mu\left(\Delta\eta^\theta\right)^2}\right] \quad (8)$$

Here the bearing is assumed to work in the full film lubrication condition regime, where the boundary conditions are zero pressure at the periphery of the pad and a constant pressure of 2000 is maintained at the leading and trailing edges. The fluid film thickness is expressed as a function of θ; and is given as;

$$H=\left[\frac{H_1-H_0}{\beta}\right]\theta+H_0 \quad (9)$$

Where the internal and external radii being chosen as 0.5 and 1.5 respectively.

Viscosity Model

The viscosity model used for this model is pressure dependent. That is we adopted the Barus expression. $\mu=\mu_0e^{\alpha P},\alpha\geq 0$.

Axial Load

Integrating the pressure over the bearing surface gives the load. 441 values of pressure are generated. These values of pressure are dependent on the radius r and angle θ. R software programming language, a software environment for statistical computing and graphics is used to fit a parametric function for $P(r,\theta)$ and hence integration to obtain the load is done. Mathematically the load is expressed as;

$$W_t=\frac{2\pi}{\beta}\int_\alpha^\beta\int_a^b P(r,\theta)rdrd\theta \quad (10)$$

IV. RESULTS AND DISCUSSION

A glimpse on the outlook of the numerical solutions of pressure and axial load is provided below.

In figure 3 (a) five pads are used, in figure 3 (b) ten pads are used while in figure 3 (c) fifteen pads are used. The Stribeck number is maintained at 18. This is to ensure a hydrodynamic lubrication is maintained. The pressure distributions for different number of pads have discrepancies in that the as we the number of pads used is increased, a higher maximum pressure is attained. This because the increase in the number of pads results to a decrease in fluid film thickness and hence the pressure within the fluid mass increases. There is a good agreement in the pressure build up from the leading edge up to around 75 percent of the pad, followed by a drop pressure towards the trailing edge. This is because as the fluid enters the pad from the leading edge from the grooves, there is a decrease in the velocity that causes a decrease in minimum film thickness hence pressure

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increases. The sharp drop in pressure is due to the increase in velocity as the fluid escapes from the pads back to the grooves resulting to an increase in film thickness thus decrease in pressure.

In figure (4) curves are shown for simulation results for pressure profiles various number of pads for r=1, the central radius within the pad. The variations of the pressure profiles are due to changes in inertia at the edges of the grooves resulting in reduced pressure at the leading edge of the groove and a rise in pressure at the trailing edge. This is as a result of the recirculation flow in the groove accelerating the oil entering the groove.

Figure (5) shows the pressure distribution against the external load carried by the bearing. Accruing from the results, distribution matches with the load carrying capacity of the bearing. This is because the load carrying capacity of a bearing is determined by the magnitude of the pressure distribution that is as a result of the relative movement between the two surfaces of the bearing which generates a lubricant velocity field.

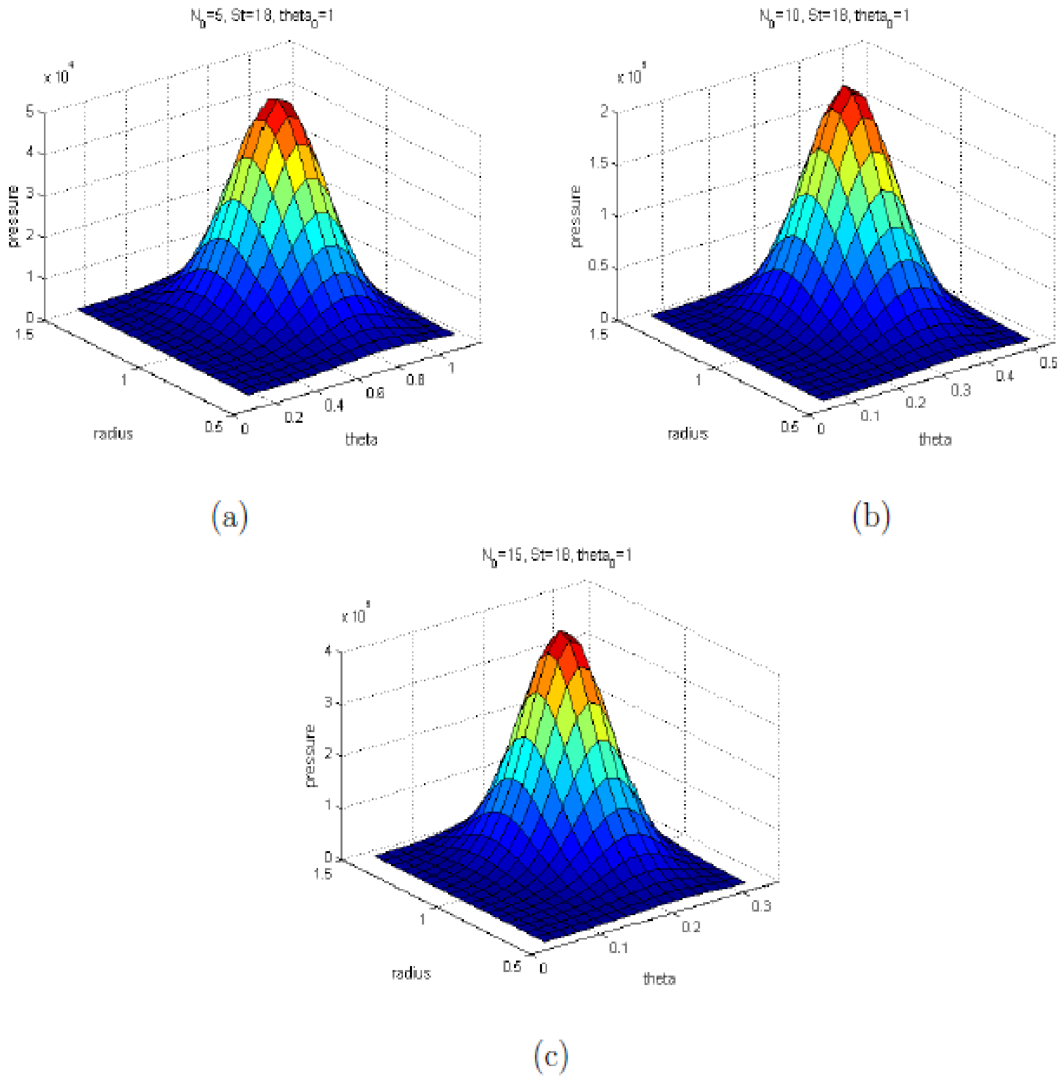

Fig 3: A comparison of simulation results for the pressure profiles for various numbers of pads. In (a) five pads are used, in (b) ten pads are used while in (c) fifteen pads are used. The Stribeck number is maintained at 18.

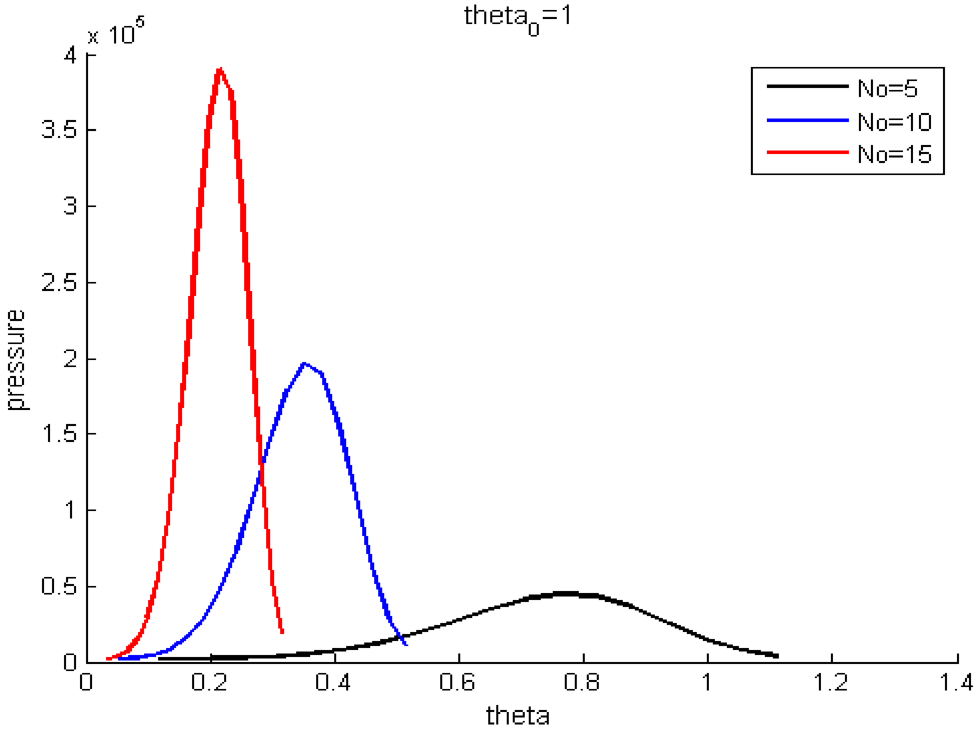

Fig 4: A comparison of simulation results for pressure profiles various number of pads for r=1.}

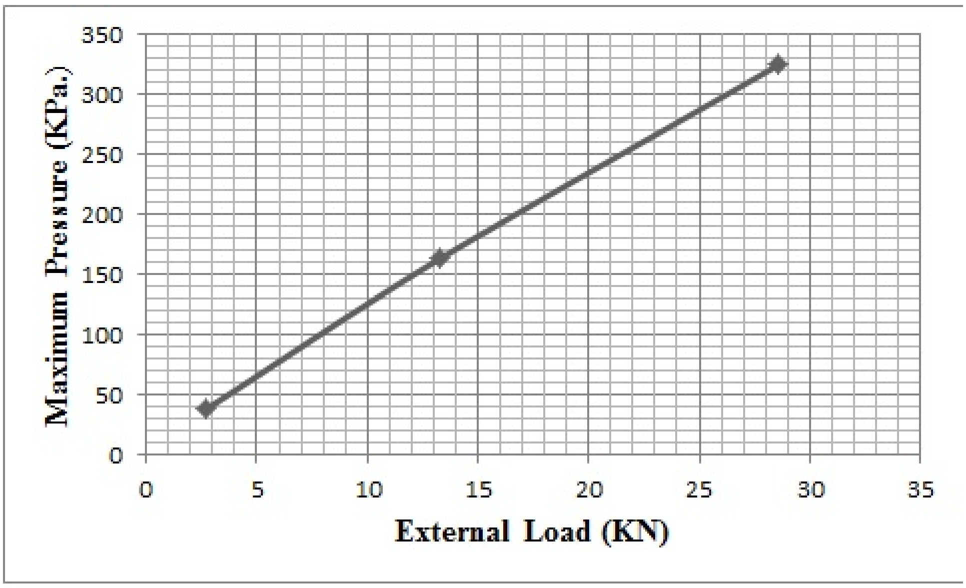

Fig 5: External Load - Maximum Pressure

IV. CONCLUSION

A two dimensional Reynolds equation is modified into conservative form and a finite difference method based solution procedure for computing pressure values is verified. Parametric regression is thus used to determine pressure as a function of the radius r and the circumferential distance θ and on integration of these pressure points gives the load. There is good agreement between the results of this model and the results published earlier by different authors. This proves the fidelity of our model and that of the software package that was used determine the load. The authors however recommend an improvement of the model to include a temperature dependent viscosity.

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NOMENCLATURE

r : Radial coordinate, m

Δr : Division of grid along radial direction, m

μ : Viscosity of oil, Pa.s

θ : Angle from the leading edge to the trailing edge, rad

$\Delta\theta$: Angular division of the grid, rad

W_t : Bearing load, N

P: Pressure, Pa

H: oil film thickness, m

H_0, H_1 : Oil film thickness in the leading and trailing edges, m

H_1 : Oil film thickness in the trailing edge, m

i, j : Index of node in radial and circumferential directions

β : Circumferential length of the pad, rad

S_t : Stribeck number

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