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The Behaviour of Buckled CSFS Isotropic Rectangular Plate Using Polynomial Series Shape Function on Ritz Method

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Abstract*—The behavior of a buckled isotropic rectangular plate of Clamed-Simply-Free-Simple (CSFS) plate was critically examined in this paper; this was accomplished by truncating the polynomial series at the fifth orthogonal terms, which satisfied the boundary condition leading to a particular shape function that was applied in the Ritz method. This shape function was substituted into the total potential energy functional, which was subsequently minimized and the critical buckling load was obtained. The resulting values of non dimensional parameters of the buckling load were compared with the previous research works of Iyengar, Timoshenko and Ibearugbulem within the range of aspect ratios from 0.1 to 2.0. A graph of critical buckling load against aspect ratio was plotted. The behavior of the graph shows that as the aspect ratios increases from 0.1 to 2.0, the critical buckling load decreases. For aspect ratios of 0.2; 0.4; 0.8; 1.0, the values of non dimensional parameters of the buckling load were 25.35; 6.61; 2.08; 1.63 respectively. It was discovered that for the aspect ratio of 0.2; 0.4; 0.8, the percentage difference between the critical buckling load of the present study and those of Iyengar and Ibearugbulem are -0.00394%; 4.3217%; -3.9352% and 37.2442%, 8.6043%, -14.6256% respectively. For aspect ratio of 1.0, the percentage difference between the present study and Timoshenko is -4.1765%.

***Keywords*—Critical Buckling Load, Shape Function, Polynomial Series, Total Potential Energy Functional, Boundary Condition, Isotropic Rectangular Plate.**

I. INTRODUCTION

The buckling behaviour of an isotropic rectangular plate occurs when compression plus bending loads are applied to a structural member, plate elements of the member can be subjected to in-plane stresses which vary along the loaded edges of the plate. The elastic critical buckling of the plate stresses is dependent on the edge-support condition and the ratio of bending stress to uniform compression stress. It is common, though artificial, to use the elastic critical buckling stress as a benchmark for delineating different forms of plate buckling. Yu and Schafer [1] analyzed the effect of longitudinal stress gradients on the elastic buckling of thin isolated plate. They observed two types of thin plate, which are (i) a plate simply supported on all four edges and rotationally restrained on two longitudinal edges (ii) a plate simply supported on three edges with one longitudinal edge free and the opposite longitudinal edge rotationally restrained. Wang, Wang and Shi [2] used the new version of differential quadrature method to obtain buckling loads of thin rectangular plates under non-uniform distributed in-plane loading.

Iberugbulem, O. M [3] solved a problem of compressed SSSS thin rectangular plate using Taylor- Mclaurin series as the shape function of the buckled plate. Timoshenko and Woinowsky-Krieger [4], Ugural [5], Ye [6], used Fourier Series to analyze rectangular plate. Plaut and Guran [7] studied buckling of rectangular plate under uniaxial loading by using plates and loads that are uniform. The loaded edges were restrained in position and direction while the unloaded edges were simply supported. Seung-Eockkim, Huu-Tai Thai, Jachong Lee [8], used the two variable refined plate theories to study the buckling analysis of isotropic and orthotropic plate. Their theory takes account of transverse shear effects and parabolic Distribution of the transverse shear strains through the thickness of the plate.

This paper used polynomial shape function in Ritz method to analyze the critical buckling load of a plate with clamped-simply-free-simply support. It satisfied the non-essential (force, dynamic or mechanical) boundary conditions for a rectangular plate subjected to in-plane load along one axis (x-axis) of the principal plane. The schematic diagram of the plate is shown in figure 1.

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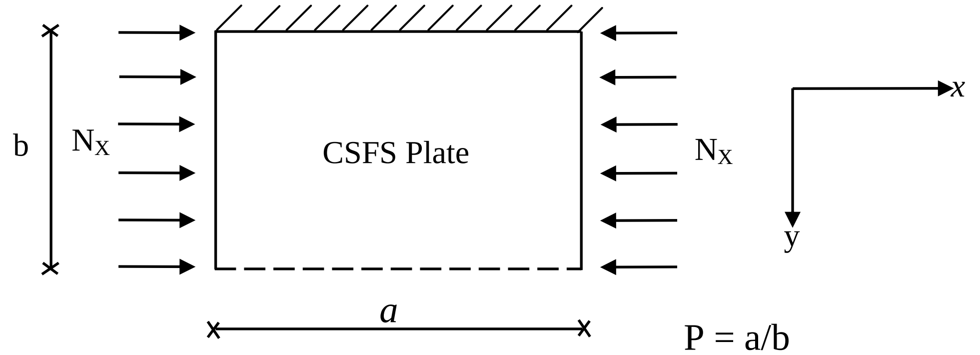


Fig 1: Schematic representation of in-plane loaded CSFS plate

NOTATION

W is the shape function (deflection function).

V is the shear force.

μ is Poisson's ratio

First partial derivative of deflection with respect to R is: $w'^{R} = \frac{\partial w}{\partial R}$.

First partial derivative of deflection with respect to Q is: $w'^{Q} = \frac{\partial w}{\partial Q}$

Second partial derivative of deflection with respect to R and Q is: $w'^{R'Q} = \frac{\partial^2 w}{\partial R \partial Q}$

Second partial derivative of deflection with respect to R is: $w''^{R} = \frac{\partial^2 w}{\partial R^2}$

Second partial derivative of deflection with respect to Q is: $w''^{Q} = \frac{\partial^2 w}{\partial Q^2}$

Third partial derivative of deflection with respect to R and Q is: $w''^{R'Q} = \frac{\partial^3 w}{\partial R^2 \partial Q}$

II. TOTAL POTENTIAL ENERGY FUNCTIONAL

Ibearugbulem (2011) derived the total potential energy functional for a rectangular isotropic plate subjected to in-plane load in x – direction and polynomial shape function as follows:

$$\pi_x = \frac{D}{2b^2}\int_0^1\int_0^1\left[\frac{b^3}{a^3}\left(w''^{R}\right)^2 + \frac{a}{b}\left(w''^{Q}\right)^2 + \frac{2b}{a}\left(w'^{R'Q}\right)^2\right]\partial R\,\partial Q$$

$$-\frac{bN_x}{2a}\int_0^1\int_0^1\left(w'^{R}\right)^2\partial R\,\partial Q \qquad (1)$$

$$W = \sum_{m=0}^{4}\sum_{n=0}^{4} a_m R^m . b_n Q^n \qquad (2)$$

Where "a" and "b" are plate dimensions in x and y directions. Nx is the in-plane load in x direction flexural rigidity, W is the shape function. D is the flexural rigidity.

$\pi$is the total potential energy functional along x axis, $R = \frac{x}{a}$; $Q = \frac{y}{b}$; $0 \le R \le 1$; $0 \le Q \le 1$ where R and Q are dimensionless quantities.

III. SHAPE FUNCTION FROM POLYNOMIAL SERIES

The boundary conditions for CSFS plate are:

$w_{(R=0)} = 0$; $w_{(Q=0)} = 0$; $w_{(R=1)} = 0$ (3)

$w''^{R}_{(R=0)} = 0$, $w'^{Q}_{(Q=0)} = 0$ (4)

$w''^{R}_{(R=1)} = 0$, $V_{Q(Q=1)} = [w'''^{Q} + (2-\mu)w''^{R'Q}]_{(Q=1)} = 0$ (5)

Applying these boundary conditions in equation (2) gave:

$$w = A(R - 2R^3 + R^4)(4Q^2 - 4Q^3 + Q^4) \quad (6)$$

IV. APPLICATION OF RITZ METHOD

Differentiating equation (6) partially with respect to either R or Q or both gave the following equations.

$$\omega'^R = A(1 - 6R^2 + 4R^3)(4Q^2 - 4Q^3 + Q^4) \quad (7)$$
$$\omega''^R = A(-12R + 12R^2)(4Q^2 - 4Q^3 + Q^4) \quad (8)$$
$$\omega'^Q = A(R - 2R^3 + R^4)(8Q - 12Q^2 + 4Q^3) \quad (9)$$
$$\omega''^Q = A(R - 2R^3 + R^4)(8 - 24Q + 12Q^2) \quad (10)$$
$$\omega'^{R/Q} = A(1 - 6R^2 + 4R^3)(8Q - 12Q^2 + 4Q^3)(11)$$

Integrating equations (7), (8), (10), and (11) partially with respect to R and Q in a closed domain respectively gave:

$$\int_0^1\int_0^1 (\omega'^R)^2\, \partial R\, \partial Q = A^2(0.485714285)(0.406349206) = 0.197369614A^2 \quad (12)$$
$$\int_0^1\int_0^1 (\omega''^R)^2\, \partial R\, \partial Q = A^2(4.8)(0.406349206) = 1.95047619A^2 \quad (13)$$
$$\int_0^1\int_0^1 (\omega^{11Q})^2\, \partial R\, \partial Q = A^2(0.049206349)(12.8) = 0.629841267A^2 \quad (14)$$
$$\int_0^1\int_0^1 (\omega'^{R/Q})^2\, \partial R\, \partial Q = A^2(0.485714285)(1.219047619) = 0.592108842A^2 (15)$$

Substituting equations (12), (13) and (14), (15) into equation (1), and differentiating partially with respect to A gave;

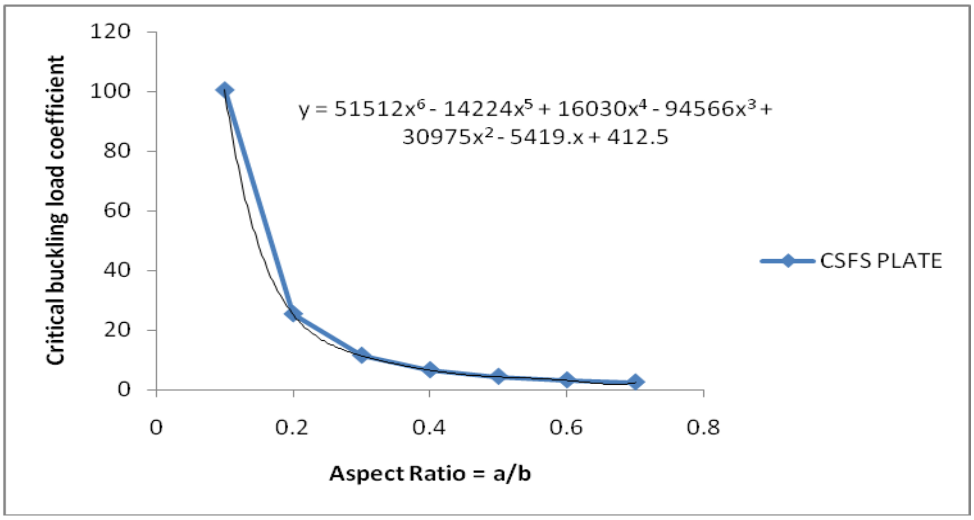
$$\frac{\pi_x}{\partial A} = \frac{DA^2}{Pa^2}(1.95047619 + 0.629841267P^4 + 0.592108842P^2) - \frac{N_xA^2}{2P}(0.197369614) = 0 \quad (16)$$

Where the aspect ratio, $P = \frac{a}{b}$(17)

Making N_x the subject of the equation

$$N_x = \frac{D\pi^2}{b^2}\left(\frac{1.0012917}{P^2} + 0.323333776P^2 + 0.30396355\right) (18)$$

V. RESULT AND DISCUSSION



(a) First segment

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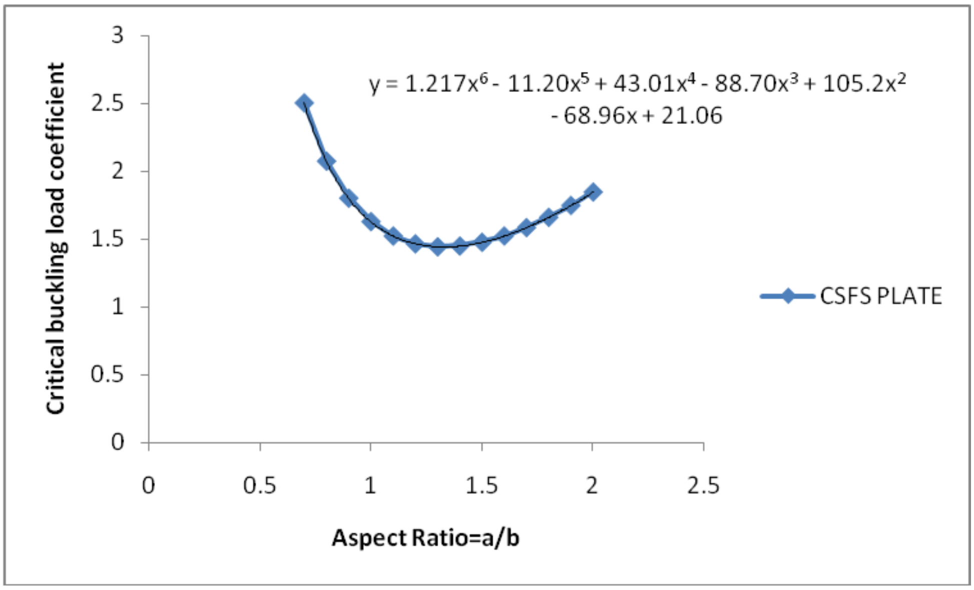

(b) Second segment

Fig. 1: Graphs of critical buckling load against aspect ratio

To examine the critical buckling load of CSFS plate, a numerical calculation was carried out with the aid of a polynomial series as shape function. A graph of critical buckling load against aspect ratio was plotted. It was discovered from the movement of the graph that as the aspect ratios increases from 0.1 to 2.0, the critical buckling load decreases. The graph was divided into two segments ranging from 0.1 to 0.7 in the first segment and from 0.7 to 2.0 in the second. The graph of both segments are of 6^{th} degree polynomial with an equations of Y= 51512X6 – 14224X5 + 16030X4 – 94566X3 + 30975X2 – 5419X + 412.5. In the second section, the graph is of 6 degree polynomial with an equation of Y = 1.217X6- 11.20X5 + 43.01X4 – 88.70X3 + 105.2X2 – 68.96X + 21.06. It was discovered that for the aspect ratio of 0.2, 0.4 and 0.8, the percentage difference between the critical buckling load of the present study and that of Iyengar and Ibearugbulem were-0.00394%, 4.3217%, -3.9352% and 37.2442%, 8.6043%, -14.6265% respectively. For aspect ratio of 1.0, the percentage difference between the present study and Timoshenko was -4.1765% as shown in table 1.

Table 1: K from different aspect ratios for CSFS rectangular plate

Aspect Ratio, P = a/b	K from Present Study A	K from Iyengar (1988) B	K from Timoshenko (1936) C	K from Ibearugbulem (2012) D	Percentage Difference Between A&B	Percentage Difference Between A &C	Percentage Difference Between A&D
0.2	25.349	25.35	*	18.47	-0.00394		37.2442
0.4	6.614	6.34	*	6.09	4.3217		8.6043
0.8	2.075	2.16	*	2.42	-3.9352		-14.2562
1.0	1.629		1.7	*	*	-4.1765	

*Means not available.

VI. CONCLUSION

The polynomial series used as shape function has been applied in this paper for the buckling behavior of an isotropic rectangular plate. The polynomial series as a shape function satisfies all the non-essential (dynamic) boundary conditions of the plate. Hence, it is s very good approximation of the exact shape function for the plate.It is therefore recommended that the present theory can accurately predict the critical buckling load of isotropic plates.

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