ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 2, Issue 5, September 2013

Numerical Study of Functions of Potential Energy in Dynamic of Particles and Systems

Carlos Figueroa Navarro,Raúl Riera Aroche

***Abstract*— The purpose of this work is to obtain the numerical solution of the most used potential energy functions in the explanation of physical phenomena and particle systems by using Matlab. The main idea of our contribution is to generate plots which lead to have a greater understanding of the concept. With this software the potential behaviors more common in the literature are generated, such as the harmonic oscillator, the Coulombian and the called central force field; then others well-known as the Lennard Jones, Yukawa and Morse potential are defined. Finally, it presents a Dirac Delta function and Gaussian potential. This article aims to apply Matlab to the study of dynamics in order to be useful in the teaching of quantum mechanics, gravitation and numerical methods.**

Index Terms— **Central field force, Dirac Delta function, Lennard Jones potential, Morse potential, Yukawa potential.**

I. INTRODUCTION

In the study of vectorial mechanics or basic quantum physics, there are available many books containing solved problems, the reading of these has helped complicated topics for students alien to physics as engineers, mathematicians or chemists. However, the material can be such a higher educational impact, and even better if it is studied from a computational perspective. The set of selected problems to discuss here have the characteristic that they lack of the graphic demonstration of the solution, we complement the discussion through numerical processing figure.

There are studies in the literature containing lessons from the point of view of Matlab; for example, [1] and [2] are two good books that contain many developed exercises. Another good set of problems is [3], such references are a source of exercises that can be discussed with other variants of interest and cannot be ignored in this kind of study. Potentials included here are among the most studied in classical physics and some are used in quantum mechanics, for example, the potential of a simple harmonic motion is very valuable because its link to the principle of conservation of energy. The study of the model is essential because it serves to model problems of springs in mechanical engineering and molecules in solid state physics. A valuable bookthat exemplifies potential problems is Benguriaand Depassier[4]; it develops calculus and analysis for springs as example of energy conservation.

On the other hand, there are abundant works on the subject of the Coulomb potential and the gravitational potential. Both have properties that are significant parts of the study of electromagnetism and planetary dynamic, as well as in the study of atomic models. In astronomy planetary dynamics we can find interesting works as in [5]. We only focus on the Kepler problem.

There are others of higher complexity as Lennard Jones potential; such potential is very useful in molecular dynamics. Here we use the work of [6] that approximates by using mathematical constants such as the Golden Section. Another potential applied in nuclear physics is the Yukawa potential, here we utilize it in the terms used by [7]. The so called Morse potential isa key function in diatomic molecules; here, the result of [8] is reproduced with Matlab support.

Other potentials to consider are the Gaussian potential and the Dirac delta function potential that are easy to describe the curve. These functions, also called unit impulse, are used to describe narrow and deep attractive potential [9]. Gaussians and other functions have as limit a delta Dirac, the plot helps to show this analytic problem. Other no less important for quantum mechanics is the Bohm potential, which is not included in this study since its graph is not obtained directly.

Two key concepts are noteworthy: the kinetic energy depends on the change in position whereas the potential energy depends only on the position. In our work we consider one-dimensional motion in scalar form treatment. Our idea is then rethinking the potential energy asmain part of the dynamic study by implementing Matlab. The main idea of this work is to use Matlab as a way to demonstrate the behavior of the curves of the functions and also be shaping the always difficult task of a numerical solution to energy problems.

ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 2, Issue 5, September 2013

II. SIMPLE HARMONIC OSCILLATOR POTENTIAL

The one dimensional harmonic oscillator is the easiest way to detect the energy conservation since it is considered no dissipative forces are involved in, this makes constant the total energy, the mechanical energy is the kinetic K plus potential U.

$$K+U=\frac{1}{2}mv_x^2+\frac{1}{2}kx^2=cte\ . \qquad (1)$$

If we check the potential is easy to detect that it comes from a given force $F=-kx$ applied on the relation with r is vector in x,y,z direction, here only x.

$$V(r)=-\int F(r)\,dr\ . \qquad (2)$$

When studying simple harmonic motion, a differential equation is solved that leads to a solution type

$$x=A\cos(\omega t+\phi)\ . \qquad (3)$$

Where: A is the wave amplitude, ω is related to the oscillation frequency, t is the time in seconds, and ϕ is called phase angle. When replacing the solution, $v_x=\frac{dx}{dt}$ is generated; then

$$K=\frac{1}{2}mv_x^2=\frac{1}{2}mA^2\omega^2 sen^2(\omega t+\phi)$$
$$U=\frac{1}{2}kx^2=\frac{1}{2}kA^2\cos^2(\omega t+\phi)\ . \qquad (4)$$

Considering that $k=m\omega^2$, the total energy is

$$E=\frac{1}{2}kA^2\left[\cos^2(\omega t+\phi)+\sin^2(\omega t+\phi)\right]\ . \qquad (5)$$

$$E=\frac{1}{2}kA^2\ . \qquad (6)$$

These are plotted by using Matlab for the following example of J. P. Campillo [10]:
A particle of 2 kg of mass is clamped to the end of a dock and it moves according to the equation $x(t)=2\cos(10t)$; the period for this motion is given by

$$\omega=\frac{2\pi}{T} \quad \rightarrow \quad 10=\frac{2\pi}{T} \quad T=0.20\pi \text{ sec} \quad A=2\ .$$

Likewise using

$$\omega=\sqrt{\frac{k}{m}} \quad 10=\sqrt{\frac{k}{2}} \quad k=200\ N/m\ .$$

The kinetic energy is of the form

$$K=\frac{1}{2}mA^2\omega^2\sin^2(\omega t+\phi)$$
$$K=\frac{1}{2}(2)(2)^2(10)^2\sin^2(10t) \qquad (7)$$
$$K=400\sin^2(10t)\ .$$

The corresponding potential energy

ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 2, Issue 5, September 2013

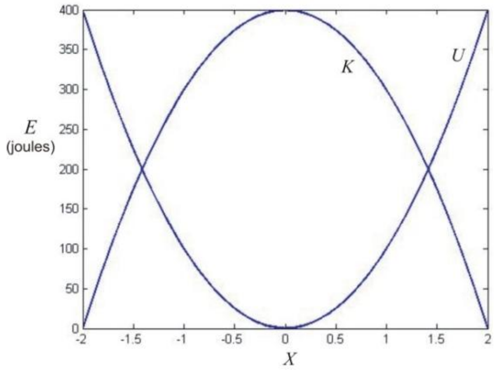

Fig. 1. Kinetic and potential energy. Conservative system

$$U = \int_0^x kx dx = \frac{1}{2}kx^2 = \frac{1}{2}kA^2 \cos^2(10t)$$

$$U = \frac{1}{2}(200)(2)^2 \cos^2(10t) = 400\cos^2(10t) \ . \qquad (8)$$

The total energy is

$$E = \frac{1}{2}kA^2 = \frac{1}{2}(200)(2)^2 = 400\, joules \ .$$

Here, Campillo calculus finishes [10]. Following, both energies are generated andpresented in Fig. 1

III. COULOMB POTENTIAL

In the case of Coulomb potential, that is generated from the determined force for two electric charges point q_1, q_2 separated by a distance r

$$F = k\frac{q_1 q_2}{r^2} \qquad k = \frac{1}{4\pi\varepsilon_0} \ . \qquad (9)$$

The force is attractive $F < 0$ or repulsive $F > 0$. Also $\varepsilon_0 = 8.8541 \times 10^9 \ N m^2 / C^2$ called permittivity constant, in vector form Coulomb law is expressed:

$$\vec{F} = -k\frac{q_1 q_2}{r^2}\hat{r} \ . \qquad (10)$$

Just like gravitational force, the electrostatic force is conservative; therefore a potential can be associated to the system so that

$$U_b - U_a = -\frac{1}{4\pi\varepsilon_0} q_1 q_2 \int_{r_a}^{r_b} \frac{dr}{r^2} = \frac{1}{4\pi\varepsilon_0}\left(\frac{1}{r_b} - \frac{1}{r_a}\right) \ . \qquad (11)$$

If chosen a reference point a such that r_a corresponds to an infinite separation among particles, we have

$$U(r) = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} \ . \qquad (12)$$

Following the potential energy function of a hydrogen atom is plotted, where the radial distance between the electron and the proton is measured in terms of the Bohr radius. In this case, the charge $e = 1.6 \times 10^{-19}\ coulombs$ is used so that

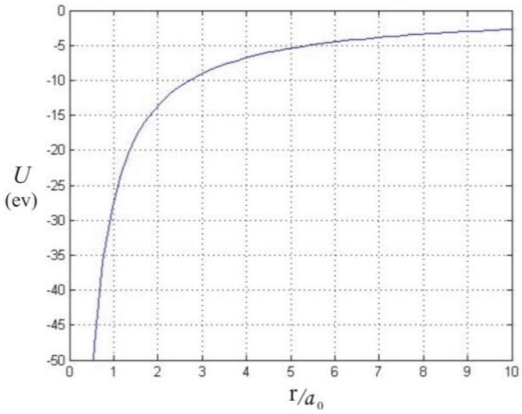

Fig. 2. Coulombian potential of Hydrogen atom, from Halliday, Krane and Resnick

$$U(r) = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}. \tag{13}$$

Whose graph is shown in Fig. 2 which reproduces the result of [11].

IV. GRAVITATIONAL POTENTIAL

For evaluating the potential energy of two bodies that act through gravitational force, it is known that a body which is moving to a height h near the ground surface is given by $U = -mgh$, but this applies to small changes in height compared with distance to the center of the earth and the gravitational force is more or less constant, but for example, the altitude of a satellite in orbit, we need a more general way.

Demonstrate that gravitational force is conservative can be done by analyzing the different paths and checking that energy is always the same, which makes it clear that fulfills the principle of independence of the path. Let a particle of mass m moving from one position a to b radially. The particle mass M, resting on the origin, exerts a gravitational force in m, then

$$U = -\int F(r)dr = -\int_a^b \frac{GMm}{r^2}dr = GMm\left(\frac{1}{r_a} - \frac{1}{r_b}\right). \tag{14}$$

At distant points $r_b = \infty$ and let r the distance between particles, we have

$$U(r) = -\frac{GMm}{r}. \tag{15}$$

V. POTENTIAL OF CENTRAL FORCE FIELD

The gravitational interactions governing the behavior of celestial bodies or the Coulombian interaction between a pair of point charges are considered central force fields.

Newton force and gravitational potential are considered for an application of planetary or Keplerian motion. Let M the mass of the solar system and m that of a moving body in the solar system. The study of the problem deserves a review of the path type (hyperbola, elliptical or circular) likewise laws of Kepler. The energy of the system is

$$E = \frac{1}{2}mv^2 - G\frac{mM}{r}. \tag{16}$$

Using the Binet formula and after some mathematical developments we obtain an effective potential, given by

$$U_{eff} = -G\frac{Mm}{r} + \frac{l^2}{2mr^2} \ . \tag{17}$$

For example, to obtain the effective potential of the Earth planet from the Sun such that

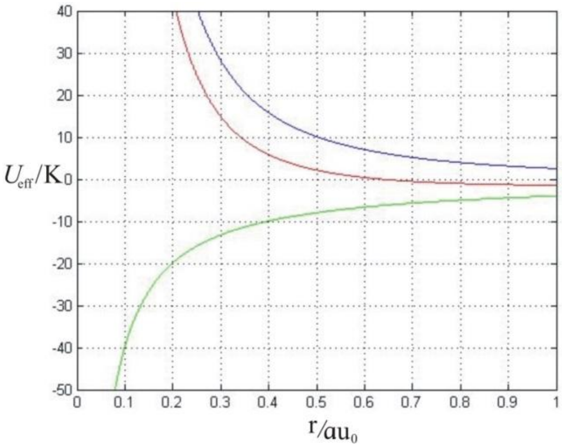

Fig. 3. Earth's gravitational potential from the Sun.

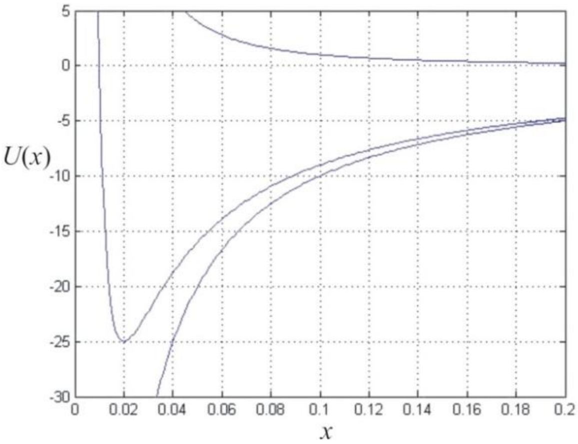

Fig. 4. Central field system, fromD. Strauch.

$M_{sun} = 1.99\times10^{30}\ kg$ $m_{earth} = 5.98\times10^{24}\ kg$. Additionally, using $l = m_{earth} r^2 \omega$, where $\omega = 2\pi/T$ is frequency, so then the respective curves are obtained (Fig. 3).
In order to obtain the stable states we can proceed as follows:

$$b = GMm \quad a = \frac{l^2}{2m} \tag{18}$$

Now the equation has the form

$$U(r) = \frac{a}{r^2} - \frac{b}{r}$$

$$\frac{dU(r)}{dr} = -\frac{2a}{r^3} + \frac{b}{r^2}\bigg|_{r=r_0} = 0 \tag{20}$$

$$r_0 = \frac{2a}{b} = \frac{l^2}{2GMm^2} \ .$$

One way of showing this is by using a function of the same type such as the yield of [12], plotted in Fig. 4.

$$U(x) = -\frac{1}{x} + \frac{0.1^2}{x^2} \ .$$

Another central field model is the Bohr atom for hydrogen, which assumes that the electron and proton spin in circular orbits and considering the Coulomb interaction of the respective electric charges, it is possible to calculate the first orbit (Bohr radius).
There are potentials $V(r)$ from other isotropic central forces [12], for example

$$\vec{F} = \frac{V_0}{r^2}\left(1 + \frac{r}{\lambda}\right)e^{-r/\lambda}\hat{r} \qquad \vec{F} = \left(\frac{a}{r^{\alpha+1}} - \frac{b}{r^{\beta+1}}\right)\hat{r} \qquad k, V_0, \lambda, a, b, \alpha, \beta \ \text{ctes} \ . \tag{21}$$

VI. YUKAWA POTENTIAL

In particles and atomic physics a Yukawa potential, also called a screened Coulomb potential, is generated by a force

ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 2, Issue 5, September 2013

$$\vec{F}=\frac{V_0}{r^2}\left(1+\frac{r}{\lambda}\right)e^{-r/\lambda}\hat{r}\ . \qquad (22)$$

Which whereas calculating the associated potential has then [13]

$$V(r)=-V_0\int\frac{1}{r^2}\left(1+\frac{r}{\lambda}\right)e^{-r/\lambda}dr=V_0\int\left(\frac{d}{dr}\frac{e^{-r/\lambda}}{r}\right)dr=\frac{V_0}{r}e^{-r/\lambda}\ . \qquad (23)$$

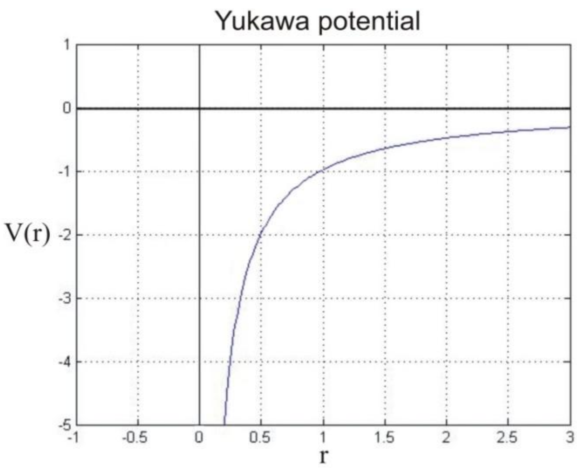

Fig. 5. Yukawa potential, from De Lange and Pierrus

De Lange and Pierrus yield arrives tothe function $V(r)$; ithas the following behavior, shown in Fig. 5

Other potentials are calculated in Dehen work [7] that relates to a gravitational potential of the form type Yukawa

$$V(r)\approx\frac{1}{r}\left(1+\alpha e^{-r/\lambda}\right) \qquad (24)$$

$$\alpha=-7.2\times10^{-3}\quad \lambda=200\,m\ .$$

Whose function is described in Fig. 6.

VII. LENNARD JONES POTENTIAL

Although there is no universal interaction potential among particles there is a very suitable expression when reproducing a potential for interaction among the experimentally derived atoms called Lennard Jones potential has the form of

$$U(r)=U_0\left[\left(\frac{r_0}{r}\right)^{12}-2\left(\frac{r_0}{r}\right)^{6}\right]. \qquad (25)$$

Where U_0 is given in joules and r_0 in angstroms. They are characteristic parameters of the system. This is a binary interaction system, which means that it considers interactions between pairs of molecules

It is also possible to generate its scheme with the following type of Lennard Jones potential described in Ashcroft and Mermin [14]. Fig. 7 describes such potential.

ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 2, Issue 5, September 2013

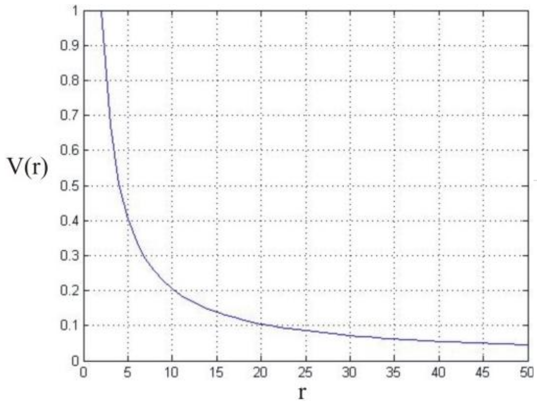

Fig. 6. Yukawa-type gravitational potential

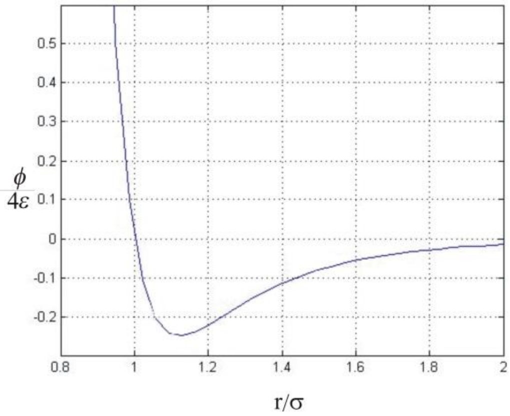

Fig. 7. Lennard Jones for Neon, from Ashcroft and Mermin

$$\phi(r) = -\frac{A}{r^6} + \frac{B}{r^{12}} \qquad \sigma = \left(\frac{B}{A}\right)^{1/6}$$
$$\phi(r) = 4\varepsilon\left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6}\right] \qquad \varepsilon = \frac{A^2}{4B}. \tag{26}$$

Other work about Lennard Jones is the Lim Cheng Teik [6]. It sets the approximation through known constants as Golden Section, this last associated with the Fibonacci sequence: In order to reproduce this result, is used

$$U_F = D\left[\frac{5}{8}\left(\frac{R}{r}\right)^{13} - \frac{13}{8}\left(\frac{R}{r}\right)^{5}\right]. \tag{27}$$

By using $D = 143.22,\ R = 0.3759\,\text{nm}$ then we have a Lennard Jones potential approximated through the Fibonacci series, Fig. 8.

VIII. Morse Potential

For a two-particle system the potential energy is a function of the distance between them [8]. Morse potential is frequently used to study the interaction between atoms which leads to the formation of a single molecule, this potential has the form

$$V(r) = D\left(1 - e^{-\beta(r - r_0)}\right)^2. \tag{28}$$

Where D, β, r_0 are positive constants that depend on the system under study, for example for the H_2 molecule the values of these constants are

$D = 7.61\times10^{-19}\,J,\ \beta = 0.0931\,pm^{-1},\ r_0 = 74.1\,pm$.

The behavior is shown below in Fig. 9.

IX. Dirac Delta Function And Gaussian Potential

The unit impulse function and Dirac delta function are related as noted below. Many physical phenomena are explained by using this kind of potentials. Then, we have firstly unit impulse function.

ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 2, Issue 5, September 2013

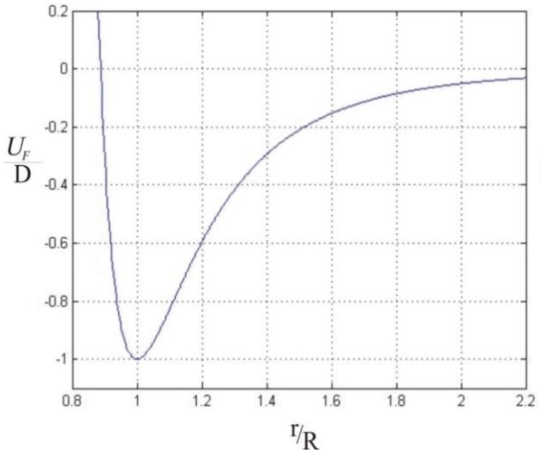

Fig. 8. Lennard Jones potential, fromLim Ch. Teik

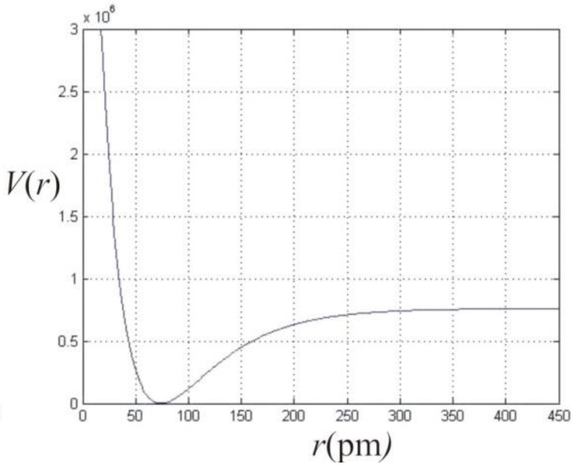

Fig. 9. Morse potential

$$\delta_b(t-a)=\begin{cases}0 & t<a-b\\ \dfrac{1}{2b} & a-b\le t\le a+b\\ 0 & t>a+b\end{cases} . \tag{29}$$

It is called unit impulse for this property

$$\int_{-\infty}^{\infty}\delta_b(t-a)\,dt=1 .$$

The Dirac delta function results as performing this limit.

$$\delta(t-a)=\lim_{b\to 0}\delta_b(t-a)=\begin{cases}\infty & t=a\\ 0 & t\neq a\end{cases}$$

$$\int_{-\infty}^{\infty}\delta(t-a)\,dt=1 .$$

Both equations define the Dirac delta function. Here we can approach by usingCarlos Trallero propose[15].

$$\delta_n(x)=\lim_{n\to 0}\frac{n}{\pi\left(1+n^2x^2\right)} . \tag{30}$$

$$\delta(x)=\lim_{L\to\infty}\frac{\sin(xL)}{\pi x} . \tag{31}$$

In Fig. 10, the limit of function (30) is plotted. Equation (31) is another representation of the Dirac delta function; it is presented in Fig. 11.
In the other hand, it can be shown that a Gaussian function which has the following property is demonstrated in the theoretical book of Luis de la Peña [9]. Here, we focus its graphical behavior.

$$\lim_{\sigma\to 0}\frac{1}{\sqrt{2\pi\sigma}}e^{-(x-a)^2/2\sigma^2}=\delta(x-a) . \tag{32}$$

Gaussian potential for three σ values is plotted in Fig. 12.
Carlos Trallero [15] shows a variety of potential for Schrodinger equation, one of them has the shape of the Gauss bell curve, such as that presented below.

$$U(x)=\frac{U_0}{\cosh^2(x/a)} . \tag{33}$$

ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 2, Issue 5, September 2013

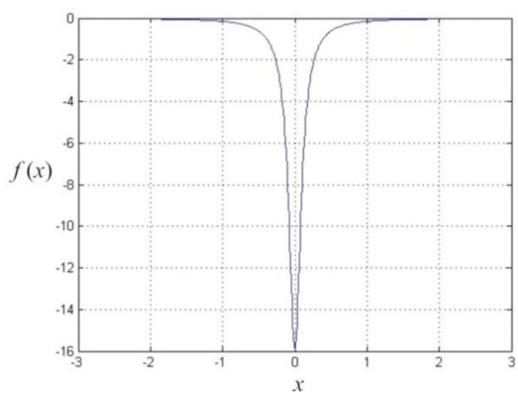

Fig. 10. Function that approximates the Dirac delta.

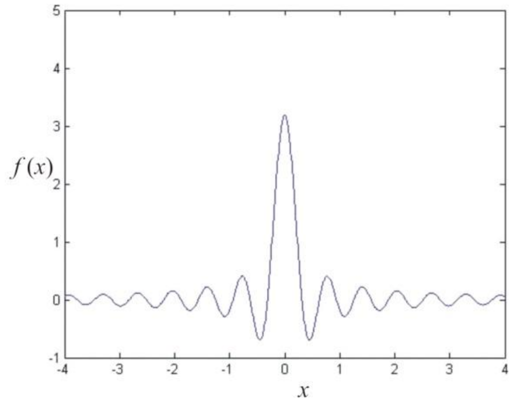

Fig. 11. Function that approximates the Dirac

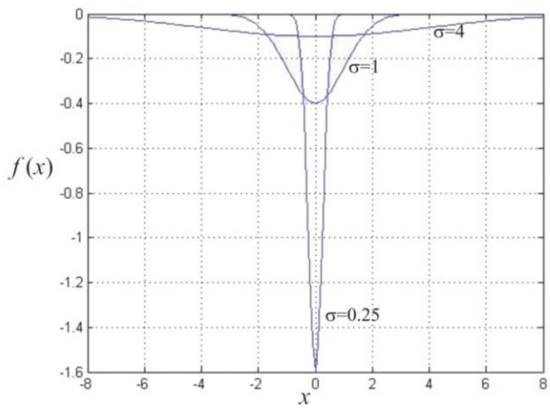

Fig. 12. Gaussian potential.

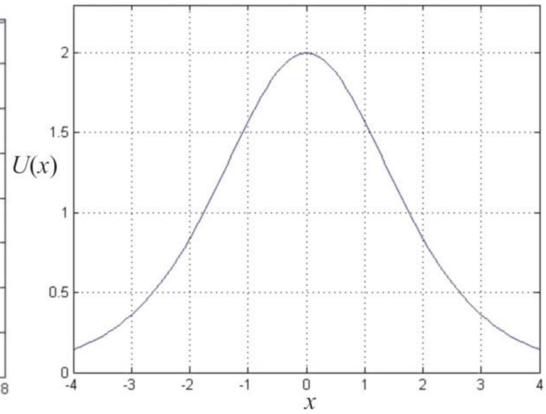

Fig. 13. Potential Gaussian kind.

The graphical behavior is shown in Fig. 13.
Finally it would be a challenge generating a graphical Bohm potential that has the expression of the form

$$U=-\frac{\hbar^2}{2m}\frac{\nabla^2 R}{R}\ . \qquad (34)$$

The position is R and its expression depends on the curve, also known as quantum potential. There are papers in the literature dealing with the Bohm potential, e.g. the De la Peña work [9] who contains the mathematical developments, whereas there are works where it is solved so clearly as [16,17].

X. Discussion Of Results

The harmonic oscillator example is key because it explains well and easily the principle of conservation of energy. An important feature of Fig. 1 is the x-axis which has implicitly the equation (3). On the other hand, the Coulombian potential plot in the hydrogen atom, Fig. 2, approaches well the [11].
The Bohr model is an excellent example of basic physics and cannot be ignored. In the event of the gravitational potential study we attempted to generate a chart of the potential role of the Sun and of the Earth as a central field problem. Missed to adjust the curve to observe the points of stability, but this problem deserves entering the world of the Keplerian model for the solar system, it is also associated with angular momentum and eccentricity of the orbits. The Fig. 4 has differences and not seemed the reference [12].
For the Yukawa potential the equation (23) is reproduced in Fig. 6, which is a good analysis made by [7]. A good result is presented by the Lennard Jones potential in Fig. 7, since it approximates well to Ashcroft and Mermin yields. Factors A and B in equation (20) were calculated based on data from the same source. Similarly, we can say that Fig. 8 correctly fit Teik result from [6]. For example, equation (27) uses Fibonacci sequence as polynomial factors. The other curves of the potential Morse functions, in Fig. 9, are useful for explaining systems of diatomic molecules and the presented curve gives a good approximation to the reference. The Dirac delta

ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 2, Issue 5, September 2013
function and Gaussian potential can bind when developing the scheme in Fig. 12. The relation between hyperbolic functions and Gaussian curve explain the approximation of Fig. 13.

XI. CONCLUSION

It is possible, by using software, to get a better understanding of the types of energy (kinetic and potential) present in dynamics. Courses such as classical mechanics or vectorial dynamic may rely on numerically using programs like *Mathematica* or *Matlab*.The examples discussed herein have influence to do mathematics applied in physics. In that sense, our proposal for a computational perspective may be a more effective alternative to helping learn somewhat complicated topics for the student. For example, the principle of conservation can be solved in a better way if it is propitiated the development of its graphical behavior likewise with the problem of central force field or the Dirac delta function, where is experienced that the approximation function fulfills the original profile. Furthermore, our computational idea opens up other possibilities some of which are pointed out next.
Some interesting calculation are, for example: a) In gravitation it is possible to study the solar system, by calculating the kinetic and potential energies of all the planets; b) The Coulombian potential, can be obtained for Helium and other simple elements; c) Regarding Lennard Jones potential, it can be calculated for other molecules and noble gases; and finally, d) Matlab can be very useful in explaining conservative forces. Therefore, this paper has positive expectations in the numerical study of the systems. Wethink that if we focus in lessons of potential energy asmain function, by using the remake of plot yield, has value in the process of explaining the physics concept.The treatment computational perspective proposed here can have educational advantage.
Finally, it is worth recognizing that our approach is limited to the study of one type of energy, and this amounts to a partial view of the problem, because the kinetic energy is excluded. However, we propose this kind of study, seeking to reduce the difficulties of learning; it is true that generalization is lost in this way, but the balance is positive as it gains in facilitating access to system dynamics and particles. Furthermore, this approach may assist the available set of books and studies of solved problems, some of them included in the references. This gives value to thistype of works and researches when complementing the purpose of this sort of literature, and this work is part of that goal.

REFERENCES

[1] V. Rovenski, “Modeling of curves and surfaces with Matlab,” Edit. Springer, USA 2010.

[2] Gander and D. Hrebicek, “Solving problems in scientific computing using Maple and Matlab,” Edit. Springer, USA 2004.

[3] Quarteroni and F. Saleri, “Cálculo científico con Matlab,” ISBN 978-88 470-0504-4. Edit. Springer, USA 2004.

[4] R.Benguria and M.C. Depassier, “Problemas resueltos de mecánica clásica”. II edición. Edit. Alfaomega Chile (1999)

[5] T. Oswalt and H., “Bond Planets, Stars and Stellar Systems 2,” Chapter of Astronomical Techniques, Software and data, USA 2013.

[6] T. Ch. Lim, Journal of Mathematical Chemistry 41 4, 2007.

[7] H. Dehen and F. Ghaboussi, International Journal of Theoretical Physics, 26 pp. 5, 1987.

[8] J. Hernandez T., “Fundamentos de espectroscopia,” Web: depa.fquim.unam.mx/jesusht/Fourier_ejemplo_sierra.pdf, 2013.

[9] De la Peña and M. Villavicencio, “Problemas de Mecánica Cuántica,” Edit. Fondo de Cultura Económica, México 2003.

[10] Juan P. Campillo, “Problemas de selectividad,” Web: jcampillo.com/upload/fisica-problems-pdf, 2013.

[11] D. Halliday, R. Resnick and K. Krane, “Física 2,” Edit Cecsa, pp. 538, USA 1997.

[12] Dieter Strauch, “Central Potential and the Kepler Problem,” Classical Mechanics chapter, USA 2009.

[13] O.L. De Lange and J. Pierrus, “Solved Problems in Classical Mechanics,” Edit. Oxford, pp. 113, USA 2010.

[14] N. Ashcroft and N.D. Mermin, “Solid state physics,” ISBN. 0030839939, USA 1976.

[15] C. Trallero and M. Leyva, “Problemas de Mecánica Cuántica,” Edit. Pueblo, Cuba 1989.

[16] Mostacci and V. Molinari, “Effects of Bohm Potential on a charged gas,” Central European Journal of Physics, 2010.

[17] L.S. Kuzmenkov and S.G. Maksimov, “Quantum Hydrodynamics of particles with Coulomb interaction and Bohm Potential,” Theoretical and mathematical Physics, 1999.

ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 2, Issue 5, September 2013

AUTHOR BIOGRAPHY

Carlos Figueroa Navarro. Ph.D. in Physics. Universidad de Sonora, México 2010. Departamento de Ingeniería Industrial Universidad de Sonora. México. Research work in numerical analysis and computational physics.

Raúl Riera Aroche. Ph.D.. in Physics. Universidad de la Habana Cuba 1994. Departamento de Investigación en Física Universidad de Sonora. México. Research work in superconductivity.