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Experimental Form-Finding for Möbius Strip and Ennerper Minimal Surfaces Using Soap Film Models

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Abstract—This paper describes an effort to produce soap film models with boundaries corresponding to those defined by Möbius strip and Enneper minimal surfaces. Soap film models with surfaces deviating from mathematically defined minimal surfaces of Möbius strip and Enneper are shown.

***Index Terms*— Form-finding, *S*oap film model, Möbius strip, Enneper.**

I. INTRODUCTION

Soap film model is used to find the surface form that minimizes area subject to appreciate constraints. Ireland [1] has presented some puzzling measurements of the tension between soap film and a solid surface. Moulton and Pelesko[2] have investigated Catenoid soap film model subjected to an axially symmetric electric field. The experimental and theoretical analysis for this Field Driven Mean Curvature Surface (FDMC) surface provides a step in understanding how electric fields interact with surfaces driven by surface tension. Such interactions may be of great benefit in micro scale systems such as micro electro mechanical systems (MEMS) and self-assembly. Brakke[3] has treated the area minimization problem with boundary constraints. Koiso and Palmer[4] have studied a variational problem whose solutions are a geometric model for thin films with gravity which is partially supported by a given contour. Boudaoud and Amar[5] have studied Helicoid soap film. The vibration equation shows that the Helicoid is the stable surface when its winding number is small. Catenoid is locally isometric to Helicoid so that their vibration spectra are strongly related. The normal forms of the bifurcations confirm the analysis. Huff[6] has considered soap films spanning rectangular prisms with regular *n*-gon bases. As the number of edges *n* varies, there are significant changes in the qualitative properties of the spanning soap films as well as a change in the number of spanning soap films.

It can be seen from the previous paragraph that soap film model has been used in many different studies. The main reason for its use is due to the fact that the surface formed can be observed physically. Another main reason is that experimental study of surface form using soap film model is relatively easy and simple. In the field of shell and spatial structures, form-finding analysis is needed in order to determine surface form that can be obtained under certain given boundary constraint and pre-stressed pattern. One particular case where form-finding analysis is needed is related to tensioned fabric structures. Tensioned fabric structures are structures where very thin architectural fabrics in tensioned are used to span large column-free space. This category of structure can be used to form a variety of surface due to the flexibility of the material used. In order to determine surface form of tensioned fabric structures, form-finding can be carried out using computational or experimental method. In this paper, form-finding using soap film model is carried out in order to determine surface form under boundaries corresponding to shape defined by mathematical equations for the Mobius Strip surface and Enneper minimal surface. In comparison with other known minimal surfaces, Enneper minimal surface and Mobius Strip surface are less studied. Soap film models obtained through this study can be used to verify the accuracy of the mathematically defined surface. The outcome of verification can also be used for the purpose of verification of accuracy of form-finding analysis using computational method.

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II. MÖBIUS STRIP SURFACE AND ENNEPER MINIMAL SURFACE

A. *Möbius strip*

A Möbius strip of half-width *W* with mid-circle of radius *R* at height *z* = 0 as shown in Figure 1 can be represented parametrically by the following set of equations ([7]):

$$X = (R + S\cos\frac{\theta}{2})\cos\theta, Y = (R + S\cos\frac{\theta}{2})\sin\theta, Z = S\sin(\frac{\theta}{2}) \quad (1)$$

for {*S*:-*W*,*W*}and {θ:0,2π}.

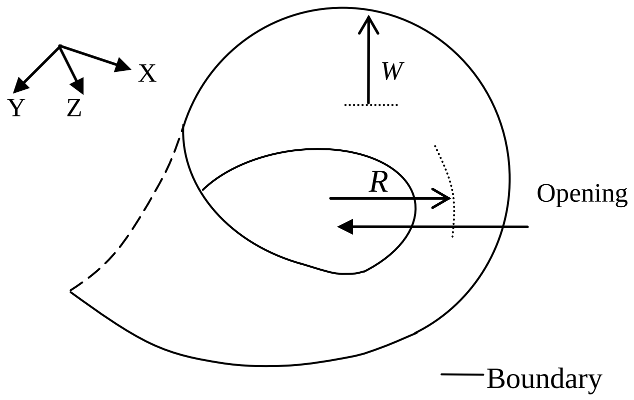

Fig 1: Möbius strip surface

B. *Enneper*

Enneper surface in Figure 2 can be obtained by using the following Equation (2) ([8]):

$$X = u - \frac{u^3}{3} + uv^2, Y = -v + \frac{v^3}{3} - vu^2, Z = u^2 - v^2 \quad (2)$$

for *u* and *v* = variables.

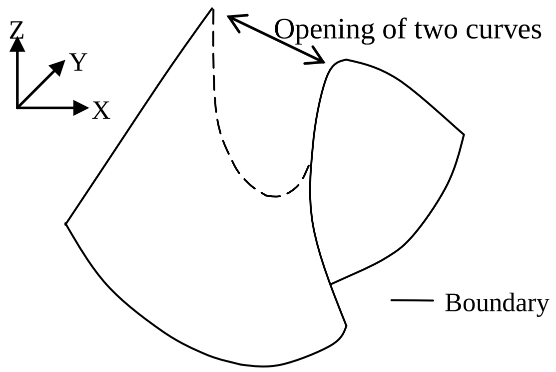

Fig 2: Enneper surface

III. PREPARATION OF SOAP FILM MODEL

The actual size of the Möbius strip *R/W*=1 and *R/W*=2 soap film model are height, H=15cm; width, B=22.5cm and

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H=6.5cm; B=16cm, respectively. The actual size of the Enneper u=v=0.86 and 1.25 soap film model are H=21cm; B=12cm and H=26cm; B=16cm, respectively.

The boundary of models are built using aluminum wire. Other materials used are steel, plywood, rubber band and super glue as shown in Figure 3.

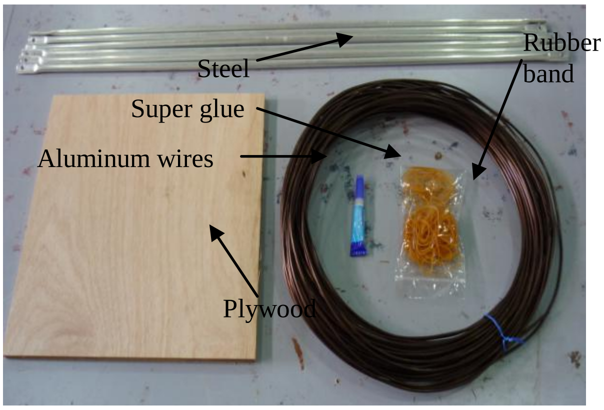

Fig 3: Materials used to build the boundary of the soap film model

The making of boundary frame or wire frame involves the use of a wooden base where x and y coordinates of the boundary of the soap film model are measured (Figure 4). As shown in Figure 4, standing steel rods are used to support the wire frame at the desired height corresponding to z coordinate of the wire frame. The wire is secured to the steel rod by rubber band. In this way, boundary frames in the forms defined by (1) to (2) are realized.

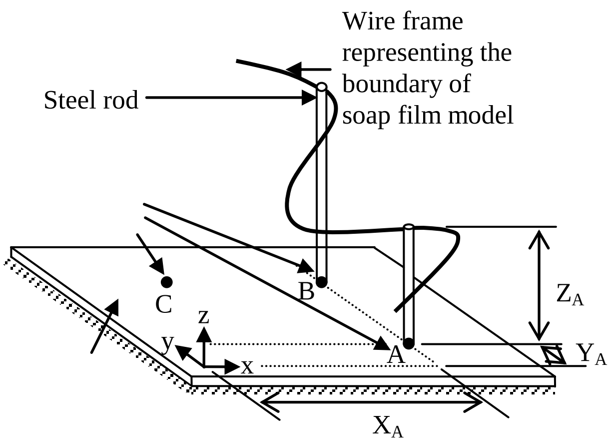

Fig 4: Making of boundary wire frame

Figure 5 shows the set-up used to produce soap film model. Soap film model is produced by dipping the wire-frame into a box containing soap solution with the help of a jack. The composition of the soap solution used is 25.7% of glycerin, 22.8% of concentrated car detergent and 51.5% of distilled water. The soap solution is allowed to set for one day to allow the alcohol which may exist in the detergent to evaporate. Theodolite is used to check the horizontal alignment as illustrated in Figure 6 and plan position alignment in Figure 7. θ in Figure 7 is used for checking if soap film model is in correct plan position. This is to ensure that the soap film model is suspended in such a way that the orientation of image taken using camera can be compared with mathematically defined surface.

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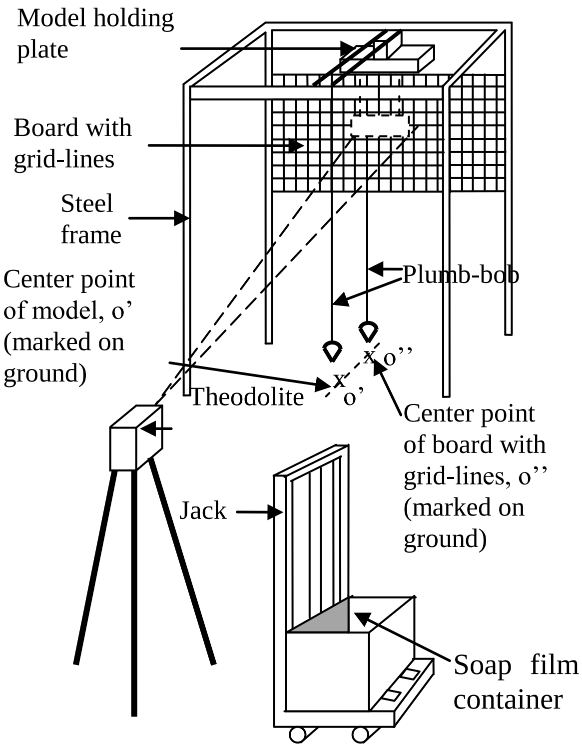

Fig 5: Set-up for the soap film model experiment

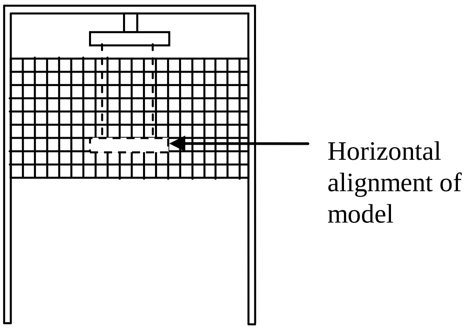

Fig 6: Horizontal alignment of the soap film model

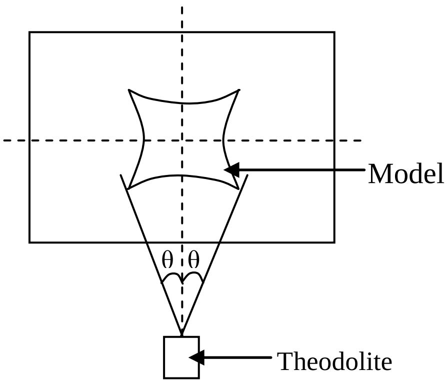

Fig 7: Plan position of the soap film model

The following procedures are followed for the checking of alignment of soap film model using theodolite:

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(a) Plumb-bob is used to determine the projection of center point of model holding plate on the ground as shown in Figure 5. Point o' is marked on the ground.

(b) Plumb-bob is used to determine the projection of center line of board with grid-lines on the ground as shown in Figure 5. Point o'' is marked on the ground. These two points lie along the same straight line as shown in Figure 5.

(c) Theodolite is positioned at suitable distance from the frame with its view aligned along line o'-o''.

(d) Horizontal alignment of wire-frame model is checked using the theodolite.

(e) Plan alignment of wire-frame model by turning the model holding plate is checked by using the theodolite. The position of the camera is set on the place of theodolite.

(f) After checking of proper alignment of the soap film model, the theodolite is removed. A camera is then placed at the same location as that of the theodolite to capture image of the soap film model.

IV. RESULTS OF SOAP FILM MODELS

The soap film experiment set up is shown in Figure 8. In the following sections, soap film models showing good agreement with surfaces defined by (1) to (2) are first presented. This is then followed by those models where deviation in the surface form between soap film models and mathematically defined surfaces has been observed.

Fig 8: Soap film experimental set up

A. Möbius strip (R/W=2)

Figure 9 shows the plan view of mathematically defined Möbius strip with *R/W*=2. Figure 10 shows the corresponding soap film model. Comparison of Figures 9 and 10 shows that the result obtained from soap film model is the same with mathematically defined surface which has the characteristic opening at the center.

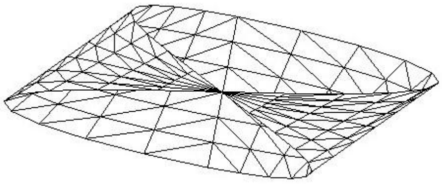

Fig 9: Mathematically defined Möbius strip (W/R=2)

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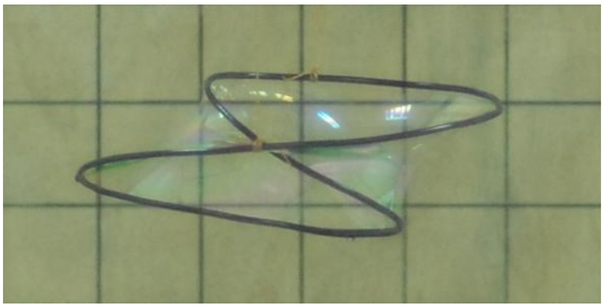

Fig 10: Möbius strip soap film model (W/R=2)

B. Enneper (u=v=0.86)

Figure 11 shows the mathematically defined Enneper surface with u=v=0.86. The corresponding result obtained through soap film experiment is shown in Figure 12. Comparison of the two surfaces shows that they are in good agreement.

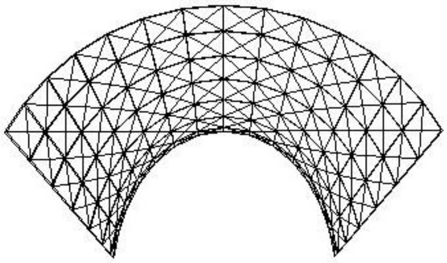

Fig 11: Mathematically defined Enneper surface (u=v=0.86)

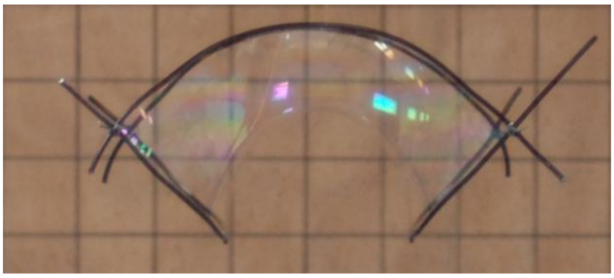

Fig 12: Enneper soap film model (u=v=0.86)

C. Möbius strip (R/W=1)

Figure 13a shows the mathematically defined surface of Möbius strip with *R/W*=1. The corresponding Möbius strip soap film model as shown in Figure 13b does not show the characteristics of Möbius strip with opening at the center.

(a) (b)

Fig 13: (a) Mathematically defined surface of Möbius strip (R/W=1) with opening at the center (b)Soap film model (Möbius strip, R/W=1)

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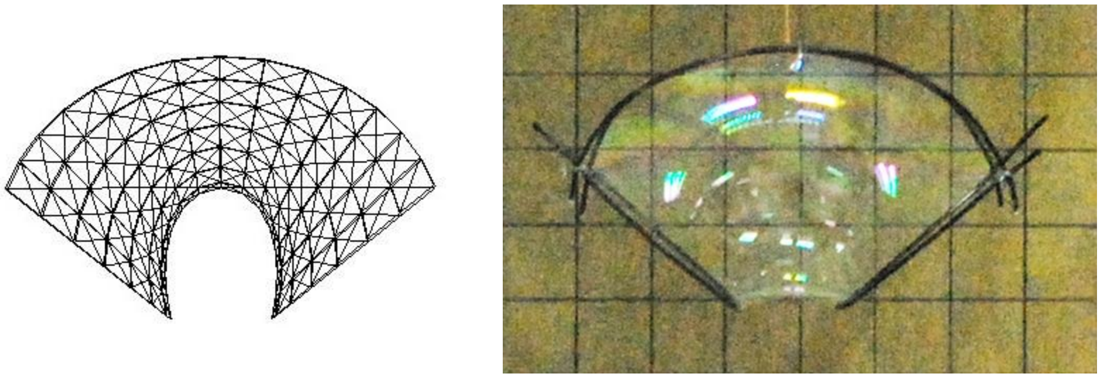
D. Enneper (u=v=1.25)

Enneper surface with *u*=*v*=1.25 defined by (2) is shown in Figure 14a. The corresponding soap film model is shown in Figure 14b. Comparison of Figures 14a and 14b clearly shows that the deviation of the mathematically defined Enneper minimal surface from that of soap film model.

V. DISCUSSION

Möbius strip surface with *R/W*=2, Enneper surface with parameters *u*=*v*=0.86 are found to be in good agreement with the mathematical defined surface. For Möbius strip with *R/W=2*, the form obtained after soap film experiment is similar to the Möbius strip surface with opening at the center of the surface. When *R/W*=1, the soap film model is found to be different from the form of Möbius strip surface defined by (1). The opening of the center is missing. Such incapability of surface topology to be kept in the form of a disc under continuous changes of the boundary circle has been mentioned by [9]. This results indicates that there is a specific value of *R/W* ratios above which the surface of Möbius strip surface maintains the topological characteristic of disc with a central opening. Mathematically defined Enneper surface with parameters *u*=*v*=0.86 is a stable minimal surface. The reason for the deviation of mathematically defined Enneper surface from the corresponding soap film model is probably due to the reason that for parameter *u*=*v*=1.25, Enneper surface defined by (2) no longer corresponds to stable minimal surface ([10]). Soap film model shows the corresponding stable shape under the parameter values of (*u*,*v*) = (1.25,1.25) which is different from that defined by (2).



(a) (b)

Fig 14: (a) Mathematically defined Enneper (u=v=1.25) (b) Soap film Enneper (u=v=1.25)

VI. CONCLUSION

Soap film model with surface shape in close agreement with mathematically defined Möbius strip and Enneper surfaces have been produced. Deviations between the two sets of surfaces have been shown to occur when *R/W*=1 in Möbius strip and parameters *u*=*v*=1.25 in the case of Enneper surfaces.

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