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Solving Fuzzy Multi Objective Non-linear Programming Problem Using Fuzzy Programming Technique

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***Abstract*— The aim of this paper is to present a method in which a fuzzy multi objective non-programming problem is reduced to crisp using ranking function and then the crisp problem is solved by fuzzy programming technique.**

Keywords: **Multi Objective Non-linear Programming Problem, Fuzzy Multi Objective Non-linear Programming Problem, trapezoidal fuzzy Numbers, crisp problem.**

I. INTRODUCTION

Most of the real world Problems is inherently characterized by multiple, conflicting and incommensurate aspects of evaluation. These axes of evaluation are generally operationalized by objective functions to be optimized in framework of multiple objective linear programming models. Furthermore, when addressing real world problems, frequently the parameters are imprecise numerical quantities. Fuzzy quantities are very adequate for modeling these situations. Bellman and Zadeh [1] introduced the concept of fuzzy quantities and also the concept of fuzzy decision making. The most common approach to solve fuzzy linear programming problem is to change them into corresponding deterministic linear programme. Some methods based on comparison of fuzzy numbers have been suggested by H.R.Maleki [8], A. Ebrahimnejad, S.H. Nasseri [6], F. Roubens [7], A.Munoz. Zimmermann [2] has introduced fuzzy programming approach to solve crisp multi objective linear programming problem. Recently H.M.Nehi et .al . [9] used ranking function suggested by Delgodo et.al. [5] to solve fuzzy MOLPP.
In this paper , we introduced a method in which a fuzzy multi objective non-linear programming problem (FMONLPP) is first reduced to crisp MONLPP using ranking function suggested by F.Roubens [7] and the resulting one is solved by partial modification of fuzzy programming technique of Zimmermann [3] . The coefficients of all objective functions as well as the constraints are fuzzy in nature. A numerical example is given to illustrate the procedure.

II. MULTI OBJECTIVE NON- LINEAR PROGRAMMING

The problem to optimize multiple conflicting non linear objective functions simultaneously under given constraints is called multi objective Non- linear programming problem and can be formulated as the following optimization problem.

$$\text{Max } f(x) = (f_1(x),\ f_2(x) \ldots\ldots\ldots\ldots\ f_k(x))^T$$

$$\text{s.t } \quad x \in X = \{ x \in R^n \mid g_j(x) \leq 0 ,\ j= 1,2 \ldots m \} \quad \ldots\ldots \quad (2.1)$$

where $f_1(x)$, $f_2(x)$ …………… $f_k(x)$ are k distinct non linear objective functions of the decision variables and X is the feasible set of constrained decision.

Definition 2.1

X* is said to be a complete optimal solution for (1) if there exist $x^* \in X$ such that

$$f_i(x^*) \geq f_i(x) \ , \quad I = 1, 2 \ldots k \text{ for all } x \in X .$$

III. RANKING FUNCTION FOR FUZZY NUMBERS

Definition: 3.1

Let A be a fuzzy number whose membership function can generally be defined as

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$$\mu_A(x) = \begin{cases} \mu_A^L(x) & a^1 \le x \le a^2 \\ 1 & a^2 \le x \le a^3 \\ \mu_A^R(x) & a^3 \le x \le a^4 \\ 0 & otherwise \end{cases}$$

Where $\mu A^L(x) : [a^1, a^2] \rightarrow [0, 1]$ and $\mu A^R(x) : [a^3, a^4] \rightarrow [0,1]$ are strictly monotonic and continuous mappings. Then it is considered as left right fuzzy number. If the membership function $\mu_A(x)$ is piecewise linear, then it is referred to as a trapezoidal fuzzy number and is usually denoted by $A = (a^1, a^2, a^3, a^4)$. If $a^2 = a^3$ the trapezoidal fuzzy number is turned into a triangular fuzzy number $A = (a^1, a^3, a^4)$

A fuzzy number A = (a , b, c) is said to be a triangular fuzzy number if its membership function is given by

$$\mu_A(x) = \begin{cases} \frac{x-a}{b-a} & a \le x \le b \\ 1 & x = b \\ \frac{x-b}{b-c} & b \le x \le c \\ 0 & otherwise \end{cases}$$

Assume that $R: F(\mathcal{R}) \rightarrow \mathcal{R}$ be linear ordered function that maps each fuzzy number in to the real number, in which $F(\mathcal{R})$ denotes the whole fuzzy numbers. Accordingly for any two fuzzy numbers $\tilde{a}$ and $\tilde{b}$ we have

$\tilde{a} \underset{R}{\geq} \tilde{b}$ iff $R(\tilde{a}) \ge R(\tilde{b})$

$\tilde{a} \underset{R}{>} \tilde{b}$ iff $R(\tilde{a}) > R(\tilde{b})$

$\tilde{a} \underset{R}{=} \tilde{b}$ iff $R(\tilde{a}) = R(\tilde{b})$

We restrict our attention to linear ranking function, that is a ranking function R such that $R(k\tilde{a} + \tilde{b}) = k R(\tilde{a}) + R(\tilde{b})$ for any $\tilde{a}$ and $\tilde{b}$ in F (R) and any $k \in R$.

Roubens 's ranking function:

The ranking function proposed by F. Roubens is defined by

$$R(\tilde{a}) = \frac{1}{2}\int_0^1 (inf\ \tilde{a}_\alpha + \sup \tilde{a}_\alpha)\, d\alpha.$$

Which reduces to

$$R(\tilde{a}) = \frac{1}{2}\left(a^L + a^U + \frac{1}{2}(\beta - \alpha)\right) \quad \ldots\ \ldots \quad (3.1)$$

For a trapezoidal number

$$\tilde{a} = (a^L - \alpha,\ a^L,\ a^U,\ a^U + \beta)$$

Solving Fuzzy Multi Objective Non-linear Programming Problem. (FMONLPP)

A fuzzy multi objective linear programming problem is defined as follows

$$\max\ \tilde{z}_r = \sum_j \tilde{c}_{rj} x_j^{\alpha_j} \qquad r = 1,2. .. .\ q$$

$$\text{s.t} \quad \sum_j \tilde{a}_{ij} x_j \lesssim \tilde{b}_i\ , \qquad i = 1,2 \ldots\ m \qquad \ldots\ldots \quad (3.2)$$

$$x_j \ge 0$$

where $\tilde{a}_{ij}$ and $\tilde{c}_{rj}$ in the above relation are in the trapezoidal form as

$\tilde{a}_{ij} = (a_{ij}^1, a_{ij}^2, a_{ij}^3, a_{ij}^4)$

$\tilde{c}_{rj} = (c_{rj}^1, c_{rj}^2, c_{rj}^3, c_{rj}^4)$

Definition 3.2

$x \in X$ is said to be a feasible solution to the FMONLP problem (3.2) if it satisfies constraints of (3.2).

Definition3.3

$x^* \in X$ is said to be an optimal solution to the FMONLP problem (3.2) if there does not exist another $x \in X$ such that $\tilde{z}_i(x) \ge \tilde{z}_i(x^*)$ for all i= 1,2,. . . q .

Now the FMOLNP can be easily transformed to a classic form of a MONLP by considering R as a linear ranking function. By implementing the R on the above model, (3.2)we obtain the classical form of MONLP problem:

$$\max\ R(\tilde{z}_r) = \sum_j R(\tilde{c}_{rj})\, x_j^{\alpha_j} \qquad r = 1,2. .. .\ q$$

$$\text{s.t} \qquad \sum_j R(\tilde{a}_{ij}) x_j \lesssim R(\tilde{b}_i) \qquad i = 1,2. . . . .\ m$$

$$x_j \ge 0$$

so we have

$$\max\ z_r' = \sum_j c'_{rj} x_j^{\alpha_j} \qquad r = 1,2,. . . . ..\ q$$

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s.t : $\sum_j a'_{ij}\, x_j \le$ b'$_i$ i= 1,2, ... m . . . . (3.3) $x_j \ge 0$

where a_{ij}',b_i',c_j' are real numbers corresponding to the fuzzy numbers $\tilde{a}_{ij}$, $\tilde{b}_i$, $\tilde{c}_j$ with respect to linear ranking function R, respectively.

Lemma 3.4

The optimal solutions of (3.2) and (3.3) are equivalent.

Proof.

Let M_1 , M_2 be sets of all feasible solutions of (3.2) and (3.3) respectively

Then x ∈ M_1 iff $\sum_j (\tilde{a}_{ij}) x_j \lesssim (\tilde{b}_i)$, i = 1,2, ... m

By considering R as a linear ranking function, we have

$$\sum_j R(\tilde{a}_{ij}) x_j \lesssim R(\tilde{b}_i)$$

i = 1,2. . . . . m

$\Rightarrow \sum_j a'_{ij}\, x_j \le$ b'$_i$

Hence x ∈ M_2

Thus $M_1 = M_2$

Let x*∈ X be the complete optimal solution of (3.2)

Then $\tilde{z}_r(x^*) \ge \tilde{z}_r$(x) , for all x ∈ X

Where X is feasible set of solutions.

$\Rightarrow$ R($\tilde{z}_r(x^*)) \ge R(\tilde{z}_r$(x))

$\Rightarrow R\left(\sum \tilde{c}_{rj} x_j^{*\alpha_j}\right) \ge R\left(\sum c_{rj} x_j^{\alpha_j}\right)$

$\Rightarrow \left(\sum R(\tilde{c}_{rj}) x_j^{*\alpha_j}\right) \ge \sum R(c_{rj}) x_j^{\alpha_j}$, ∀ j = 1,2,. . . q

$\Rightarrow \sum c'_{rj}\, x_j^{*\alpha_j} \ge \sum c'_{rj} x_j^{\alpha_j}$, ∀ j = 1,2. . . . . . . .q

$\Rightarrow$ Z_r' (x*) ≥ Z_r' (x) , ∀ x

$$\mu_{z_r}(x) = \begin{cases} 0 & if\ \ Z_r \le L_r \\ \frac{Z_r - L_r}{U_r - L_r} & if\ \ L_r < Z_r < U_r \\ 1 & if\ \ Z_r \ge U_r \end{cases}$$

Step -4

Using the above membership functions we formulate a crisp model by introducing an augmented variable λ as:

Min : λ

Subject to

$$\sum C'_{rj} x_j^{\alpha_j} + (U_r - L_r)\lambda \ge U_r \quad , \quad r = 1,2,. . . . q$$
$$\sum a'_{ij}\, x_j \le b'_i \quad , \quad i = 1,2, ...m \quad . . . . . . \ (4.2)$$
$$\lambda \ge 0, \quad x_j \ge 0 \quad , \quad j = 1,2,. . . . .n$$

Step- 5

Here the crisp model (4.2) is a non –linear programming problem with non linear constraints. This is solved using separable programming method .Thus we get compromise solution.

IV. FUZZY PROGRAMMING TECHNIQUE

We have to solve the MONLPP

max $z_r' = \sum_j c'_{rj}\, x_j^{\alpha_j}$ r = 1,2,. . . . .. q

s.t : $\sum_j a'_{ij}\, x_j \le$ b'$_I$ i = 1,2, ... m . . . . (4.1)

$x_j \ge 0$

In partial modification of Zimmermann's fuzzy programming technique we formed a technique to solve multi objective non linear programming problem. The method is presented briefly in the following steps.

Step- 1

Solve the multi objective Non- linear programming problem by considering one objective function at a time and ignoring all others. Repeat the process q times for q different objective functions. Let X^1, X^2,. . . . . . . X^q be the ideal solutions for respective functions.

Step- 2

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Using all the above ideal q solutions in step – 1 construct a pay – off matrix of size q by q . Then form the pay-off matrix find the lower bound (L_r) and upper bound (U_r) for the objective function Z_r' as:

$L_r \leq Z_r' \leq U_r$ r = 1,2,. . . . . q

Step- 3

Define fuzzy linear membership function $\mu_{z_r}(x)$ for the r th objective function Z'_r , r = 1,2 . . .q as

$$\mu_{z_r}(x) = \begin{cases} 0 & if \;\; Z_r \leq L_r \\ \frac{Z_r - L_r}{U_r - L_r} & if \;\; L_r < Z_r < U_r \\ 1 & if \;\; Z_r \geq U_r \end{cases}$$

Step -4

Using the above membership functions we formulate a crisp model by introducing an augmented variable λ as:

Min : λ

Subject to

$$\sum C'_{rj} x_j^{\alpha_j} + (U_r - L_r)\lambda \geq U_r \quad , \quad r = 1,2,\ldots q$$
$$\sum a'_{ij} x_j \leq b'_i \quad , \quad i = 1,2,\ldots m \qquad \ldots\ldots \quad (4.2)$$
$$\lambda \geq 0, \quad x_j \geq 0 \quad , \quad j = 1,2,\ldots n$$

Step- 5

Here the crisp model (4.2) is a non –linear programming problem with non linear constraints. This is solved using separable programming method .Thus we get compromise solution.

V. Numerical Example

$$\max : \tilde{z_1}(x) = \tilde{2}x_1 + \tilde{3}x_2 - \tilde{2}x_1^2$$
$$\max : \tilde{z_2}(x) = \tilde{3}x_1 + \tilde{4}x_2 - \tilde{5}x_1^2$$

s.t

$$\tilde{1}x_1 + \tilde{4}x_2 \leq \tilde{4}$$
$$\tilde{1}x_1 + \tilde{1}x_2 \leq \tilde{2} \qquad \ldots\ldots\ldots \quad (4.3)$$
$$x_1, x_2 \geq 0$$

Where

$\tilde{2}$ = (1.9,2.1,2.2,2.6)
$\tilde{3}$ = (2.2,2.3,3.3,3.8)
$\tilde{2}$ = (1.2,1.3,2.3,2.8)
$\tilde{3}$ = (2.3,2.5,3.3,3.5)
$\tilde{4}$ = (3.2,3.4,4.2,4.8)
$\tilde{5}$ = (4.3,4.4,5.2,5.7)
$\tilde{1}$ = (0.8,0.9,1.1,1.5)
$\tilde{4}$ = (3.2, 4.0,4.4)
$\tilde{1}$ = (0.7, 0.9,1.1,1.3)
$\tilde{1}$ = (0.6,0.8, 1.3,1.7)
$\tilde{4}$ = (3.3,3.4,4.1,4.4)
$\tilde{2}$ = (1.8,1.9,2.2,2.5)

Using ranking function suggested by Roubens [7] the problem reduces to

$$\max : z'_1(x) = R(\tilde{2})x_1 + R(\tilde{3})x_2 - R(\tilde{2})x_1^2$$
$$\max : z_2'(x) = R(\tilde{3})x_1 + R(\tilde{4})x_2 - R(\tilde{5})x_1^2$$

s.t

$$R(\tilde{1})x_1 + R(\tilde{4})x_2 \leq R(\tilde{4})$$
$$R(\tilde{1})x_1 + R(\tilde{1})x_2 \leq R(\tilde{2})$$
$$x_1, x_2 \geq 0$$

$$\Rightarrow \quad \max : z'_1(x) = 2.2x_1 + 2.9x_2 - 1.9x_1^2 \qquad \ldots\ldots\ldots \quad (1')$$
$$\max : z'_2(x) = 2.9x_1 + 3.9x_2 - 4.9x_1^2 \qquad \ldots\ldots\ldots \quad (2')$$

s.t

$$1.1x_1 + 3.9x_2 \leq 3.8$$
$$1.0x_1 + 1.1x_2 \leq 2.1 \qquad \ldots\ldots\ldots \quad (3')$$

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Volume 2, Issue 5, September 2013

$x_1, x_2, \geq 0$

Solving (1') with (3') by Wolf's method

$x_1 = \frac{539}{1482} = 0.3637$, $x_2 = \frac{50387}{257798} = .8718$

Solving (2 ') with (3') by Wolf's method

$x_1 = \frac{9}{49} = 0.18$, $x_2 = \frac{1763}{1911} = 0.9226$

The lower bound (L.B) and upper bound (U.B) of objective functions z_1' and z_2' have been computed as follows

Function	L.B	U.B
Z_1'	3.0655	3.0770
Z_2'	3.8065	3.9653

As per step -4, let us solve

min : λ

s.t

$$2.2x_1 + 2.9x_2 - 1.9x_1^2 + 0.115 \geq 3.0770$$
$$2.9x_1 + 3.9x_2 - +4.9x_1^2 + 0.1588 \geq 3.953$$
$$1.1x_1 + 3.9x_2 \leq 3.8 \qquad \ldots\ldots\ldots\ldots\ldots \qquad (4.4)$$
$$1.0x_1 + 1.1x_2 \leq 2.1$$
$$x_1, x_2 \geq 0$$

Solving (4.4) the optimal solution of the problem is obtained as :

$x_1^* = 0.4518$

$x_2^* = 0.8469$

Now the optimal values of the objective functions of FMONLPP (4.3) become

$$\tilde{z}_1^* = \tilde{2}x_1^* + \tilde{3}x_2^* - \tilde{2}x_1^2$$
$$= (1.9,2.1,2.2,2.6)x_1^* + (2.2,2.3,3.3,3.8)x_2^* - (1.2,1.3,2.3,2.8)x_1^{2*}$$
$$= (2.4767,2.6314,3.3194,3.8214)$$
$$\tilde{z}_2^* = \tilde{3}x_1^* + \tilde{4}x_2^* - \tilde{5}x_1^{2*}$$
$$= (2.3,2.5,3.3,3.5)x_1^* + (3.2,3.4,4.2,4.8)x_2^* - (4.3,4.4,5.2,5.7)x_1^{2*}$$
$$= (2.8716,3.1109,3.9866,4.4831)$$

The membership functions corresponding to the fuzzy objective functions are as follows.

$$\mu_{\tilde{z}_1}(x) = \begin{cases} 0 & x \leq 2.4767 \\ \frac{x-2.4767}{0.1597} & 2.4767 < x \leq 2.6314 \\ 1 & 2.6314 < x \leq 3.3194 \\ \frac{3.8214-x}{0.502} & 3.3194 < x \leq 3.8214 \\ 0 & x > 3.8214 \end{cases}$$

$$\mu_{\tilde{z}_2}(x) = \begin{cases} 0 & x \leq 2.8616 \\ \frac{x-2.8616}{0.249} & 2.8616 < x \leq 3.1109 \\ 1 & 3.1109 < x \leq 3.9866 \\ \frac{4.4831-x}{0.4965} & 3.9866 < x \leq 4.4831 \\ 0 & x > 4.4831 \end{cases}$$

VI. Conclusion

In this paper we considered fuzzy multi objective Non linear programming problem in which the objective function is nonlinear but the constraints are linear. The method can be applied to problems when both objective functions and constraints are nonlinear.

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