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Kinds of Pre A*-lattices

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Abstract:* *In this paper classify the kinds of Pre A*-lattices like distributive Pre A*-lattice, modular Pre A*-lattice and establish some theorems related with these. This paper analyzes the concept of Lattice on Pre A*-algebra. Define the lattice on Pre A*-algebra and call such a defined lattice on Pre A*-algebra as Pre A*-lattice and derive the properties of the Lattice on Pre A*-algebra. Define greatest lower bound and least upper bound on Pre A*-algebra .We observe that 2 acts as greatest lower bound(g.l.b) with respect to meet whereas 2 acts as least upper bound(l.u.b) with respect to join. Define atoms, dual atoms, irreducible elements with respect to meet as well as join on Pre A*-algebra. Obtain various theorems on these atoms, dual atoms, irreducible elements on Pre A*-algebra establish the atomic lattices, dual atomic lattices on Pre A*-algebra.

Key Words: **Pre A* - Algebra, Pre A* -Lattice, Complemented Pre A^{*-} lattice.**

I. INTRODUCTION

In a draft paper [4], The Equational theory of Disjoint Alternatives, around 1989, E.G. Manes introduced the concept of Ada (Algebra of disjoint alternatives) $(A, \wedge, V, (-)^I, (-)_\pi, 0, 1, 2)$(Where $\wedge$,V are binary operations on A, $(-)^I$, $(-)_\pi$ are unary operations and 0,1,2 are distinguished elements on A)which is however differ from the definition of the Ada of his later paper[5] Adas and the equational theory of if-then-else in 1993. While the Ada of the earlier draft seems to be based on extending the If-Then-Else concept more on the basis of Boolean algebras and the later concept is based on C-algebras $(A, \wedge, V, (-)^{\sim})$)(where $\wedge$,V are binary operations on A,$(-)^{\sim}$ is a unary operation) introduced by Fernando Guzman and Craig C. Squir[2]. In 1994, P.Koteswara Rao[3] first introduced the concept of A*-algebra $(A, \wedge, V, *, (-)^{\sim}, (-)_\pi, 0, 1, 2)$)(where $\wedge$,V,$*$ are binary operations on A, $(-)^{\sim}$,$(-)_\pi$ are unary operations and 0,1,2 are distinguished elements on A) not only studied the equivalence with Ada, C-algebra, Ada's connection with 3-Ring, Stone type representation but also introduced the concept of A*-clone, the If-Then-Else structure over A*-algebra and Ideal of A*-algebra. In 2000, J.Venkateswara Rao[6] introduced the concept Pre A*-algebra $(A, \vee, \wedge, (-)^{\sim})$(where $\wedge$,V are binary operations on A,$(-)^{\sim}$ is a unary operation on A)analogous to C-algebra as a reduct of A*- algebra, studied their sub direct representations, obtained the results that **2**= {0, 1} and **3**= {0, 1, 2} are the sub directly irreducible Pre-A*-algebras and every Pre-A*-algebra can be imbedded in $\mathbf{3}^X$ **(** where $\mathbf{3}^X$ is the set of all mappings from a nonempty set X into **3**= {0, 1, 2}**).**

1. Pre A* - Algebra [6]

1.1 Definition:-

An algebra $(A, \vee, \wedge, (-)^{\sim})$ satisfying

(a) $(x^{\sim})^{\sim} = x, \forall x \in A$

(b) $x \wedge x = x, \forall x \in A$

(c) $x \wedge y = y \wedge x, \forall x,y \in A$

(d) $(x \wedge y)^{\sim} = x^{\sim} \vee y^{\sim}, \forall x,y \in A$

(e) $x \wedge (y \wedge z) = (x \wedge y) \wedge z, \forall x,y,z \in A$

(f) $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z), \forall x,y,z \in A$

(g) $x \wedge y = x \wedge (x^{\sim} \vee y), \forall x,y \in A$

is called a Pre A* - algebra.

1.2 Definition of a greatest lower bound of an element on Pre A^* - algebra [8]:

Let A be a Pre A^* - algebra. An element $a \in A$ is said to be a lower bound if $a \le x, \forall x \in A$. And a is said to be a greatest lower bound if there exists a lower bound b such that $b \le a$. The greatest lower bound (g.l.b) of the elements a, b is denoted by $a \wedge b$.

1.3 Example: Since in a Pre A^* - algebra, $0 \wedge 1 = 0$ so 0 is g.l.b of {0, 1}. Also since $2 \wedge 0 = 2$ so 2 is g.l.b of {0,2}. H**ence in** a Pre A^* - algebra, 2 if exists then it is the g.l.b.

1.4 Definition of least upper bound of an element on Pre A^* - algebra:

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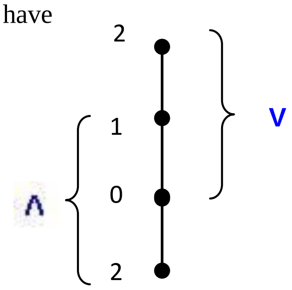
Let A be a Pre A^* - algebra. An element $a \in A$ is said to be an upper bound if $x \leq a$, for all $x \in A$. And a is said to be a least upper bound if there exists an upper bound b such that $a \leq b$. The least upper bound (l.u.b) of the elements a, b is denoted by $a \vee b$.

1.5 Example: Since in a Pre A^* - algebra, $0 \vee 1 = 1$. So 1 is l.u.b of {0, 1}. Also since $2 \vee 0 = 2$, 2 is l.u.b of {0, 2**}.** Hence in a Pre A^* - algebra, 2 if exists then it is the l.u.b.

1.6 Note: Since in a Pre A∗–algebra. We have



Observe that element 2 acts as the least element with respect to "meet", whereas the same element (2) acts as the greatest element with respect to "join"

2.Pre A^* - lattice

In this section we define the lattice on Pre A*-algebra and we call such a defined lattice on Pre A*-algebra as Pre A*-lattice and we derive the properties of the Lattice on Pre A*-algebra. We define greatest lower bound and least upper bound on Pre A*-algebra

2.1 Pre A^* - lattice (Definition) [8]:

Let A be a Pre A^* - algebra. A non-empty subset L of a Pre A^*-algebra A in which for each pair of elements $a \in A$, $b \in B(A)$ in L has greatest lower bound $a \wedge b$ and least upper bound $a \vee b$ exists in L .Such a defined set L in Pre A^* - algebra is said to be a Pre A^* -lattice.

2.2 Pre A^* -Lattice as algebraic system (Definition):

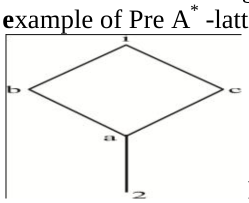
Let A be a Pre A^* - algebra. A non-empty subset L of a Pre A^*-algebra A, equipped with two binary operations meet ($\wedge$) and join ($\vee$) which assign to every pair $a \in A$, $b \in B(A)$ of the elements of L, uniquely an element $a \wedge b$ as well as an element $a \vee b$ in L in such a way that the following axioms holds.

(i) $a \wedge (b \wedge c) = (a \wedge b) \wedge c$, $\forall$ a, b, c $\in$ L (associative)
(ii) $a \wedge b = b \wedge a$, $\forall$ a, b $\in$ L (commutative)
(iii) $a \wedge (a \vee b) = a$, $\forall$ a, b $\in$ L (absorption law)

2.3 Note: The above axioms (i), (ii), (iii) holds with respect to $\vee$ also**.**

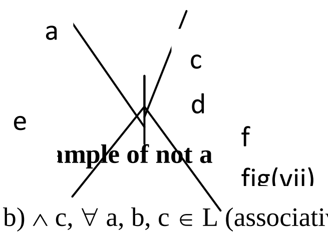
2.4 Examples of Pre A^* - lattices:

1. Let A be a Pre A^*-algebra and 2 = {0, 1}is a subset of A then 2 ={0,1}is a Pre A^* -lattice
2. 3= {0, 1, 2} is a subset of a Pre A^*-algebra then 3= {0, 1, 2}is a Pre A^* -lattice
3. Fig(vi) is an **e**xample of Pre A^* -lattice



Fig(vi) Example of Pre A^* -lattice

4. Example of a poset which is shown in Fig(vii) is not a Pre A^*-lattice:



2.5 Properties of Pre A* -lat

(i)$a \wedge (b \wedge c) = (a \wedge b) \wedge c$, $\forall$ a, b, c $\in$ L (associative)
(ii) $a \wedge b = b \wedge a$, $\forall$ a, b $\in$ L (commutative)
(iii) $a \wedge (a \vee b) = a$, $\forall$ a, b $\in$ L (absorption law)
(iv)$a \wedge a = a$, $\forall$ $a \in$ L (idempotent)

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2.6 Theorem[9]: Let A be a Pre A∗–algebra. L is a subset of A Then (L, $\wedge$, $\vee$) is a Pre A* - lattice

3. Kinds of Pre A*-lattices [9] :

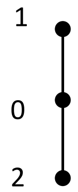
In this section we define some kinds of Pre A*-lattices like distributive Pre A*-lattice, modular Pre A*-lattice and we prove some theorems related with this.

3.1 Distributive Pre A^* -lattice (Definition): Let A be a Pre A^* – algebra and L is subset of A. Then a Pre A^* -lattice (L, $\wedge$,$\vee$) is said to be distributive Pre A^* -lattice if any elements a, b, c in L we have the distributive law

$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c), \forall$ a, b, c $\in$ L

3.2 Example of a distributive Pre A^* -Lattice:

The chain shown in Fig(v) is a distributive Pre A^* -lattice



Fig(v)

3.3 Theorem: Let A be a Pre A* - algebra and L is a subset of A which is Pre A^*- lattice.Then L becomes a distributive Pre A* -Lattice

Proof: Since A is a Pre A∗–algebra, and L is a subset of A which is a Pre A* -lattice (by 3.2.6)and since distributive law holds in a Pre A∗–algebra,

$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c), \forall$ a, b, c $\in$ L.

Hence L becomes a distributive Pre A* -lattice.

3.3 Semi-*-complement of an element on a Pre A∗–algebra (Definition):

Let A be a Pre A∗ – algebra with the least element α and x∈A. An element $x^{\sim}$ ∈A is said to be semi-*****-complement of x if $x \wedge x^{\sim} = \alpha$

3.4 Example: Let A be a Pre A∗ – algebra with least element with respect to $\vee$ is 0 and since $0 \wedge 1 = 0$,so 1 is semi-*****-complement of 0

Since$1 \wedge 0 = 0$, 0 is the semi-*****-complement of 1

3.5 Example: Let A be a Pre A∗ – algebra with least element with respect to $\wedge$ is 2 and 2∈A , since $2 \wedge 2 = 2$, hence 2 is semi-*****-complement of 2

3.6 Definition: Let A be a Pre A∗ – algebra with least element is α. The semi-*****-complements of an element other than the least element α is called a proper semi-*****-complement in A. If in addition the proper semi-*****-complement is maximal then it is called maximal proper semi- *****-complement in A.

3.7 Example 1: In a Pre A^* – algebra with least element with respect to $\vee$,

Since $0 \wedge 1 = 0$, $1 \wedge 0 = 0$, so 0, 1 are semi-*****-complements to one another and 1 is the proper semi-*****-complement

Example 2: Let A be a Pre A∗ – algebra with least element with respect to $\wedge$(i.e.,2) , since $2 \wedge 2 = 2$, 2 is semi-*****-complement but proper semi-*****-complement not exists for 2

3.8 Semi-*-complemented Pre A∗-lattice (Definition): Let A be a Pre A∗ – algebra with least element α .Then A is said to be semi-*****-complemented Pre A∗ -lattice if every inner element (other than least and greatest elements in A) has at least one proper semi-*****-complement.

3.9 Example: Let **3**={0,1,2} be a Pre A∗ – algebra with least element with respect to $\vee$ is 0 and greatest element with respect to $\vee$ is 2 which is a semi-*****-complemented Pre A∗ lattice since 1 has proper semi-*****-complement.

3.10 Note: Let A be a Pre A∗ – algebra with least element. Then every *****-complement of an element is a semi-*****-complement but semi-*****-complement need not be *****-complement

3.11 Definition: (*-complement or complement of an element on a Pre A∗–algebra):

Let A be a Pre A∗ – algebra with t**he** l**e**ast element α and the greatest element β.

Then $a^{\sim} \in$ A is said to be complement of a∈ A if $a \wedge a^{\sim} = \alpha$, $a \vee a^{\sim} = \beta$ .Such a defined complement $a^{\sim}$ of the element a on a Pre A∗ – algebra is said to be the *-complement

3.12 Note: If α is **l**east element with respect to $\vee$ is 0, β is greatest element with respect to $\wedge$ is 1 then $a \wedge a^{\sim} = \alpha$, $a \vee a^{\sim} = \beta$ therefore $0 \wedge 1 = 0$, $0 \vee 1 = 1$,so 0 is the complement of 1 Similarly $1 \wedge 0 = 0$, $1 \vee 0 = 1$, so 1 is the complement of 0.

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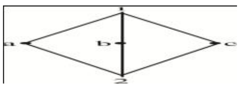
3.13 Note: Let A be a Pre A∗ – algebra. If α is **the l**east element with respect to $\wedge$ i.e.,2, β is the greatest element with respect to $\vee$ i.e.,2 since $2 \wedge 2 = 2$, $2 \vee 2 = 2$
So 2 is the complement of 2

3.14 *-Complemented Pre A* -lattice (Definition):
Let A be a Pre A∗ – algebra then A is said to be *-complemented **Pre A* -**lattice if each element has a *-complement in it.

3.15 Example: The Pre A***** **-**lattice shown in the fig(i) is a *-complemented Pre A***** **-**lattice.Here every element has a *-complement but these are not unique.



Fig(i) *-complemented Pre A* **-lattice**
Here b,c are *-complements of a
Here a,c are *-complements of b
Here a,b are *-complements of c

3.16 Example: fig(ii), is an example of a Pre A* -lattice which is not *-complementedIn fig(ii), the elements a,e c,d have *-complements but the element b has no *-complement.

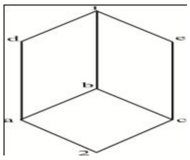


Fig (ii) **not *-complemented** Pre A* **-lattice**

3.17 Theorem: Let A be a Pre A∗–algebra then for any subset L of A, a Pre A*- lattice L becomes a complemented distributive Pre A* -Lattice**.**

Proof: Since A is a Pre A∗–algebra, then for any subset L of A, L is a distributive Pre -A* -lattice (by theorem 3.3). Since each element in L has complement in it.
Hence L is a complemented distributive Pre A* -lattice.

3.18 Lemma: In the Poset (A, $\leq$), if $a \leq b \Rightarrow a \vee (b \wedge c) = b \wedge (a \vee c)$, $\forall$ a, b, c $\in$ A

Proof: Define $\leq$ in A as $a \leq b \Leftrightarrow a \wedge b = a$ (i.e., $a \vee b = b$)
Suppose $a \leq b$ then $b \wedge a = a$.
Now $b \wedge (a \vee c) = (b \wedge a) \vee (b \wedge c) = a \vee (b \wedge c)$ (by 1..1 (f))

3.19 Definition (Modular Pre A* -Lattice): Let A be a Pre A∗ – algebra and L be subset of A. Then a Pre A* -lattice L is said to be a modular Pre A* -lattice if $a \leq b \Rightarrow$
$a \vee (b \wedge c) = b \wedge (a \vee c), \forall$ a,b,c $\in$L

3.20 Example: The Pre A***** **-l**attice shown in Fig (ii) , is the modular Pre A* **-**lattice
Since $2 \leq a$, $2 \vee (a \wedge i) = a \wedge (2 \vee i)$.

3.21 Theorem: Let A be a Pre A^*–algebra. Then a sub set L of A is a modular Pre A^* -lattice
Proof: Since (L, $\leq$)is a Pre A^* -lattice. By lemma 3.3.12, if $a \leq b \Rightarrow a \vee (b \wedge c)$
$= b \wedge (a \vee c)$, $\forall$ a, b, c $\in$ L. Hence L is a modular Pre A^* -lattice**.**

3.22 Unique *-complement of an element on a Pre A∗–algebra(Definition): Let A be a Pre A∗ – algebra. Then a$\in$ A is said to be a unique *-complement if a has exactly one *- complement in A

.3.23 Uniquely *-complemented Pre A* -lattice(Definition):
Let A be a Pre A∗ – algebra and L be a subset of A Then L is said to be uniquely *-complemented Pre A***** **-**lattice if each element in L has unique *-complement in L.

3.24 Example: The Pre A*****-lattice shown in fig (viii) is uniquely *-complemented

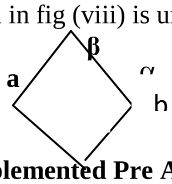


Fig (iii) Example of a uniquely *-complemented Pre A*-lattice

3.25 Relative *-complement on a Pre A∗–algebra (Definition):
Let A be a Pre A∗ – algebra and L be a subset of A. Let [a, b]$\in$ L and u is an element of [a, b].An element $x^{\sim}$ of L is said to be relative *-complement of u in[a, b]
if $x \wedge x^{\sim} = a$, $x \vee x^{\sim} = b$

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3.26 Note: If x is a relative *-complement of u in [a, b] then we have x $\in$[a, b] and x is *-complement of u in[a,b]

3.27 Example: 0 $\wedge$ 1 =0, 0 $\vee$ 1 =1, so 0 is the relative *-complement in [0, 1]

1 $\wedge$ 0 =0 , 1 $\vee$0 =1,so 1 is the relative *-complement in [0,1]

Example: If a=2, b=2 since 2 $\wedge$ 2 =2 , 2 $\vee$2 =2, so 2 is the relative *-complement of 2

3.28 Relatively *-complemented Pre A* -lattice (Definition):

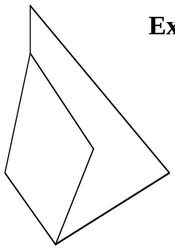
Let A be a Pre A∗ – algebra. Then a subset L of A is said to be relatively *-complemented **Pre A* -**lattice if for any triplet of elements a, b, u$\in$ L such that a ≤ u ≤ b there exists at least one *-complement of u in [a,b]. That is every interval of L is a *-complemented Pre A* -lattice of A.

3.29 Example: The Pre A***** **-**lattice shown in figure (i) is an example of a Pre A***** **-**lattice which is *-complemented as well as relatively *-complemented

3.30 Note: Let A be a Pre A∗ – algebra then every bounded relatively *-complemented Pre A* -lattice in A is *-complemented but converse is not true. That is a *-complemented Pre A***** **-**lattice may or may not be relatively *-complemented Pre A* -lattice.

3.31 Example: The Pre A***** **-**lattice shown in fig (iii) is *-complemented but not relatively *-complemented. Since [α, b], [a, β] are not *-complemented Pre A***** **-**lattice since a has no *-complements in [α, b].

3.32 Example: The Pre A* -lattice shown in fig (iii) is not relatively *-complemented



Example of Pre A* -lattice which is not relatively *-complemented

fig (iv)

Since [a, β] ={a, d, β} is not a *-complemented Pre A***** -lattice since a has no *-complement hence it is not relatively *-complemented.

3.33 Section *-complemented Pre A* **-lattice (Definition):**

Let A be a Pre A∗ – algebra with least element α and L be a subset of A Then L is said to be section *-complemented Pre A* **-**lattice if every interval of the form [α, a] (a$\in$L)is a *-complemented Pre A***** **-**lattice of A.

That is for each pair of elements a, x with x ≤a there exists an element $x^{\sim}$ $\in$A such that $x \wedge x^{\sim} = \alpha$, $x \vee x^{\sim} = a$

3.34 Example: The Pre A***** **-**lattice shown in this fig (iv) is not section *-complemented because [0, b] is a not *-complemented Pre A***** **-**lattice

3.35 Example: The **Fig. (i)** 2 is an example of a Pre A***** **-**lattice which is section *-complemented

3.36 Theorem: Let A be a Pre A∗–algebra. Then every relatively *-complemented Pre A***** **-**lattice in A is Section *-complemented.

Proof: Since A is a Pre A∗–algebra, if L is a relatively *-complemented Pre A* -lattice then by the definition L every interval of A is a *-complemented Pre A***** **-**lattice of A.

Hence L is a section *-complemented Pre A***** **-**lattice .

3.37 Note: Let A be a Pre A∗ – algebra every relatively *-complemented Pre A***** **-**lattice in A is section *-complemented but converse is not true.

For example, in Fig(iii) is section *-complemented but not relatively *-complemented

Since [a, β] ={a, d, β} is not a *-complemented Pre A***** -lattice since a has no *-complement hence it is not relatively *-complemented.

3.38 Semi-*-complemented Pre A* -lattice (Definition): Let A be a Pre A∗ – algebra with least element α and L be a subset of A.Then L is said to be semi-*-complemented Pre A***** -lattice if every inner element (other than least and greatest elements in A) has at least one proper semi-*-complement.

3.39 Weakly *-complemented Pre A* -lattice (Definition): Let A be a Pre A∗ – algebra with least element α and L be a subset of A Then L is said to be weakly *-complemented Pre A***** **-**lattice if any pair of elements a, b (a < b) of L a has semi *-complement, that is however not a semi *-complement of b. That is $x^{\sim}$ is semi-*-complement of a but not semi- *-complement of b.

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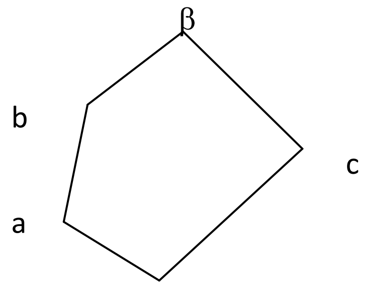
3.40 Example: The Pre A***** **-**lattice shown in fig (iv) of is example of a Pre A***** **-**lattice which is not weakly *-complemented. This is not weakly *-complemented since a<b and c is semi *-complement of both a and b



α **Fig. (v) Example of a semi lattice which is not weakly *-complemented**

3.42 Theorem: Let A be a Pre A*–algebra. Then every weakly *-complemented Pre A***** **-**lattice in A is semi-*-complemented.

3.43 Theorem: Let A be a Pre A*–algebra with least element α .Then every section *-complemented Pre A***** **-**lattice in A is weakly *-complemented.

3.44Theorem: Let A be a Pre A*-algebra with least element α , greatest element β. Then every uniquely *-complemented Pre A***** **-**lattice in A is weakly *-complemented

4 Atoms and dual atoms for Pre A*-lattice [9]**:**

In this section we define least and greatest elements in a Pre A* -lattice**,** we define atoms, dual atoms, irreducible elements in Pre A* - algebra and we prove some theorems in these. We define atomic, dual atomic Pre A*- lattice. We give some examples of these.

4.1 Definition of least and greatest elements on a Pre A* -lattice: Let A be a Pre A* – algebra and L be any Pre A* -lattice in A. An element $\alpha \in$ L is called least element if $\alpha \leq x, \ \forall x \in L$. Similarly $\beta \in L$ is called greatest element if $x \leq \beta, \ \forall x \in L$.

4.2 Definition of atom for Pre A*-lattice: Let L be a subset of a Pre A* - algebra A. Then an element p of a bounded below Pre A*-lattice L with least element α is called an atom, if $\alpha \prec p$ (α is covered by p).

If there exists an atom p, for each element $a \neq \alpha$ of L such that p≤a. Then we say that L is atomic Pre A* -lattice

That is, in a Pre A* - lattice, if (S, ∧) is a semi lattice with least element **2**,then 2 is atom with respect to ∧ if $2 \prec p$

if **(S,** ∨**)** is a semi lattice with least element **0**,then 0 is atom with respect to ∨

if $0 \prec p$.

4.3 Example: In the Pre A* -lattice shown in fig (i) a, b, c are atoms and this is atomic.

In the Pre A* -lattice shown in fig (vi) a is the only one atom and this Pre A*- lattice is also atomic**.**

In the Pre A* -lattice shown in fig (ii) a, c are atoms and this Pre A* -lattice is also atomic.

4.4 Theorem: Let A be a Pre A*-algebra and L be a subset of A then every finite Pre A* **-lattice**, which is bounded below is atomic.

Proof: Let L be a subset of a Pre A* - algebra A.

Then an element p of a bounded below Pre A* **-**lattice L with least element α is called an atom, if $\alpha \prec p$ (α is covered by p).

Since in a Pre A* - algebra A,if **(S,** ∧**)** is a semilattice with least element **2**,then 2 is atom with respect to ∧ if $2 \prec p$

If there exists an atom p, for each element $a \neq \alpha$ of S such that p≤a. Then we say that L is atomic

It is true for every such a finite Pre A***** **-**lattice L

4.5 Dual atom for Pre A* -lattice (Definition):

Let L be a subset of a Pre A* - algebra A. Then an element p of a bounded above Pre A* -lattice with greatest element β is called dual atom if if $q \prec \beta$ (q is covered by β)

If there exists a dual atom q for any element $a \neq \beta$ of L such that $a \leq q$. Then we say that L is dual atomic Pre A* -lattice

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4.6 Note: That is, in a Pre A* - lattice, if **(S, $\vee$)** is a semilattice with greatest element **2**,then 2 is dual atom with respect to $\vee$ if 2 $\prec$ p

That is, in a Pre A* - lattice, if **(S, $\wedge$)** is a semilattice with greatest element **1**,then 1 is dual atom with respect to $\wedge$ if 1 $\prec$ p

4.7 Example: Consider the diagrams: fig (i), fig (vi), fig (ii)

In fig (i), a, b, c are dual atoms; In fig (vi), b, c are dual atoms.

In fig (ii), d, e, b are the dual atoms. All these are dual atomic Pre A* -lattices.

4.8 Join irreducible elements for Pre A* - lattice (Definition): Let A be a Pre A*- algebra and Lbe a subset of A which is Pre A* -lattice in A with a lower bound α. An element a in L is said to be join irreducible if a = x $\vee$ y $\Rightarrow$ a = x or a = y.

4.9 Example 1: 2 is join irreducible in a Pre A* - lattice.

Example 2: In Fig(v) , every element in this chain is join irreducible.

4.13 Meet irreducible elements for Pre A* - lattice (Definition):

Let A be the Pre A* - algebra and a subset L of A be a Pre A* -lattice in A with an upper bound β. An element a in L is said to be meet irreducible

if a = x $\wedge$ y $\Rightarrow$ a = x or a = y.

4.14 Example: 2 is meet irreducible in a Pre A* - lattice.

Example: In Fig(v) ,

Every element in this chain is meet irreducible

II. CONCLUSION

Established the lattice structure on Pre A*-algebra and call such a defined lattice on Pre A*-algebra as Pre A*-lattice and established the properties of the Lattice on Pre A*-algebra. Defined greatest lower bound and least upper bound on Pre A*-algebra. Observe that 2 act as greatest lower bound (g.l.b) with respect to meet whereas 2 act as least upper bound (l.u.b) with respect to join. Studies atoms, dual atoms, irreducible elements with respect to meet as well as join on Pre A*-algebra. Obtained various theorems on these atoms, dual atoms, irreducible elements on Pre A*-algebra. Established the atomic lattices, dual atomic lattices *on* Pre A*-algebra.

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