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Multifractal Analysis of Annual Run-Off: Using WTMM Method

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Abstract*—The study analyses the multi fractal properties of the river run off records from four hydrological stations in the Bharthapuzha river basin using WTMM analysis. The multifractal analysis was performed for each month of each year and also for the whole year.The pattern exhibited multi scaling structure among months with localized effect on one side of the spectrum. that correspond to lower frequencies. Multi fractal analysis revealed that during the monsoon season, with moderate weather conditions, larger discharge rates were observed with low fluctuation exponent. These fluctuating exponents vary from one basin to another basin.. For all run off records, the prevalent multi fractality is consistent. Further the width of the spectrum tends to be inversely propotional to the width of the basin area being analysed.

***Index Terms*—Wavelet Transform, Multi Fractal Analysis, Fractal Dimension, Singularity Spectrum.**

I. INTRODUCTION

In recent times ,fractal structures found in many physical, hydrological, astrological, and hydro physical processes, etc, modeled as a mechanical dissipative many body system with nonlinearity, had attracted a great deal of interest[1-3]. Chaotic behavior in the sense of time bounded attractor in phase space, governed by fully deterministic evolution equations, with sensitive dependence on initial conditions might therefore occur in these types of systems. It is well known that an attractor is a strange attractor (or fractal), if its dimension is fractional (.i.e., non integer). Fractal objects like time series often have self similar structure at different levels of magnification that reveals details at arbitrary small scales. For uniform fractals (mono fractals), the scaling is uniquely described by one exponent called the fractal dimension D_0.It has been found that similar large scaled fractal structures (with $D_0 \approx 4/3$) with percolation character, can be realized in turbulence, galaxies, wind storms, oceans etc[5-6]. The analysis of river flows has got a long standing as regards run off and precipitation records [4]. Regular monitoring of surface run-off of a river provides valuable information on the eco-hydrologic condition of a river basin.viz; it provides valuable insights in to the spatio -temporal variation of water quality and quantity that is a measure of the health of a river.

Multi fractal signals, on the other hand, are signals with regularity, which do often change abruptly from one point to the next [7]. In recent years it has been realized that a multi fractal description is required to fully characterize the run off records. The objective of this study is to examine the applicability of multi fractal analysis for describing and quantifying the water quality spatial variability of various river basins of Bharthapuzha that flows through Kerala, one of the populous states in India. The multi fractal description, due to Wavelet Transform Modulus Maxima (WTMM Method), endowed with varying moments can serve as an efficient test for the state of the art of run-off precipitation of the records.

Kerala located in the south west corner of India ($8.5^{0} – 11^{0}$ N & 76^{0} -77 ^{0}E), is rich in water bodies with a forest cover (≈10.336 km^2 of western Ghats), that are the origin points of many rivers joining Arabian sea. The Bharthapuzha, (209 km long with basin area of 6186 km^2) flowing between $10^{0} 25^{1}$- $11^{0}15^{1}$ N and $75^{0}50^{1}$ -76^{0} 55^{1} E is the second largest west flowing perennial rivers of the state with well developed flood plain and fluvial terrace. Even though this river has nine dams with several small check dams, and is the crucial water source for the residents of malappuram, trichur, palakkad and Coimbatore districts, the basin is facing severe water scarcity, and drought, due to great temporal and spatial non linear variability of precipitation limits.

II. MATERIALS AND METHODS

A. *Multi Fractal Analysis: Theory*

The word ‘fractal’ was coined *by* Benoit. B Mandelbrot in 1975, and it refers to geometric structures with self similar (scale invariant) property, i.e., it is a pattern of variation that can be divided in to parts, each of which is, at least statistically or approximately a scaled down copy of the whole. Scaling refers to the invariant properties in any process over a range of scales. Scaling systems can be described by fractal and multifractal theories, the latter being an evolution that results from the former. Multifractal theory deals with multi scaling that helps one to generalize the scaling properties associated with a process. Although, fractals imply self similar objects, exact self similarity is only seen in mathematical objects like Sierpinski gasket and Minkowsi curve. However, natural fractals, show this self similar behavior under arbitrary scaling.i.e., Multi fractals are objects with interwoven sets of varying fractal dimension (distribution of singularities)[7-8].These multi fractal sets have been characterized on the basis of generalized dimensions D_q and the associated spectrum of singularities f($\propto$) [9]. Multifractal spectra are graphs of how a pattern behaves if resolved/amplified in certain ways and are finger prints characterizing fractal objects. Differences in multifractal spectrum imply different scale structures which may affect the process and it characterizes the hidden order in apparently random processes. Various multi fractal formalisms have been developed to describe statistical properties of the singular measures of signals. Wavelets transform modulus maxima [WTMM method], is a recent multi resolution tool in implementing the multi fractal formalism on singular signals with complex time series [10]. It can determine the singularity spectrum directly from the scaling behavior of the partition function, and account for the multi scale nature of the time series (A time series represents scalar measurements of some quantity, which depends on the current state of the system, taken at multiples of sampling time (Δt)) of nonlinear phenomena obtained. In this approach, the numerical quantification of time series is done by the so called singularity spectrum f($\propto$) characterized by holder exponent $\propto$, a measure of local measure quantifying the strength/ smoothness of a distribution measure.

For a continuous process with spectral density Γ_x (υ), the wavelet transform is in general a complex valued function given by

$$T\psi\, |f\,(x_0, a)\,| = \frac{1}{a}\int_{-\infty}^{+\infty} f(x)\,\psi\left(\frac{x-x0}{a}\right) dx \qquad (1)$$

Where a is the scale parameter and f (x_0) is the location parameter with ψ defining the family of wavelets constructed by dilations and translations of a single function. The local singular behavior of f around x_0 is characterized by the power law behavior:

$$T\psi\, |f\,(x_0, a)| \sim \propto^{\tau(q)}, \qquad (2)$$

With the scaling exponent $\tau(q)$. Further, all the singularities of signal can be detected by local maxima of Tψ |f (x_0, a)| at a given scale a .i.e., the wavelet transform modulus maxima{WTMM). The partition function for computing the singularity spectrum of fractal signals scales as:

$$\chi\,(q,a) = \propto^{(q-1)Dq} \qquad (3)$$

where Dq is the set of generalized dimensions [10], characterizing the non uniformity of the fractal measure. For positive values of q, the partition function χ (q,a) characterize the scaling of large fluctuations in the data (strong singularities).For negative values of q, it reflects the weak singularities. It can be shown that the singularity strength $\propto$ and the parameter f ($\propto$) and D_q are related by a Legendre Transformation,

$$f\,(\propto) = q\,\frac{d}{dq}\,[\,(q-1)D_q\,] - (q-1)D_q \qquad (4)$$

$$\propto(q) = \frac{d}{dq}\,[\,(q-1)D_q\,] \qquad (5)$$

Application of WTMM method, to time series data allows us to characterize correlations of different types, with $0 < q < 0.5$ implying the presence of anti correlated behavior and $q > 0.5$ reflecting correlated dynamics. A plot of f ($\propto(q)$) vs $\propto$ (q) for a range of q values is called a multi fractal spectrum.

B. *Methodology*

For this study, data on water quality for 10 water years [1998-2009] for the four river gauge stations under CWC, Govt of India (taken from IHDB for non classified basins) were converted into time series using Mat Lab software.WTMM technique was used to examine the spatio temporal behavior of water quality data at these stations. We handle in each yearly analysis a series of 36000 pts for a 0.5 hr record sampled at 20 Hz with highest resolvable frequency $\frac{1}{2\Delta}$ where Δ is the sampling interval. Block averaging the time series at 10 points /block results in 3600 points in which highest resolvable frequency is f and the lowest frequency obtainable from a given data being $\frac{1}{N\Delta}$, with N being the number of points used. Also, we use the fifth derivative of the Gaussian function as the analyzing wavelet:

$$\Psi^5(t) = d^5(e^{t2/2})/ dt^5 \qquad (6)$$

In this work, the discharge velocity v is studied in terms of the percentage of frequency distributed at different times. Thus, one value may be taken, as the fraction, p, in a certain time i. The support of this measure is the set of real numbers corresponding to t values..Thus, p_i can be interpreted as the probability of finding the discharge velocities of a certain value within interval 'i.' The structure of this measure p on the segment[t_0-t_1] obeys the scaling law,

$$P \propto \delta^{\alpha}, \text{ as } \delta \rightarrow 0 \qquad (7)$$

Where δ is the length of sub intervals in which the total segment is divided. However, p is spread over the interval in such a way that the concentration of velocities vary differently and a different behavior is observed in different positions [different spatial positions.]. That is, the singularity exponent $\propto$ is a function of the position i , many sites i may share the same exponent when a regular covering of size δ is chosen. The f($\propto$) on the other hand, counts how often specific values $\propto$ of the singularity strength occur.

III. RESULTS AND DISCUSSION

The determination of the multi fractal spectrum for the total annual water discharge during the years 1998-2009 for the tributaries at mankara, pulamanthole, kumbidy and pudur was done separately for each month and also for each year. The stations are representative for different rivers with different climatic zones. As a representative example, Fig.1.shows the 10 yrs of runoff record at pudur. In all the entire multi fractal spectrums, differences in scaling were found in the negative q values while positive q values account similar scaling. That is, for positive values q values τ(q) describes the scaling behavior of the segments with large fluctuations, characterized by a smaller scaling exponent for multifractal series. On the other hand, negative values of q, describes the scaling behavior of the segments with small fluctuations characterized by larger scaling exponent.

To study the structure presented in the time series, multifractal analysis using WTMM was done for each station to get the spectrum. The multi fractal spectrum for the year 1998 [Figs. 2(a-g)], for the tributary at pudur (taken as a Representative example) has an asymmetrically long right part with alpha reaching 1.13 and f ($\propto$) descending to as low as 0.62. High $\propto$ values indicate the presence of areas with extremely low discharge velocity field while low f($\propto$) values suggest that the measure that cover low discharge velocity areas is relatively small. The overall distribution of water discharge velocity can be described as relatively homogenous because only a few points on multi fractal spectrum have high $\propto$ values and low f($\propto$) values. The spectra of a water discharge for the years 1999-2003 are relatively similar, with the right portion just a little longer than the left portion. The spectrum of discharge rate in 1999 is wider than those of 2001 and 2002 with α ranging from 0.75 to 1.15, which indicates that the overall variability in 1999 was higher than that in the other years, consistent with high coefficient of variation in 1999. The mf spectrum for 2005 was not asymmetrical as it was in 1998, however asymmetry is still present. This indicates as in the case with 1998, the presence of a few areas with relatively low discharge [Figs .c-d].The spectrum is narrow, with alpha value 1.01. However, the mf spectrum for 2006-2009 is similar and is skewed to the left which signifies that generally high alpha values prevailed at low discharge velocities. Further It is seen from

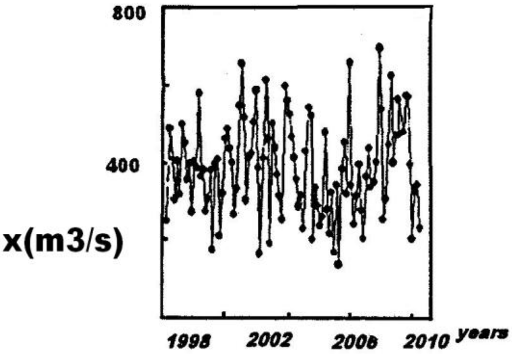
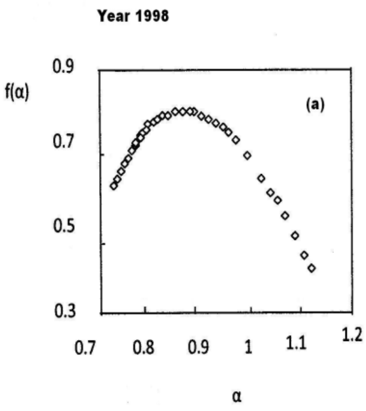
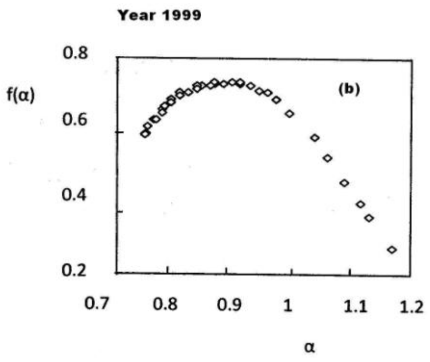
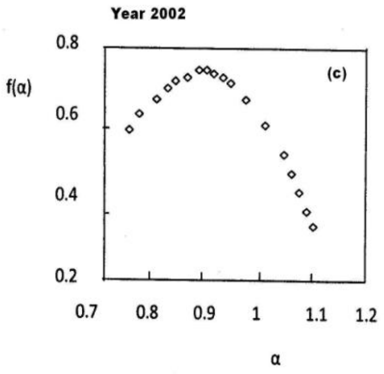


Fig.1. Time Series Of The Annual Run Off Fluctuations At Pudur River Gauge Station.

the f- α curve that the right side of the spectrum [i.e, negative q values] reflects multiple scaling while positive q values shows similar scaling. This is indicative of the low discharge velocity for daily run off, attributed by the weather patterns, especially precipitation falling with in a single day on the area of the basin. Further, the relatively high amounts of precipitation that fall on the field may not significantly affect the discharge rate. However, they provided more water for redistribution within the basin and caused more pronounced patterns of f low values.


Year 1998


Year 1999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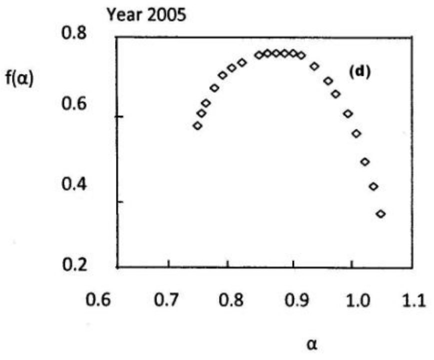
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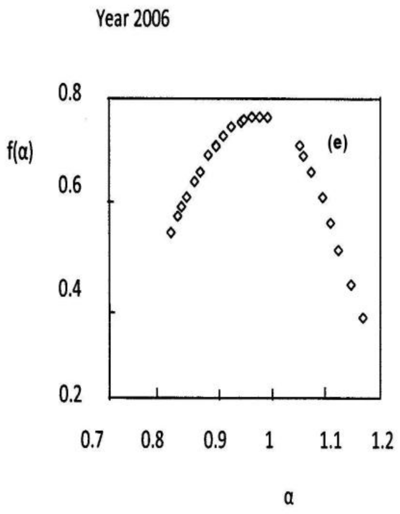
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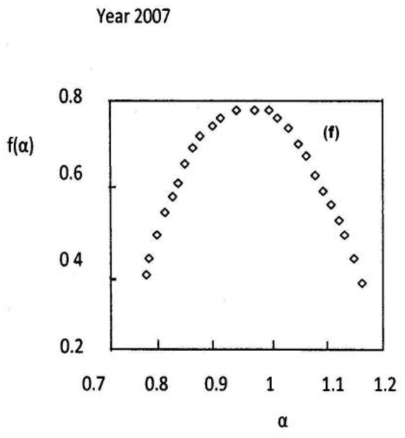
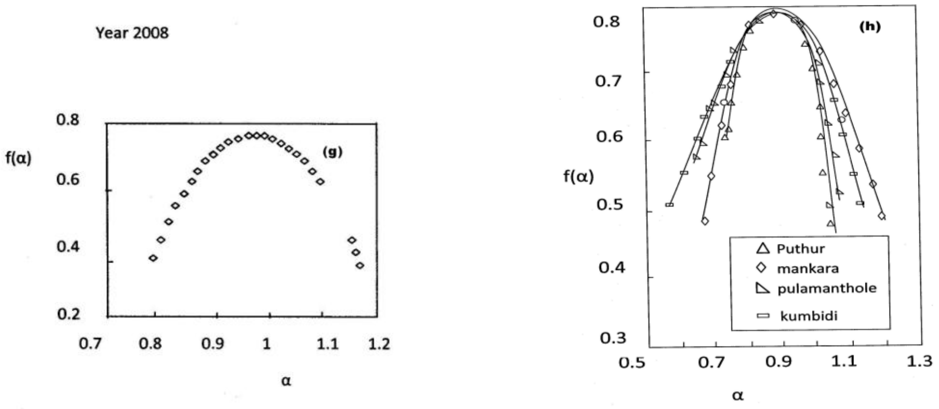
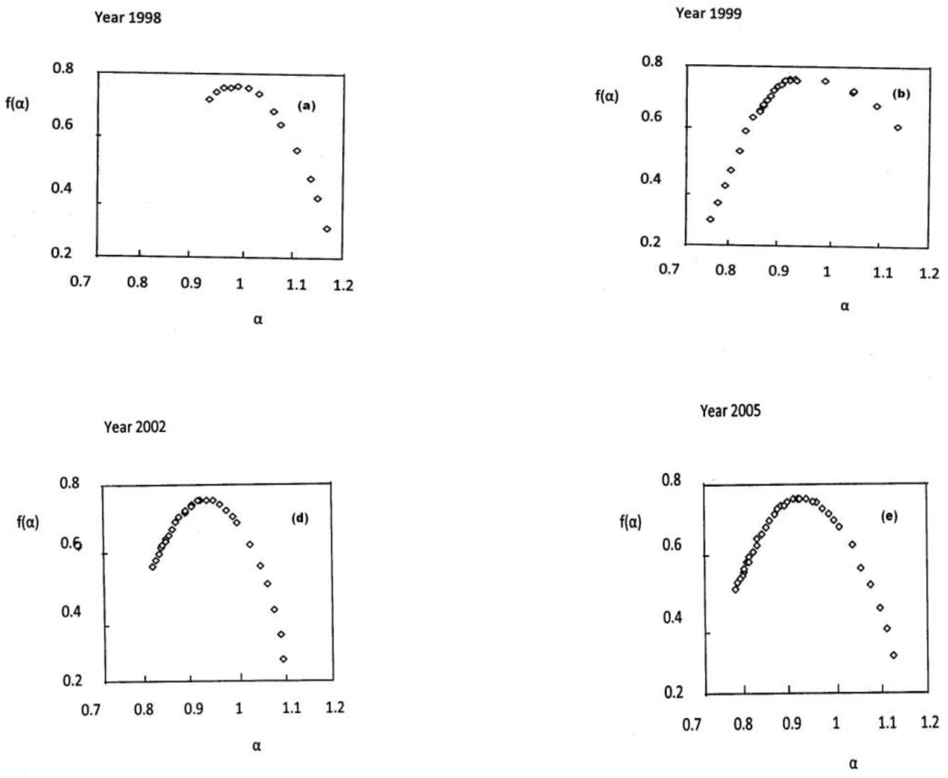

Year 2007



Figs.2.(a)-(g):The Multi fractal spectra f(∝) for the representative run off record at pudur for different Years and Fig.2.(h): The overall f(∝) spectra for each station.

The overall structure of the time series for each year for the remaining stations is almost the same. However the months that showed higher complexities are different for each station.[Fig.2(h)]. The multifractal spectrum obtained for the daily run off for the river gauge station at Kumbidy is however narrow as compared to pudur. In fact, these multifractal exponents are regarded as a fingerprint of each station with. The width of the spectrum changing inversely as the width of the basin area being analyzed. Further, the fractal dimension is different for all the studied stations with kumbidy having the highest value. Further the left shifted singularity spectrum is indicative of more irregularities as compared to all other stations.



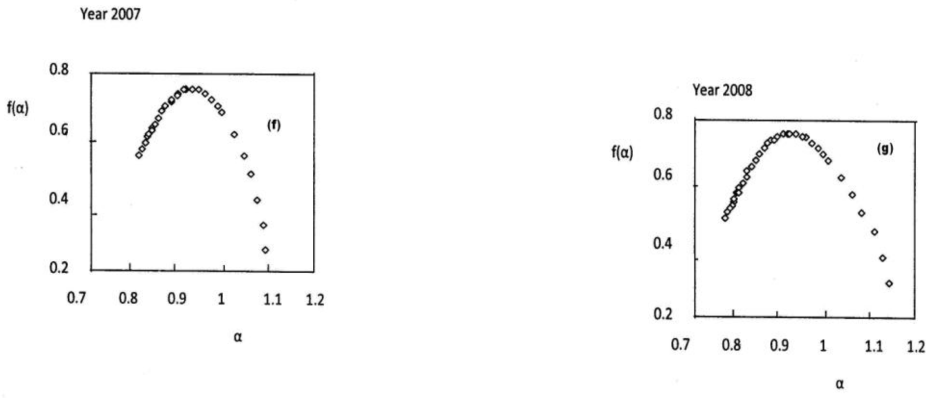


Fig.3: (a-g) The Multifractal spectra f(∝) for the representative run off record at kumbidy for different years

IV. CONCLUSION

In this study, the scaling behavior of daily run off time series of four representative river gauge stations, for ten water years (1998-2009), was done using wavelet technique (WTMM Method). The convex function f(α) varies between months signifying the richness of the structure presented in the time series. The variations in complexity between months, is accounted for by computing the difference in amplitude (.i.e., $\alpha_{max} - \alpha_{min}$) of the spectrum for each month. The month that shows a lowest complexity is across years are slightly different. The spectrum for the whole year is contained in the range of the month's spectrums. However the overall structure of the time series for each year is the same with differentiating scaling behavior at the right side of the spectrum. The sets of holder exponents and singularity spectra obtained show that each station exhibits a unique value of singularity strength and spectrum. The location and range of singularity strength vary for different river gauge stations, with the shape (height and width) of the spectra giving the time interval analysis of the run off. In all cases we found that fluctuations exhibit self similar scaling pattern with long term persistence on time scales. The fluctuation exponent varies from one gauge station to another in a wide range between 0.75 to 1.2 showing non universal scaling pattern. This is substantiated by the variation of D_1 and D_2 values across years. Smaller value of D1 and D_2 indicate domination of long range variation while higher values indicate domination of short range variability in the spatial distribution of run off pattern. Further the distance between the minimum and maximum α values for each multifractal spectrum is a signature of spatial variability distribution. Hence, our study reveals that the multi fractality in the daily run off, is due to clustering of time patterns of values on different time scales in the presence of long term correlations and trends. Based on this modeling characterization simulation of the realistic time series can be possible for an optimal water management.

REFERENCES

[1] Lukkin.F., "Fractals in Physics," 1998.

[2] Liftshifts E. M, Pitavsky. L.P., "Statistical physics-M.Nauka 1978.

[3] Lichtenberg A., M. Liebermann., Regular and stochastic Dynamics (Mir., M., 1984).

[4] Amilcare porporato, Luca Ridolfi ," Nonlinear analysis of river flow time sequences", Water Resources Research **33(6)**, pp.1353-1357.

[5] Praveen Kumar, Efi Foufoula.g, Polis.S.P. "Wavelet Analysis for Geophysical Applications", American Geophysical union, 1997.

[6] B.B. Mandelbrot, Fractals in physics , New York Press , 1975.

[7] J.Freder, Fractals, Plenum Press, New York Press, 1988.

[8] Multifractal tutorial http://www.physionet.org /tutorials/ multifractal/indes.html.

[9] Evertsz, C.J., B.B.Mandelbrot." Multifractal Measures: In chaos and Fractals, Springer Verlag D., Eds.: New York, 1992; pp. 921-953.

[10] L.Guan, Y.Du, LLi, " Wavelets in Petroleum Industry:; Past Present and Future", SPE 89952,2004

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