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Integrability over Fuzzy Number Space

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Abstract -This paper presents the integrability of fuzzy number mapping defined over fuzzy number space based on the concept of "Integration means to give the sum of." Here we try to extend the concept of integrability and Riemann integrability of functions to fuzzy number functions (mappings) on fuzzy number interval which is closed and bounded subset of real numbers, which may help to develop more ideas for fuzzy models. Also we framed the definitions and theorems for integration in view of the properties of elements in the fuzzy number interval based on their membership functions (grade).

Index Terms: **Convex Fuzzy Number Mappings, Fuzzy Integration, Fuzzy Upper and Lower Integral, Fuzzy Upper and Lower Metric, Fuzzy Riemann Integral, OFS.**

I. INTRODUCTION

A crisp set rises abruptly, making its elements totally disjoint with the other members of the universe.However, such a strict categorisation does not exist as long as the human reasoning process is concerned. Problems featuring complexity and ambiguity have been addressed subconsciously by humans since they could think; these ubiquitous features pervade most social technical and economic problems faced by the human race. The closer one looks at a real-world problem, the fuzzier becomes its solution. Fuzzy set theory provides a means of representing uncertainties. The idea of fuzzy sets proposed by Dr.Lotfi Zadeh in 1965.Zadeh extended the notion of binary membership to accommodate various degrees of membership on the real continuous interval[0,1],where end points 0 and 1 conform to no membership and full membership respectively. A fuzzy number is an ordinary number whose precise value is somewhat uncertain. Fuzzy numbers are used in Statistics, Computer Programming, Engineering and Experimental Science. Fuzzy numbers allow us to model non-probabilistic uncertainties in an easy way. Fuzzy number represents a real number interval whose boundary is fuzzy. Fuzzy numbers are fuzzy subsets of the set of real numbers satisfying some additional conditions. Arithmetic operations on fuzzy numbers have also been developed, and are based mainly on the extension principle or on interval arithmetic. When operating with fuzzy numbers, result of the calculations depends on the shape of the membership functions of these numbers. Less regular membership functions lead to more complicated calculations. Moreover, fuzzy numbers with simpler shape of membership functions often have more intuitive and more natural interpretation. Any fuzzy number can be thought of as a function whose domain is a specified set. In many respects, fuzzy numbers depict the physical world more realistically than single valued numbers [2-5].Fuzzy number is expressed as a fuzzy set defining a fuzzy interval in the real number R.Since the boundary of this interval is ambiguous, the interval is also a fuzzy set. Fuzzy number should be normalised and convex. Here the condition of normalisation implies that the maximum membership value is 1 and the convex condition is that the line by α-cut is continuous.

Integration is treated as the inverse of differentiation in an elementary treatment. But historically," To integrate" literally mean "To give the sum of" [7].Here we are trying to extend the definition of integration as the sum, on fuzzy number mappings over fuzzy number space.

II. BACKGROUND

A Fuzzy set **A** in R (real line) is defined to be a set of ordered pairs A $=\{x, f_A(x)\ /x\epsilon R\}$, where $f_A(x)$ is called membership function for the Fuzzy set **A** .A Fuzzy set is called Normal if there is at least one point $x\epsilon R$ with $f_A(x)$ =1

A Fuzzy set is Convex if for any $, y\epsilon\ R\ and\ any\ \lambda \in [0,1]$,

$f_A(\lambda x_1 + (1-\lambda))x_2 \geq min\ \{f_A(x_1), f_A(x_2)\}$

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A convex fuzzy set (Fig.1) is described by a membership function whose membership values are strictly monotonically increasing or whose membership values are strictly monotonically decreasing (Or whose membership values are strictly monotonically increasing then decreasing or whose membership values are strictly monotonically decreasing then increasing) A convex fuzzy set with maximum membership value 1 is called Convex normal fuzzy set (**Fig.1**) Also a normal fuzzy set which is not convex is a Non convex normal fuzzy set (Fig.2).

A fuzzy set **A** is bounded if and only if the set $A_\alpha = \{x / f_A(x) \geq \alpha\}$ are bounded for all $\alpha > 0$

ie, for every $\alpha > 0$,there exists finite $R(\alpha)$ such that $\|x\| \leq R(\alpha)$ for all $x \in A_\alpha$

If **A** is a convex single point normal fuzzy set defined on the real line then **A** is often termed as a fuzzy number.

A fuzzy number should be normalized and convex, condition for normalised implies that maximum membership value is 1. Generally a fuzzy number represents a real number interval whose boundary is fuzzy and the fuzzy interval is represented by two end points.

By the concept of ordered fuzzy numbers, a fuzzy number **A** can be identified with an ordered pair of continuous real functions defined on the interval [0,1]

ie, **A** = (f, g) with $f, g : [0,1] \rightarrow R$ are continuous functions where f and g ,the up and down parts of the fuzzy number **A** respectively . The continuity of both parts implies that their images are bounded intervals.

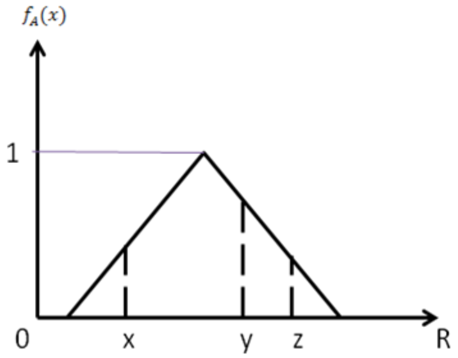

Fig 1(Convex normal fuzzy sets)

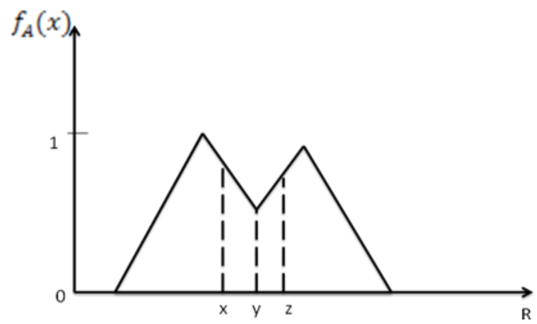

Fig 2(Non-convex normal fuzzy sets)

III. PRELIMINARIES

A Fuzzy number μ is defined as $\mu : R \rightarrow [0,1]$ which is normal, fuzzy convex, upper semi-continuous with bounded support.

Now $E = \{ \mu \mid \mu : R \rightarrow [0,1] \}$ is called a **fuzzy number space**.

Obviously, a Fuzzy set $\mu : R \rightarrow [0,1]$ is a fuzzy number if and only if $[\mu]^r$ is a closed and bounded interval for each $r \epsilon [0,1]$ and $[\mu]^1 \neq \emptyset$,null set

Also, $[\mu]^r = [\mu^-(r), \mu^+(r)], r \in [0,1]$

Moreover $[\mu]^r = \{ x \mid \mu(x) \geq r \}$

Addition and scalar multiplication of fuzzy numbers as follows

Let $\mu, \lambda \in E$ and $k \in R$, $r \in [0,1]$

$$[\mu + \lambda]^r = [\mu]^r + [\lambda]^r$$

$$[k\mu]^r = k[\mu]^r$$

$$[\mu\,\lambda]^r = [\mu]^r\ [\lambda]^r$$

A mapping $F: E \rightarrow E$ is said to be a ***Fuzzy Number Mapping***.

Also $[F(\mu)]^r = [F(\mu^-)(r)), F(\mu^+)(r))]$,

$\mu \in E$ and $r \in [0,1]$ and

$$[F(\mu)]^r = \{ x \mid F(\mu)(x) \geq r \}$$

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Let $F: E \to E$ be a Fuzzy Number Mapping

If F is bounded on all bounded subsets of E then F is bounded

If $\mu \to \mu_c \Rightarrow F(\mu) \to F(\mu_c)$ then F is continuous

ie, $|F(\mu) - F(\mu_c)| < \epsilon_0$ for $|\mu - \mu_c| < \delta_0$ for ϵ_0 , $\delta_0 > 0$

If $\mu \to \mu_c \Rightarrow F(\mu) \xrightarrow{r} F(\mu_c)$ then F is the w cut-continuous

ie, $|F(\mu)(r) - F(\mu_c)(r)| < \epsilon_0$ for $|\mu - \mu_c| < \delta_0$; ϵ_0 , $\delta_0 > 0$

If $\mu \xrightarrow{r} \mu_c \Rightarrow F(\mu) \xrightarrow{r} F(\mu_c)$ then F is the cut-continuous

ie, $|F(\mu)(r) - F(\mu_c)(r)| < \epsilon_0$ for

$|\mu(r) - \mu_c(r)| < \delta_0$; ϵ_0 , $\delta_0 > 0$

Also Continuity $\Rightarrow$the w-cut continuity and the cut- continuity $\Rightarrow$ the w-cut continuity

The lack of strict monotonicity of the branches μ^- and μ^+ ,the existency of constancy sub intervals imply that the inverse functions of μ^- and μ^+ do not exist in the classical sense. Thus to solve the above theorems we may assume that for both functions μ^- and μ^+ ,there exists a finite (or atmost countable) number of such constancy sub intervals and then the inverse functions exist in the generalized sense.

Also $F(\mu)$ itself a fuzzy number (Fig.3)

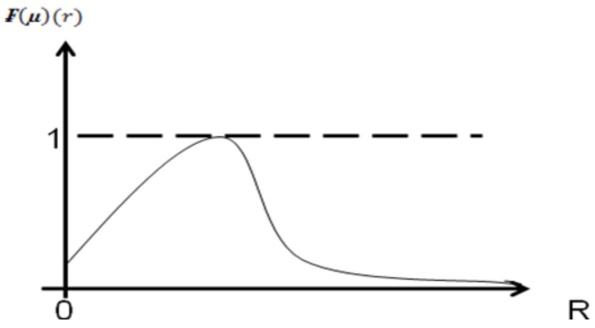

Fig.3

Let $F: E \to E$ be a Fuzzy Number Mapping, a Convex (or Concave) Fuzzy Number Mapping is defined as follows:

$F(t\mu+(1-t)\lambda) \leq t F(\mu) + (1-t)F(\lambda)$ (or$F(t\mu+(1-t)\lambda) \geq t F(\mu) (1-t)F(\lambda)$) where $\mu, \lambda \in \epsilon$ $t \in [0,1]$

In this paper we are going to develope theorems and definitions based on the membership values of the interval $[\mu]^r = [\mu^-(r), \mu^+(r)]$, $r \in [0,1]$ with monotonic(increasing) property of the lower part (Figure 4) of the interval which is lower continuous and can be partitioned. Similarly the theorems and definitions are then true for the upper part (Figure5), which is monotonically decreasing. Hence for proving the theorems in the whole interval (Figure6), it is enough to prove for one part (upper or lower)

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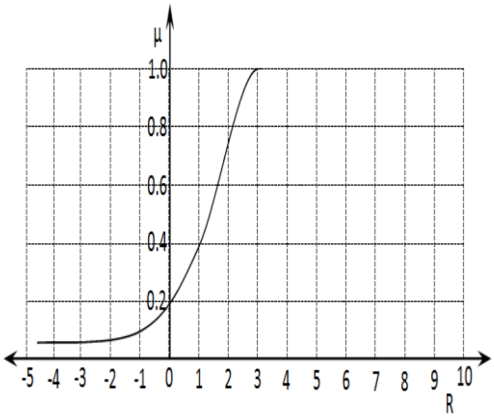

Fig 4 (Membership value increasing)

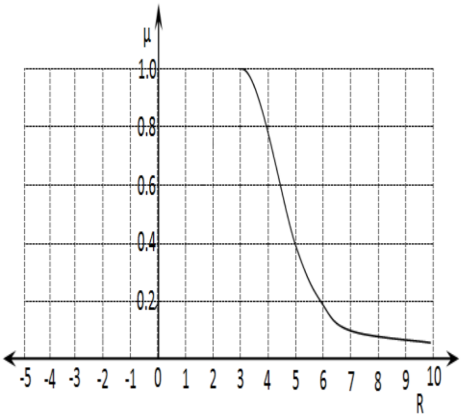

Fig 5 (Membership value decreasing)

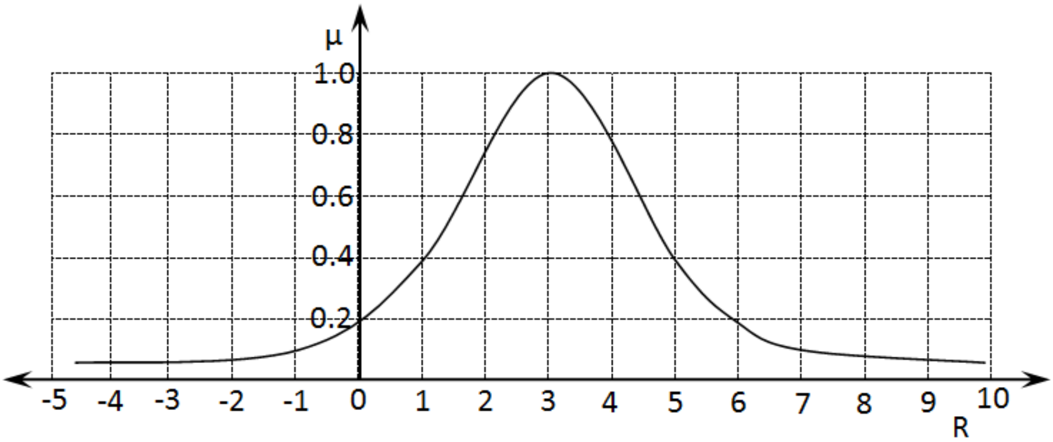

Fig 6 (Membership value on the whole interval)

IV. DISCUSSION AND RESULTS -- INTEGRABILITY OVER FUZZY NUMBER SPACE

Definition4.1

Let E be a fuzzy number space and $F: E \to E$ be a convex Fuzzy number mapping.

Consider the interval $[F(\mu)]^r$, $\mu \in E$ be a fuzzy number $0 \le r \le 1$,which is closed and bounded.

Let $\Psi = \{\mu_0(r), \mu_1(r), \ldots \ldots \ldots, \mu_n(r)\}$ be a partition of $[\mu]^r$ such that

$\mu_i(r) \le \mu_j(r)$,forall $i \le 1,2 \ldots n$(partition is only for lower part)

Now set up two fuzzy sums

Upper sum

$S_k = Max_{0 \le i \le n}\left\{ d^{-}\left(M_i F(\mu_k)(r)\right)\right\}$, for all $\mu_k \in \Psi$

Lower sum

$s_k = Max_{0 \le i \le n}\{ d^{-}(m_i F(\mu_k)(r))\}$,for all $\mu_k \in \Psi$

Where$M_i$and m_i are supremum and infemum of the class $[F(\mu_k)(r), F(\mu_i)(r)]$, i=0,1,2….n

and d^{-} is the lower metric defined on the lower part of $[F(\mu)]^r$,

$$d^{-}(M_i F(\mu_k)(r))$$

$$= Sup_{0 \le i \le n}\{M_i |F(\mu_k)(r) - F(\mu_i(r)| : \mu_k \in \Psi\}, 0 \le r \le 1,$$

$$d^{-}(m_i F(\mu_k)(r))$$

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$= Sup_{0\leq i\leq n}\{m_i|F(\mu_k)(r) - F(\mu_i(r)| : \mu_k \in \Psi\} 0 \leq r \leq 1.$

Thus $I_r^{\vee} = \lim_{k\to n} inf\, S_k$ and

$I_r^{\wedge} = \lim_{k\to n} sup\, s_k$

Where $I_r^{\vee}$ and $I_r^{\wedge}$ denotes the **upper fuzzy integrals** and **lower fuzzy integrals** respectively of fuzzy number mapping over the closed interval $[F(\mu)]^r$, $\mu \in E$ and since , $[F(\mu)]^r$, $\mu \in E$ is arbitrary,then we can extend this definition over fuzzy number space .

$F: E \to E$ said to be **fuzzy integrable** if the upper fuzzy integral($I_r^{\vee}$) and lower fuzzy integral($I_r^{\wedge}$) exists.

Definition4.2(Oscillatory Fuzzy Sum-OFS)

Consider two fuzzy sums,

upper sum $S_k = Max_{0\leq i\leq n}\{ d(M_i F(\mu_k)(r))\}$,for all $\mu_k \in \Psi$ and Lower sum $s_k = Max_{0\leq i\leq n}\{ d(m_i F(\mu_k)(r))\}$for all $\mu_k \in \Psi$, define a new sum called **Oscillatory Fuzzy Sum(OFS)** ,denoted by $\omega_k(\Psi) = Sup_k\, min\{s_k, S_k\}$, for every $k = 0,1,2, \dots n$.

Theorem 4.3

The necessary and sufficient condition for the integrability of a convex fuzzy number mapping $F: E \to E$ over the fuzzy number space E is, for every $0 \leq \epsilon \leq 1$ there exists a $0 \leq \delta \leq 1$ such that for every partition Ψ of $[\mu]^r$, $\mu \in E$,$0 \leq r \leq 1$ with $d(F(\mu_k(r)) \leq \delta$, $\mu_k \in \Psi$,for every $k = 0,1,2, \dots n$.,the **OFS**, $\omega_k(\Psi) < \epsilon$

Proof

Necessary

Suppose F is integrable on $[F(\mu)]^r$, $\mu \in E$

ie, $I_r^{\vee} = \lim_{k\to n} inf\, S_k$ and

$I_r^{\wedge} = \lim_{k\to n} sup\, s_k$ exists

Let $\Psi = \{\mu_0(r), \mu_1(r), \dots\dots\dots, \mu_n(r)\}$ be a partition of $[\mu]^r$, $0 \leq r \leq 1$ such that $\mu_i(r) \leq \mu_j(r)$,for all $i \leq j = 1,2, \dots . n$

(partition is only for lower part)

Now we have Upper sum $S_k = Max_{0\leq i\leq n}\{ d(M_i F(\mu_k)(r))\}$f or all$\mu_k \in \Psi$ and

Lower sum $s_k = Max_{0\leq i\leq n}\{ d(m_i F(\mu_k)(r))\}$,for all $\mu_k \in \Psi$

Then for $0 \leq \epsilon \leq 1$, $\mu_k \in \Psi$,for every $k = 0,1,2, \dots$ n.,we can find a $0 \leq \delta \leq 1$ such that

$d(F(\mu_k(r)) = Sup_{i=1,2,\dots n}\{|F(\mu_k(r) - F(\mu_i(r)| : \mu_k \in \Psi\} \leq \delta$(since F is bounded)

Obviously, $0 \leq s_k \leq S_k \leq 1$

This implies $min\{s_k, S_k\} < \epsilon$, for all $\mu_k \in \Psi$

Thus $Sup_k\, min\{s_k, S_k\} < \epsilon$

ie, , $\omega_k(\Psi) < \epsilon$.

Sufficient

Suppose $\omega_k(\Psi) = Sup_k Min\{s_k, S_k\} < \epsilon \text{ for } 0 \leq \epsilon \leq 1$

Now from the definition, $\lim_{k\to n} Sup_k Min\{s_k, S_k\} < \epsilon$

Then clearly

$Sup_k Min\{\lim_{k\to n} sup\, s_k , \lim_{k\to n} inf\, S_k\} < \epsilon, \ 0 \leq \epsilon \leq 1$

ie, $I_r^{\vee} = \lim_{k\to n} inf\, S_k$ and$I_r^{\wedge} = \lim_{k\to n} sup\, s_k$ exists as fuzzy integrals

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Thus F is integrable on $[\mathrm{F}(\boldsymbol{\mu})]^r$, $\mu \in \mathrm{E}$, $0 \leq r \leq 1$.

Thus the proof.

Note :If F is integrable on $[\mathrm{F}(\mu)]^r$, then it is integrable over E(since $[\mathrm{F}(\mu)]^r$ is arbitrary)

Theorem4.4

Let E be a fuzzy number space and $\mathrm{F}: \mathrm{E} \rightarrow \mathrm{E}$ be a convex Fuzzy number mapping. If F is continuous on $[\mathrm{F}(\boldsymbol{\mu})]^r$, $\mu \in \mathrm{E}$, $0 \leq r \leq 1$ then it is integrable on $[\mathrm{F}(\boldsymbol{\mu})]^r$.

Proof

Suppose F is continuous on $[\mathrm{F}(\boldsymbol{\mu})]^r$,for every $\mu \in \mathrm{E}$, $0 \leq r \leq 1$

Let $\Psi = \{\mu_0(r), \mu_1(r), \dots \dots \dots, \mu_n(r)\}$ be an arbitrary partition of $[\mu]^r$, $0 \leq r \leq 1$

Since F is continuous on $[\mathrm{F}(\boldsymbol{\mu})]^r$, for every $0 \leq \epsilon \leq 1$,there exists

$0 \leq \delta \leq 1$ such that

$\left| F(\mu)(r) - F(\mu_j)(r) \right| < \epsilon$,for all

$\mu \in \mathrm{E}$, $j = 0,1,2, \dots \mathrm{n}$.,whenever$\left| \mu(r) - \mu_j(r) \right| < \delta$

Now take $\left| \mu_k(r) - \mu_j(r) \right| < \delta$,for $\mu_k(r) \in \Psi$ then$\left| F(\mu_k)(r) - F(\mu_j)(r) \right| < \epsilon$

We have

$d(M_i F(\mu_k)(r))= Sup_{0 \leq i \leq n}\{M_i |F(\mu_k)(r) - F(\mu_i(r) | : \mu_k \in \Psi \}, 0 \leq r \leq 1,$

$\leq Sup\{M_0\epsilon, M_1\epsilon, \dots \dots, M_n\epsilon\}=M_k\epsilon$ (say)

Therefore,

$\boldsymbol{S}_k=Max_{0 \leq i \leq n}\{ d(M_i F(\mu_k)(r)) \} \leq M_r\epsilon$, where $M_k = Max\{M_0, M_1, \dots, M_n\}$

Similarly, $\boldsymbol{s}_k \leq m_r\epsilon$,

where $m_k = Max\{m_0, m_1, \dots, m_n\}$.

Thus **OFS**, $\omega_k(\Psi) < \eta$, $0 \leq \eta \leq 1$

Hence by theorem 1.1, F is integrable on $[\mathrm{F}(\boldsymbol{\mu})]^r$.

Theorem4.5

Let E be a fuzzy number space and $\mathrm{F}: \mathrm{E} \rightarrow \mathrm{E}$ be a convex Fuzzy number mapping. If F is monotonic then it is integrable on $[\mathrm{F}(\boldsymbol{\mu})]^r$.

Proof

Let $F: E \rightarrow E$ is monotonic

Suppose F is monotonically increasing

Let $\Psi = \{\mu_0(r), \mu_1(r), \dots \dots \dots, \mu_n(r)\}$ be any partition of $[\mu]^r$, $0 \leq r \leq 1$, $\mu \in E$

Since F is bounded and monotonic,

$\left| F(\mu_k)(r) - F(\mu_i(r) \right| \leq \epsilon$,for all $\mu_k, \mu_i \in \Psi$

This implies that

$Sup_{0 \leq i \leq n}\{M_i |F(\mu_k)(r) - F(\mu_i(r) | : \mu_k \in \Psi \} \leq \epsilon$

ie, $d(M_i F(\mu_k)(r)) \leq M_i\epsilon$ and so that $\boldsymbol{S}_k=Max_{0 \leq i \leq n}\{ d(M_i F(\mu_k)(r)) \} \leq M_r\epsilon$,where $M_r = Max\{M_0, M_1, \dots, M_n\}$

Also $Sup_{0 \leq i \leq n}\{m_i |F(\mu_k)(r) - F(\mu_i(r) | : \mu_k \in \Psi \} \leq \epsilon$

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ie, $d^{-}(m_i F(\mu_k)(r)) \leq m_i \epsilon$ and so that $S_k = Max_{0\leq i\leq n}\{d^{-}(m_i F(\mu_k)(r))\} \leq m_r \epsilon$,where $m_r = Max\{m_0, m_1, \ldots, m_n\}$.

Obviously , $\omega_k(\Psi) < \eta, \quad 0 \leq \eta \leq 1$

Hence F is integrable on $[F(\mu)]^r, 0 \leq r \leq 1$, μ ∈ E.

Definition4.6

When the upper fuzzy integral and lower fuzzy integral of F over every interval $[F(\mu)]^r, 0 \leq r \leq 1$, μ ∈ E are equal then F is said to be **Riemann Fuzzy Integrable** over E.

Theorem 4.7

Let E be a fuzzy number space and $F: E \to E$ be a convex Fuzzy number mapping. The condition for F to be Riemann integrable on $[F(\mu)]^r$ is that the **OFS**, $\omega_k(\Psi)$=0,for every partition Ψ of $[F(\mu)]^r, 0 \leq r \leq 1$.

Proof

From theorm 4.1,it is clear that F is integrable if $\omega_k(\Psi) < \epsilon$, where

$\Psi = \{\mu_0(r), \mu_1(r), \ldots \ldots \ldots, \mu_n(r)\}$ be any partition of $[\mu]^r$, $0 \leq r \leq 1$, μ ∈ E

Now $\omega_k(\Psi) = Sup_k Min\{s_k, S_k\} < \epsilon \; for \; 0 \leq \epsilon \leq 1$

Now $\lim_{k\to n} Sup_k Min\{s_k, S_k\} < \epsilon$,

Then clearly

$Sup_k Min\{\lim_{k\to n} sup s_k, \lim_{k\to n} inf \, S_k\} \quad < \epsilon, \; 0 \leq \epsilon \leq 1$

ie, $|\lim_{k\to n} inf \, S_k - \lim_{k\to n} sup s_k| < \epsilon \;\; 0 \leq r \leq 1$.

As $\epsilon \to 0, \lim_{k\to n} inf \, S_k = \lim_{k\to n} sup s_k$

ie, $I_r^{\vee} = I_r^{\wedge}$

Thus the proof.

Note: From the above theorem it is obvious that F is Riemann integrable over E if **OFS** with respect to each partition is zero.

V. CONCLUSION

We have studied the basic concepts of integration and developed some important ideas of fuzzy number integration in a simplest way. We hope that it may useful to apply different fields of fuzzy modelling using fuzzy numbers.

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