ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 2, Issue 1, January 2013

Using Portable Technology in the Calculus Problem Resolution

Elena Fabiola Ruiz Ledesma
Escuela Superior de Cómputo. Instituto Politécnico Nacional

***Abstract*— This paper addresses the importance of the role of new technologies in the teaching of mathematics, mainly refers to the use of graphing calculators with symbolic manipulation capabilities within the classroom on the upper level, due to its versatility and low cost compared with computers, this technology can be considered from a cognitive point of view as an organizer of the mind rather than as an amplifier of the same. It shows the resolution of a problem of maximum and minimum, where the use of graphical display allows the students the peaks and troughs of the curve, and the meaning we give the algebraic procedure achieves that used in obtaining these points, by the first derived criterion. We conclude that the use of graphing calculator allows representation related records such as algebraic and graphic, for the construction of concepts.**

Index Terms—**Calculator, Education, Functions, Graphs.**

I. INTRODUCTION

Currently there are researchers in mathematics education have been looking for alternatives to solve problems involving the teaching and learning of mathematics, as in [1], [2], and new technologies have enabled the development of innovative research in mathematics education, [3]-[5]. Examples of such technologies are integrated calculators that have software that allows symbolic manipulation and posing unusual pedagogical features that could be useful to the teacher and the student to significantly enrich the process of acquisition of mathematical knowledge. On the other hand, calculators capable of symbolic manipulation can be considered from a cognitive perspective as organizers of the mind rather than amplifying it, as noted Pea [6], the portable technology goes further to do easier or faster what we already do, also allowing multiple representations of a mathematical concept, and is expected to have radical implications both methods as in mathematics education purposes. Research In regards to the teaching of mathematics is known that current practices are based on calculation of the transmission of knowledge, which emphasizes the development of algebraic skills and neglects the search for understanding about the concepts, as in [7], [8].
The investigations that have been developed in the field of teaching-learning process of the numeracy have experienced over the last few years a significant evolution in their approaches and purposes. Transited through the studies that characterized the difficulties and obstacles in the learning of numeracy, accompanied by aspects of epistemological nature, cognitive and educational as it is mentioned in [9]-[11]. Also it has been documented in [12] that certain problems depend from the type of treatment that school conferred on the notions of function, limit, continuity, differentiation and integration. Other studies have been concerned with analyzing the underlying reasons for these difficulties and to provide effective solutions, through pedagogical proposals that are based on various theoretical frameworks. [13]- [15].

II. RESEARCH QUESTION

As previously written the following question arises: How can better serve portable technology in mathematics education of teachers and students?

III. FRAME WORK

A. Calculators as Cognitive Technologies

The Mathematics Education, such as field research, allows us to inquire about the reasons and the ways in which a student appropriates this or that concept, in addition to the development of innovative technologies for the teaching that is based on recent research results, derived from cognitive psychology, which has to do with the processing of the information in the mind of the individual (perception, memory, thought).

ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 2, Issue 1, January 2013
In [6] referred to as cognitive technologies help any means to transcend the limitations of the mind, in thinking, learning and problem-solving activities. And as “amplifiers reorganizers than the mind”. In this sense may include entities as diverse as chalk and blackboard, pencil and paper, written language and symbolic systems of mathematical notation, and lately computer systems (in this case computers and calculators).
Long before computers and calculators appeared, technical instruments such as written human intelligence expanded dramatically. According to Pea [6] , intelligence is not a quality of the mind alone but a product of the relationship between mental structures and intellect tools provided by culture, so these tools are called as "cognitive technologies".
On the other hand, Tall mentioned in [16], introduces the notion of generic organizer and this author defines it "as an environment (or micro world) that enables the student to manipulate examples (and if possible counterexamples) of a specific mathematical concept or a related system of concepts"; the intention is to help the learner to get more experience that will provide a cognitive structure on which to reflect and build more abstract concepts. A simple example is the program organizer generic Magnify from package Graphic Calculus [15], designed to allow users to amplify any part of the graph of a specified function. Most graphing calculators are capable of amplification similar to the program Magnify and can also be considered as a generic example of organizing, by way of example consider the model of the Texas Instruments TI 81, 83, 85, 86, 89, 92 and 92 Plus, it is necessary to mention that each model has some features that may also be considered as generic organizers. It is noteworthy that only the TI 89, 92 and 92 have the ability Plus symbolic manipulation, these models have an integrated version Derive.
The portable technology also can be considered as tools that allow for viewing, in terms of the definition of Zimmerman [17] the display is used to describe the processes of production or use of graphics or geometric representations of mathematical concepts, principles or problems, either hand drawn or computer generated.
It should be mentioned that graphing calculators have not the power of a computer, or the resolution of a super VGA monitor, however let you apply the above definition as to its use.
Additionally Hitt [18], mentions that the visualization of mathematical concepts is not a trivial cognitive activity: viewing is not the same as seeing. In our context, viewing is the ability to create rich mental images that the individual is able to manipulate in your mind, practicing different representations of the concept and, if necessary, use the paper or the computer to express the idea in mathematical question.
In the same order of ideas Duval [19], indicates that the mathematical visualization requires the ability to convert a problem of a semiotic system of representation to another and that recent research on the semiotic systems of representation have highlighted the importance of coordination between different representations of mathematical concepts for learning mathematics, in this sense portable technology allows different representations of mathematical concepts (algebraic, numeric and graphic).
The use of the calculator plotter allows you greater access to the multiple representations of mathematical concepts, promoting the articulation between different representations of the concepts, by facilitating the passage to a most important level of learning mathematics, Hitt [2].

B. The Calculator and the Teaching of Mathematics

In regard to the teaching of mathematics a good principle is to use the calculators only when they improve the quality of education. As it is mentioned in [12]. We can identify four ways in which they show their usefulness in education:

1. As an aid to the instructor during the class. Here the instructor can use the calculators at least three different ways, showing screens (as a slideshow), simulations and demonstrations (visual-dynamic).
2. As an aid to solve problems. This is where students learn to use technology when they do not and simultaneously cultivate a balance of skill in the use of technology and the old way of working with pen and paper.
3. As a means of encouraging students to treat mathematics as an experimental subject. Most of the classes are spent transferring an existing knowledge, without going through a paradigm that generates new knowledge. This is the primary cause of the restriction of the time: takes a long time for the student to make sufficient examples to be in a position to make a conjecture and then test it. The computer gives us the possibility to add the component to all courses of mathematics.
4. As a means to capture the spirit and excitement of the current development of mathematics. Mathematicians know that the actual math is alive, exciting, full of beauty, and constantly under development, while most of what is taught is a mathematical static, decontextualized.

ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 2, Issue 1, January 2013

Introducing technology in the classroom helps teachers to scrutinize on teaching, forcing experimenting with new ideas, revitalizing the teaching effort, allowing mathematics be taught, how to do it, and that the material is taught traditional more effectively, facilitating the introduction of non-traditional materials.

The plotter calculators with capacity of symbolic manipulation can be integrated into the classroom of mathematics as a tool to promote learning, in which the students working individually or in groups in a laboratory, they can discover mathematical principles by observing, making assumptions and verifying their conjectures. Students may be encouraged to see mathematics as an experimental science whose concepts and relationships can be better learned through exploration and reasoning. These new technologies can be seen as tools that provide graphical capabilities, calculation and symbolic manipulation. The teacher and the student serve as a resource in and outside the classroom.

It is giving a national movement to reform the teaching of mathematics at all levels, to increase their understanding, appreciation and motivation. Many students will spend their energy in practice routine manipulations, so that such skills become a primary focus of study. Students begin to perceive mathematics as a series of obstacles to overcome, with little cohesion and little relation to other situations.

All the ways in which calculators are used in research in mathematics education are available for teaching and learning of mathematics. For example, students can learn to program to address certain types of problems, or they can use software that is already integrated into the calculator as an environment for exploring ideas. The main difference between the activities of the student and the researcher is that the latter is usually cover the domains of knowledge, which are set by the most experienced members of the mathematical community, so the researcher tries to develop a new way. Of course for the student, the mathematics are new, and here there could be strong analogies with the researcher, as a large portion of the work the student has to do with the mathematics that are already part of an organized system of knowledge. The difference between a researcher and a student is about, in that the former is motivated and the second does not. This opens up possibilities for the heavy use of the calculator in mathematics education plays a motivational role, through the development of innovative research on mathematics teaching designed to help students to conceptualize mathematical ideas.

In many countries, the teaching of mathematics has taken a form of classical reading, for most teachers, and displayed in a more obvious and evident in high school and college levels. There have been numerous successful experiences, as well as significant changes in the official programs almost in all parts and materials have been produced in large quantities. But within the classroom, teachers talk, and the students listen. National exams in full knowledge test routine more independent critical thinking. But with the spread of graphing calculators, this will tend to change. Almost all the common routines will become trivial, and the machines never solve the problems: those who use them will have to think of what to do and how to interpret the deployed and the results. Computers can of course have the same effect; but these are not, and never will be as available as the graphing calculators.

Many mathematicians and educators believe that technology should be used widely in mathematics classrooms [3], [4]. The reality, however, is that the technology is not as widely used in the classroom of both high school and university. The main reason for this is of a financial nature. To the extent that there are authors who mentioned that the only activity in the school for masses is practically the teacher's explanation supported or based on a text book. So all the advantages of using technology in the mathematics classroom are not available for all students, simply because there are not enough computers in schools. This will be a problem for a long time, too, because new technology makes the old obsolete, and interesting new software does not work on old computers. Moreover, the economic situation prevents the massive acquisition of computers for schools in many countries, [3].

In [20], [21] it states that graphing calculators bring the power of visualization to all calculus students.

In reference [3] are reported 10 key activities that can be performed with the "notebook display technology" in the classroom, which are:

1. Problems numerical approach.
2. Use of analytical manipulations to solve equations and inequalities, and further support using visual methods.
3. Use of visual methods for solving equations and inequalities, and then confirm using algebraic methods.
4. Modeling, simulation and problem solving situations.
5. Use of computer generated visual scenarios to illustrate mathematical concepts.
6. Using visual methods to solve equations and inequalities, which cannot be resolved or are impractical using algebraic methods.
7. Conduct mathematical experiments, make and test conjectures.
8. Study and classify the behavior of different classes of functions.

ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 2, Issue 1, January 2013

9. Forecasting calculus concepts.
10. Investigate and explore various connections between different representations of a problem situation.

IV. OBJECTIVE

In the present article shows the importance of the use of the calculator plotter to allow the student the use of the display and not only the algebraic resolution of a problem that you are working in the classroom.

V. METODOLOGY

A. Subjects

We worked with a group of 38 students of College, students 18 years of age, in their first semester of Calculus.

B. Activity

Maximum and Minimum Topic

In the present article we show an activity using the calculator plotter in its resolution. The topic discussed was to obtain maximum and minimum in a function. First job is algebraically and then shows a passage extracted from the classroom:

1) Students were asked to calculate the local maximum and minimum of the function(1):

$f(x) = x^4 - 5x^2 + 4$ *in the interval [0,2].* (1)

Professor: *How can we know what are the maximum and minimum points of the function in the given interval?*

Student 1: *Obtain the first derivative of the function is:* $f'(x) = 4x^3 - 10x = x(4x^2 - 10)$. *(2)*

Professor: *Why do we derive the function?*

Student 2: *To calculate the values of* x, *for which the derivative is zero.*

The student proceeds to solve for x and says:

Student 2: *The derivative becomes zero if* ***x = 0*** *or if* $4x^2 - 10$ *that is to say:*

x = 0 *and* $x = \pm\frac{\sqrt{10}}{2}$. *As the derivative of the function exists for all values of the domain.*

Professor: *According to the procedure that they are following what would continue?*

Student 2: *We already have the values that are maximum and minimum, that is to say, there is a maximum when*

x= 0 , $x = \frac{\sqrt{10}}{2}$, $x = -\frac{\sqrt{10}}{2}$

Until shown here is the work carried out by the students.

VI. DISCUSSION AND ANALYSIS

In reviewing the course of action of the students in the problem, there are several aspects:

1. Students at any time refer to a graph to display the maximum and minimum points of the function.
2. Working in the field algebraic, but apparently has been learning mechanic because not give meaning to what they are doing, that is, it does not have a meaning for them.
3. Do not determine the maximum and minimum points due to not remember all the steps, allowing noted that everything has been by rote.
4. They were not given importance to the interval at which called for identifying the critical points. Due to the shortcomings of the students shown another passage that was given in the classroom, where professor explained situations where there were errors and confusions in the students:

The class continued in situations where the teacher clarified and doubts of students:

Professor: Remember that the derivative, from the geometrical point of view is the slope of the tangent line at a given point on the graph and if you derive the function equal to zero and what it means graphically is that the slope is zero, is a straight horizontally at a point of the graph and a horizontal line being then there is a maximum point or a minimum point. Look at the graph (fig.1).

ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 2, Issue 1, January 2013

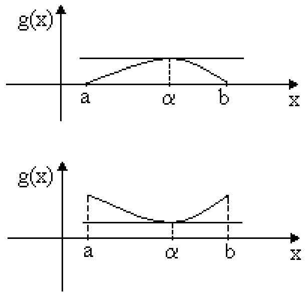

Fig. 1. Graph Representing The Slope of a Line Tangent To The Curve At A Point

Professor: In the case that we are working we have: Taking into account that the function being analyzed only in the interval **[0,2],** critical numbers are $x = 0$ and $x = \frac{\sqrt{10}}{2}$ the value $x = -\frac{\sqrt{10}}{2}$ is not considered as a critical issue because in the interval **[0,2].**

As the maximum and minimum local if there are given in the interior of the range then in **x = 0** there can be no local maximum or minimum.

Thus the only value to be considered is $x = \frac{\sqrt{10}}{2}$ This value suggests that the intervals constructed are $\left(0, \frac{\sqrt{10}}{2}\right)$ and $\left(\frac{\sqrt{10}}{2}, 2\right)$ take a test value, **k** , in each of the intervals as shown in table 1.

Table I. Monotony intervals of the function: $f(x) = x^4 - 5x^2 + 4$ **in *[0,2]***

Interval	$\left(0, \frac{\sqrt{10}}{2}\right)$	$\left(\frac{\sqrt{10}}{2}, 2\right)$
Test value ***k***	1	$\frac{18}{10}$
$f'(k)$	-6	$\frac{666}{125}$
Sign of $f'(x)$	$f'(x) < 0$ (-)	$f'(x) > 0$ (+)
Behavior of *f*	**Decrease**	***increase***

From Table I we can conclude that the graph of the function

$$f(x) = x^4 - 5x^2 + 4 \quad (1)$$

in $[0,2]$ has a local minimum in $x = \frac{\sqrt{10}}{2}$ which value of:

$$f\left(\frac{\sqrt{10}}{2}\right) = \left(\frac{\sqrt{10}}{2}\right)^4 - 5\left(\frac{\sqrt{10}}{2}\right)^2 + 4 \cong -2.25. \quad (2)$$

The graph of the function has no local maxima. But if you have an absolute maximum in ***X = 0*** which value is $f(0) = (0)^4 - 5(0)^2 + 4 = 4$ see the graph of the function shown in Figure 2.

ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 2, Issue 1, January 2013

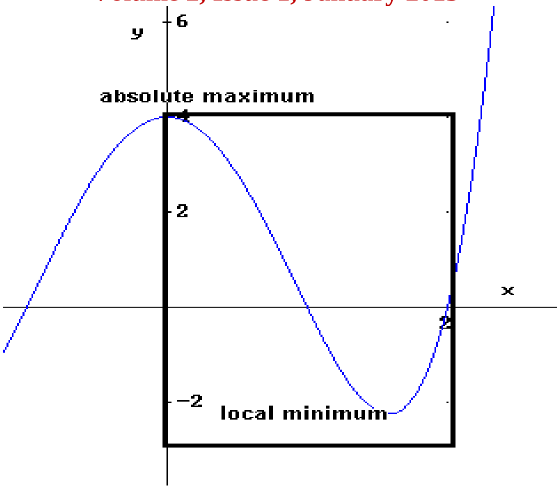

Fig. 2. Graph of $f(x) = x^4 - 5x^2 + 4$ **in [0,2]**

The graph shows that the interval is analyzed in [0, 2] and if you change the analysis interval the local endpoints can change.

VII. CONCLUSION

The fact that the plotter calculators can be widely used enables effective use of all these activities. This represents important classes of activities that are essential to obtain the goals commonly present in the mathematics of the high school and university.

ACKNOWLEDGMENT

Thank you to IPN and SIP for the economic support.

REFERENCES

[1] Ruiz, E. F. 2011 Indicadores teóricos para la Constricción de Conceptos del Cálculo Diferencial. Proyecto de investigación registrado en la Secretaría de Investigación y Posgrado (SIP), del IPN con núm. de registro CGPI 20110343. México. IPN, 2011.

[2] Hitt F. 2003. Funciones en contexto. Editorial Pearson. Educación (Prentice Hall). México. 34-56.

[3] Carvalho, J. (2006). Are Graphing Calculators the catalyzers for a real change in mathematics education? En Gomes, P. y Waits B. (Eds), Roles of Calculators in Classroom, 21-30. Una Empresa Docente & Name of Publisher. USA.

[4] Demana F. & Waits B. (1998). Pitfals in Graphical Computers or Why a Single Graph isn't enough. College Mathematics Journal 19(4): pp. 177-183.

[5] Kieran, C. (1993), Functions, Graphing and Technology: Integrating Research on Learning and Instruction in Integrating Research in the Graphical Representation of Functions, T. Carpenter, E. Fennema and T. Romberg (Eds.), First Edition, 189-237. Erlbaum Hillsdale, N.J.

[6] Pea, R. (1997). Cognitive Technologies for Mathematics Education. En Shoenfeld A. (Ed.), Cognitive Science and Mathematics Education. First Edition, 120-151. Hillsdale N.J. Lawrence Erlbaum Associates

[7] Contreras, A. (2001). El límite en el bachillerato y primer año de universidad. Perspectivas desde los enfoques epistemológicos y semióticos. Obtenido en julio 1, 2008, del sitio web de la Universidad de Granada, Grupo Didáctica de la Matemática como Disciplina Científica de la SEIEMhttp://www.ugr.es/~jgodino/siidm/huesca/limitebachillerato.pdf

[8] Zuñiga, L. (2007). El cálculo en carreras de ingeniería: un estudio cognitivo. Revisit Latinoamericana de Investigation en Mathematical Educativa 10 (1) 145–175.

[9] Aparicio, E. y Cantoral, R. (2006). Aspectos discursivos y gestuales asociados a la noción de continuidad puntual. Revista Latinoamericana de Investigación en Matemática Educativa 9(1), 1–29.

[10] Crespo, C. (2004). El concepto de continuidad y sus obstáculos epistemológicos. En L. Díaz (Ed.), Acta Latinoamericana de Matemática Educativa (volumen XVII, pp. 39–44). México: Comité Latin-American de Mathematical Educativa AC.

ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 2, Issue 1, January 2013
[11] Godino, J., Contreras, A. y Font, V. (2006). Análisis de procesos de instrucción basado en el enfoque ontológico–semiótico de la cognición matemática. Recherches en Didactique des Mathématiques 26(1), 39–88.

[12] Hitt, F. (1994). Teacher's difficulties with the construction of continuous and discontinuous functions. Focus on Learning Problems in Mathematic 16 (4), 10–20.

[13] Blázquez, S. y Ortega, T. (2002). Nueva definición de límite funcional. UNO. Revista de Didáctica de las Mathematics 30, 67–82.

[14] Tall, D. (1986). The complementary roles of short programs and prepared software for mathematics learning. Bulletin of the IMA 23, 128–133

[15] Tall, D., Blokland, P. & Kok, D. (1990). A graphic approach to the calculus (for IBM compatible computers). USA: Sunburst, Pleasantville, NY

[16] Tall, D. (1996). Functions and Calculus Education. En: A. J. Bishop et al (eds.), International Handbook of Mathematics. First Edition,:289-325. Kluwer Academic Publishers, Netherlands

[17] , Zimmermann W. (2001). Visual Thinking in Calculus. In Zimmermann W. & Cunningham S. (Eds), Visualization in Teaching and Mathematics. Vol. 2. Second Edition, 127-138. Series. USA. MAA

[18] , Hitt, F 1995. Intuición Primera versus Pensamiento Analítico. Dificultades en el Paso de una Representación Gráfica a un Contexto Real y Viceversa, Educación Matemática, Grupo Editorial Iberoamérica, México, 7(1): 63-75.

[19] Duval, R. (1998). Registros de representación semiótica y funciona- miento cognitivo del pensamiento. In: F. Hitt (Ed.), Investigaciones en matemática educativa II., Primera Edición, 101- 120. Grupo Editorial Iberoamérica. Cd. De México

[20] Goldin, G. & Kaput, J. (1996). A joint perspective on the idea of representation in learning and doing mathematics. Theories of Mathematical Learning Hillsdale, NJ: Lawrence Erlbaum Associates 4(2): 397-430.

[21] Ben-Chaim, D., Lappan, G., & Houang, R. (2005). The Role of Visualization in the Middle School Mathematics Curriculum. Focus on Learning Problems in Mathematics, Winter Edition. 11(1): 49-60.

AUTHOR BIOGRAPHY

I Obtained my Ph. D in Sciences in Educational Mathematics in the" Centro de Investigación y Estudios Avanzados del Instituto Politécnico Nacional" in 2002. The Master Degree in Educational Mathematics in 1994. The degree of Mathematics in 1991. I have 1 book, I have a chapter in a book, I have around 100 publications in journals and proceedings about education, mathematics and technology. My research work is Problem solving, Educational Mathematics and Computing Education.