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Performance Analysis of Constant Modulus Algorithm (CMA) Blind Adaptive Algorithm for Smart Antennas in a W-CDMA Network

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Abstract- Smart Antenna is recognized as an important technique for increasing the user capacity of a Wireless Communication network, this is due to its capabilities of steering nulls to reduce co-channel interference and pointing independent beams toward various users as well as its ability to provide estimates of directions of radiating sources. The core of Smart Antenna is the selection of smart adaptive beam forming algorithm. An adaptive algorithm is said to be "blind" if it does not require a known training sequence. Blind adaptive algorithm is important because, it is common practice to switch to a blind mode once the training sequence has been processed in order to track a changing environment, and blind adaptive algorithm can completely eliminate the need for a training sequence. This is desirable since the use of a training sequence reduces the number of bits available for transmitting information. This paper analyzes the Constant Modulus Algorithm (CMA) as a blind adaptive algorithm developed for Smart Antenna application in a W-CDMA communication network. The computer simulation carried out in MATLAB platform shows the signal processing technique optimally combines the components in such a way that it maximizes array gain in the desired direction simultaneously minimize it in the direction of interference.

Keywords: **Smart Antenna, Adaptive Algorithm, Beam forming, Signal Nulling, Antenna Array.**

I. INTRODUCTION

The use of an antenna array adds an extra dimension and makes the utilization of spatial diversity possible. This is due to the fact that the interferences rarely have the same geographical location as the user and therefore they are spatially separated. Different from Omni-directional antenna system and sectored antenna system, adaptive antenna array (Smart Antenna) system combine an antenna array and a digital signal processor to receive and transmit signals in a directional manner. Thus the beam pattern at the base station can be adaptively changed.

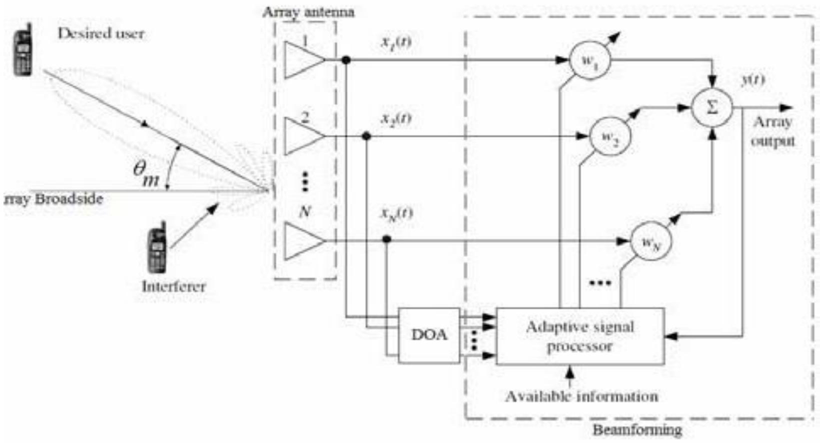

Fig 1 A Functional Block Diagram Of A Smart Antenna System [1, 2].

Smart antennas may be used to provide significant advantages and improved performance in almost all wireless communications systems. These Smart antennas dynamically adapt to changing traffic requirements. Smart antennas are usually employed at the base stations and they radiate narrow beams to serve different users. The complex weight computations based on different criteria are incorporated in the digital signal processor in the form of software algorithms. Smart antenna system consists of antennas and the associated digital control system which provide the beam-forming "intelligent" hence the term 'smart antenna'. These systems require array signal processing that involves manipulation of signal induced on various antenna elements. Their capabilities of steering nulls to reduce co-channel interference and pointing independent beams towards various radiating sources as well as their ability to provide estimates of directions of signal arrivals make them very

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attractive for commercial telecommunications system, military applications [3] and new solutions for future radio telescopes.
Signal processing aspects for this type of systems have concentrated on the development of efficient algorithm for direction of arrival (DOA) estimation [4] and adaptive beam forming. Adaptive algorithms form the heart of the array processing network. Several algorithms have been developed based on different criteria to compute the complex weights. They have their own advantages and disadvantages as far as the convergence speed, complexity and other aspects are concerned. There is still a room for an improvement in this regard to improve the performance of the whole adaptive system by improving present algorithms.
Rani, Subbaiah, and Reddy, in [5] discussed the adaptive beam forming approach for Smart antennas and adaptive algorithms used to compute the complex weights in a W-CMDA mobile environment. Bahri and Bendimerad in [6] proposed a downlink multiple-input multiple-output multiple-carrier code division multiple access system with the Least Mean Square adaptive algorithm for Smart antennas. In [7], Shubair, Mahmoud, and Samhan developed a setup for the evaluation of the MUSIC and LMS algorithms for a Smart antenna system. The authors presented a practical design of a Smart antenna system based on direction-of-arrival estimation and adaptive beam forming. Susmita Das in his work [8] provides description, comparative analysis and utility of various reference signal based algorithms as well as blind adaptive algorithms.
Dominique Godard [9] was the first to capitalize on the constant modulus (CM) property of the incoming signal in order to create a family of Blind adaptive algorithm. Specifically, Godard algorithm applies to phase modulating waveforms. One severe disadvantage of the Godard CMA algorithm is the slow convergence time. This limits the usefulness of the algorithm in a dynamic environment. A faster blind algorithm was developed by Agee [10] using the method of nonlinear Least Squares. This algorithm is known as LSCMA. Gooch and Lundell proposed the Orthongonalized CMA (O-CMA) algorithm [11], this algorithm is similar in form to the Recursive Least Squares (RLS) algorithm. It is based on the CMA cost function. Thomas Biedka [12] presented a framework for the development and analysis of blind adaptive beam forming algorithms for Smart antenna system. The authors give an exclusive summary of concepts, measurements, and parameters and validate results from research conducted within the scope of their work.
This research is aimed at analyzing the Constant Modulus Algorithm (CMA) blind adaptive algorithm developed for Smart Antenna application in a W-CDMA communication network, by estimating its performance against fixed real-time constraints. The computer simulation carried out in MATLAB platform shows the signal processing technique optimally combines the components in such a way that it maximizes array gain in the desired direction simultaneously minimize it in the direction of interference.

II. ADAPTIVE BEAMFORMING

Adaptive beam forming combines the inputs of multiple antennas (from an antenna array) to form very narrow beams toward individual users in a cell. An adaptive beam former is a device that is able to separate signals collocated in the frequency band but separated in the spatial domain. This provides a means for separating a desired signal from interfering signal. An adaptive beam former is able to automatically optimize the array pattern by adjusting the elements control weights until a prescribed objective function is satisfied. The means by which the optimization is achieved is specified by an algorithm designed for that purpose.

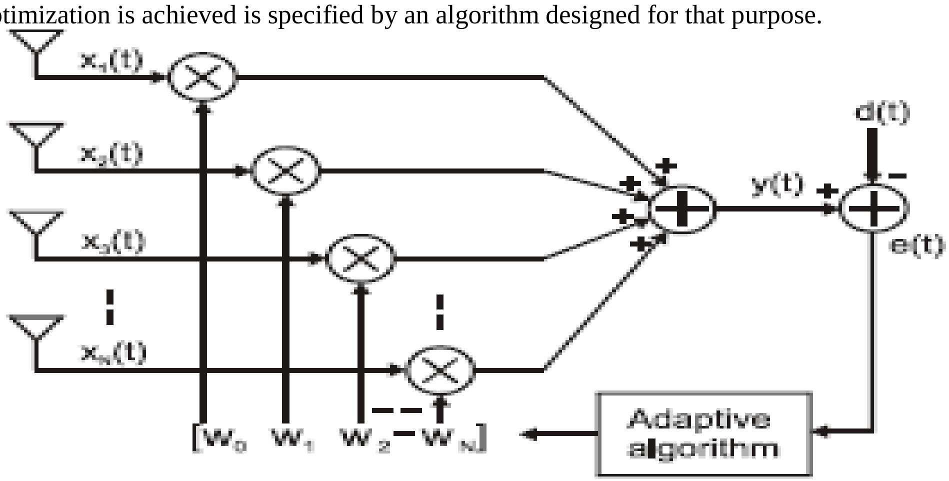

Fig 2 Adaptive Beam forming

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The digital signal processor interprets the incoming data information, determines the complex weights (amplitude and phase information) and multiplies the weights to each element output to optimize the array pattern. The output response of the uniform linear array is given as,

$$y(t) = \boldsymbol{W}^H \boldsymbol{X}(t) \qquad (1)$$

Where $\boldsymbol{W}$ is the complex weights vector and $\boldsymbol{X}$ is the received signal vector.

The complex weights vector is obtained using an adaptive beam forming algorithm. Adaptive beam forming algorithms are classified as Direction of Arrival (DOA)-based, temporal-reference-based or signal-structure-based. In DOA-based beam forming, the direction of arrival algorithm passes the DOA information to the beam former. The beam forming algorithm is then used to form radiation patterns, with the main beam directed towards the signal of interest and with nulls in the directions of the interferers.

On the other hand, temporal-reference-based beam forming uses a known training sequence to adjust the weights and to form a radiation pattern with a maximum towards the signal of interest. If $d(t)$ denotes the referenced sequence or the training symbol known a prior at the receiver at time t, an error $\epsilon(t)$ is formed as

$$\epsilon(t) = d(t) - \boldsymbol{W}^H \boldsymbol{X}(\boldsymbol{t}) \qquad (2)$$

This error signal is used by the beam former to adaptively adjust the complex weights vector, so that the Mean Square Error (MSE) is minimized. The choice of weights that minimize the MSE is such that the radiation pattern has a beam in the direction of the source that is transmitting the reference signal, and that there are nulls in the radiation pattern in the directions of the interferers.

Using the information supplied by the DOA, the adaptive algorithm computes the appropriate complex weights to direct the maximum radiation of the antenna pattern toward the desired user and places nulls toward the directions of the interferers.

There are several adaptive algorithms used for Smart antenna system, they are typically characterized in terms of their convergence properties and computational complexity. Adaptive algorithm that requires a known training sequence is known as non-blind adaptive algorithm this includes;

- Direct Matrix Inversion (DMI) Algorithm.
- Least Mean Square (LMS) Algorithm
- Recursive Least Square Algorithm (RLS)
- Normalized Least Square Algorithm (NLMS)

III. BLIND ADAPTIVE ALGORITHM

An adaptive algorithm is described as “blind” if it does not require a known training sequence. The use of training sequence places several requirements on the receiver that can be difficult to meet in practice. Such as, the timing of the training sequence must be known. That is the receiver may know that the transmitter periodically transmits a certain sequence of training symbols but how does the receiver know when to expect the sequence. Equally, the carrier frequency must be known at least for classic approaches that seek to directly minimize the Mean Square Error (MSE). The carrier and timing synchronization are very difficult to obtain in low SINR and multipath environments which are the environments where adaptive antenna arrays offer the most benefits. Therefore, the use of algorithms that do not require carrier or timing synchronization (initially) is highly desirable. One way to avoid the need for such synchronization is to exploit some property that the desired signal is known to exhibit. And by optimizing an appropriate cost function based on this property, a high quality estimate of the desired signal can be obtained. One of such important property is the constant modulus property exhibited by phase and frequency modulated signals. Another property is the second-order cyclostationarity exhibited by many communication signals at cycle frequency, such as the baud rate, double carrier frequency and the sum and differences of these [13]. A brief overview of some blind adaptive algorithms are presented, the blind adaptive algorithm considered include,

1. *SPECTRAL SELF-COHERENCE RESTORAL (SCORE)*

Self Coherence Restoral (SCORE) algorithms are designed to differentiate between desired signals and the interference by exploiting the second –order cyclostationarity exhibited by the desired signal [13]. A complex base band signal $s(n)$ is said to exhibit second-order cyclostationarity if the lag-product waveform $s(n)s*(n-\tau)$ contains a finite strength sine wave component or spectral line. The same signal exhibit conjugate cyclostationarity if the lag-product waveform $s(n)s(n-\tau)$ contains a spectral line [13]. The magnitude and phase of the spectral at cycle frequency α for lag τ is given by the (asymmetric) cyclic auto-correction function defined by

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$$\boldsymbol{R}_{ss}^{\alpha}(\boldsymbol{\tau}) \triangleq \langle s(n)s*(n-\tau)e^{-j2\pi\alpha n}\rangle_{\infty} \quad (3)$$

The relative strength of the spectral line is given by the cyclic correlation coefficient given by

$$\rho_{ss}^{\alpha}(\tau) \triangleq \frac{\boldsymbol{R}_{ss}^{\alpha}(\tau)}{\boldsymbol{R}_{ss}(0)} \quad (4)$$

Schemes which exploit cyclostationarity in general rely on the property that noise and interference will not,in the limit as the collect line approaches infinity, contribute to a cyclic correlation or cyclic spectrum. Cyclic correlation coefficient is an important parameter for predicting the performance of cyclostationarity exploiting algorithms. The strength of cyclic features associated with data rates is directly dependent on the excess bandwidth with low excess bandwidth signal exhibiting weak second-order cyclostationarity.

There are several different versions of SCORE, with each version behaving differently depending on the environment. These include the least square (LS-SCORE), the cross-SCORE and the phase-SCORE.

2. TIME-, FREQUENCY- AND CODE-GATED ALGORITHMS

Many signals are gated in either time, frequency or code. A push-to-talk signal for instance is gated in time. A CDMA signal becomes gated in frequency when it is dispread. The main idea of the property-gated approach is that some operations can be performed on the array data so that the desired signal is emphasized relative to the background noise and interference. This provides a great deal of information about the environment. Using the time-gated signal as an example, while the desired signal is off, the covariance matrix of the background noise and interference $\boldsymbol{R}_{off}$ is computed. When the desired signal is on, the covariance matrix of the desired signal plus the background noise and interference $\boldsymbol{R}_{on}$ is computed. Then the weight vector that maximizes the quantity

$$F = \frac{W^H \boldsymbol{R}_{on} W}{W^H \boldsymbol{R}_{off} W} \quad (5)$$

is computed. This weight vector maximizes the output SINR of the desired signal because

$$\boldsymbol{R}_{on} = \delta_s^2 a a^H + \boldsymbol{R}_{ii} \quad (6)$$

$$\boldsymbol{R}_{off} = \boldsymbol{R}_{ii} \quad (7)$$

and therefore

$$F = SINR + 1 \quad (8)$$

The weight vector that maximizes equation (5) is found by solving the generalized Eigen-equation

$$\boldsymbol{R}_{on} W = \lambda R_{off} W \quad (9)$$

For the dominant eigenvectors. When there is no change in the signal environment the dominant eigenvalue will be approximately unity. When a new signal appears in $\boldsymbol{R}_{on}$, the dominant eigenvalues will be approximately equal to the optional output SINR of the new signal.

3. CONSTANT MODULUS ALGORITHM (CMA)

Many wireless communication and radar signals are frequency or phase-modulated signals, such as FM, PSK, FSK, OAM and polyphase. This being the case, the amplitude of the signal should ideally be a constant. Thus the signal is said to have a constant magnitude or modulus. So if we know that the arriving signal of interest should have a constant modulus, we can devise algorithms that restore or equalize the amplitude of the original signal. The CMA is perhaps the most well-known blind algorithm and it is used in many practical applications because it does not require carrier synchronization. Dominique Godard [9] used a cost function called a dispersion function of order *p* which is given by,

$$J(k) = E\left[\left(|y(k)|^p - R_p\right)^q\right] \quad (10)$$

Where $y(k) = \boldsymbol{W}^H X(k)$ is the array output at the time *k* and *p* is the positive integer and *q* is a positive integer =1. The gradient of this cost function is zero when R_p is defined by

$$R_p = \frac{E[|s(k)|^{2p}]}{E[|s(k)|^p]} \quad (11)$$

Where *s(k)* is the zero-memory estimate of *y(k)*. the resulting error signal is given by

$$e(k) = y(k)|y(k)|^{p-2}\left(R_p - |y(k)|^p\right) \quad (12)$$

This error signal can replace the traditional error signal in the LMS algorithm to yield

$$W(k+1) = W(k) + \mu e*(k)X(k) \quad (13)$$

By selecting values of 1 or 2 for *p* different version of CMA may be obtained.

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The case of *p* =1, the cost function will be reduced to

$$J(k) = E[(|y(k)| - R_1)^2] \quad (14)$$

Where

$$R_1 = \frac{E[|s(k)|^2]}{E[|s(k)|]} \quad (15)$$

If we scale the output estimate *s(k)* to unity we can write the error signal of equation (12) as

$$e(k) = \left(y(k) - \frac{y(k)}{|y(k)|}\right) \quad (16)$$

Thus the weight vector becomes

$$W(k+1) = W(k) + \mu\left(1 - \frac{1}{|y(k)|}\right)y * (k)X(k) \quad (17)$$

Similarly when *p* = 2 the cost function will reduce to

$$J(k) = E[(|y(k)|^2 - R_2)^2] \quad (18)$$

Where

$$R_2 = \frac{E[|s(k)|^4]}{E[|s(k)|^2]} \quad (19)$$

If we equally scale the output of the estimate *s(k)* to unity ,we can write error signal of equation (12) as

$$e(k) = y(k)(1 - |y(k)|^2) \quad (20)$$

Thus, the weight vector becomes

$$W(k+1) = W(k) + \mu(1 - |y(k)|^2)y * (k)X(k) \quad (21)$$

One of the attractive features of the CMA is that carrier synchronization is not required; furthermore it can be applied successfully to non-constant modulus signal if the Kurtosis of the beam former output is less than two. This means that CMA can be applied to for example PSK signals that have non-rectangular pulse shape. This is important because it implies that the CMA is also robust to symbol timing error when applied to pulse-shaped PSK signals. Pulse shaping typically is used to limit the occupied bandwidth of the transmitted signal.

4. LEAST SQUARE-CONSTANT MODULUS ALGORITHM (LS-CMA)

One severe disadvantage of the Godard CMA algorithm is the slow convergence time. The slow convergence time limits the usefulness of the algorithm in dynamic environment where the signal must be captured quickly. A faster converging CMA algorithm similar in form to the Recursive Least Square (RLS) method is the Orthogonalized-CMA. Another fast converging CMA is the Least Square CMA (LS-CMA) which is a block update iterative algorithm that is guaranteed to be stable and easily implemented. At the n-iteration, *k*- temporal samples of the beam former output are generated using the current weight vector W_n . This gives

$$y_n(k) = W^H X(k) \quad (22)$$

The initial weight vector W_0 can be taken as

$$W_0 = [1 \;\; 0 \;\; 0 \;\; . \; . \;\; 0]^T \quad (23)$$

if no a priori information is available. The nth signal estimate is then hard limited to yield

$$d_n(k) = \frac{y_n(k)}{|y_n(k)|} \quad (24)$$

and a new weight vector is formed according to

$$W_{(n+1)} = R_{xx}^{-1} r_{xd} \quad (25)$$

Where,

$$R_{xx} = (X(k)X^H(k))_N \quad (26)$$

$$r_{xd} = (X(k)d_n * (k))_N \quad (27)$$

Equations (26) and (27) denote a time average over $0 \le k \le N-1$. The update weight vector $W_{(n+1)}$ minimizes the mean square error. The iteration described above continues until either the change in the weight vector is smaller than some threshold or until the envelope variance of the output signal is deemed sufficiently small. When the iteration is performed using a new block of data it is known as dynamic LSCMA. But when it is re-applied to the same block of data it is known as static LSCMA.

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IV. EVALUATION AND SIMULATION RESULT

The simulation is carried out on MATLAB platform where the Uniform Linear array with N number of element is used. We define the direct path arriving signal as a 32- bit binary chipping sequence with a value of ±1, and arriving at 35°, while the first multipath signal arrives at -30° and a second multipath signal arrives at -10°.

Fig 3 shows the arriving signal paths of the direct path, the first multipath, the second multipath and the combined path as seen by the receiver. This shows the effect of the multipath signals on the desired signal. Fig 4 shows the array factor plot and how the CMA algorithm has suppressed the multipath signals while directing maximum to the direct path signal. Fig 5 shows the effect of the algorithm on the received combined signal. Fig 6 shows that the performance of the CMA algorithm improves when the number of iteration n is increased from 25 to 50. Fig 7 shows that the LSCMA algorithm does a better job of nulling the multipath terms than the CMA Algorithm.

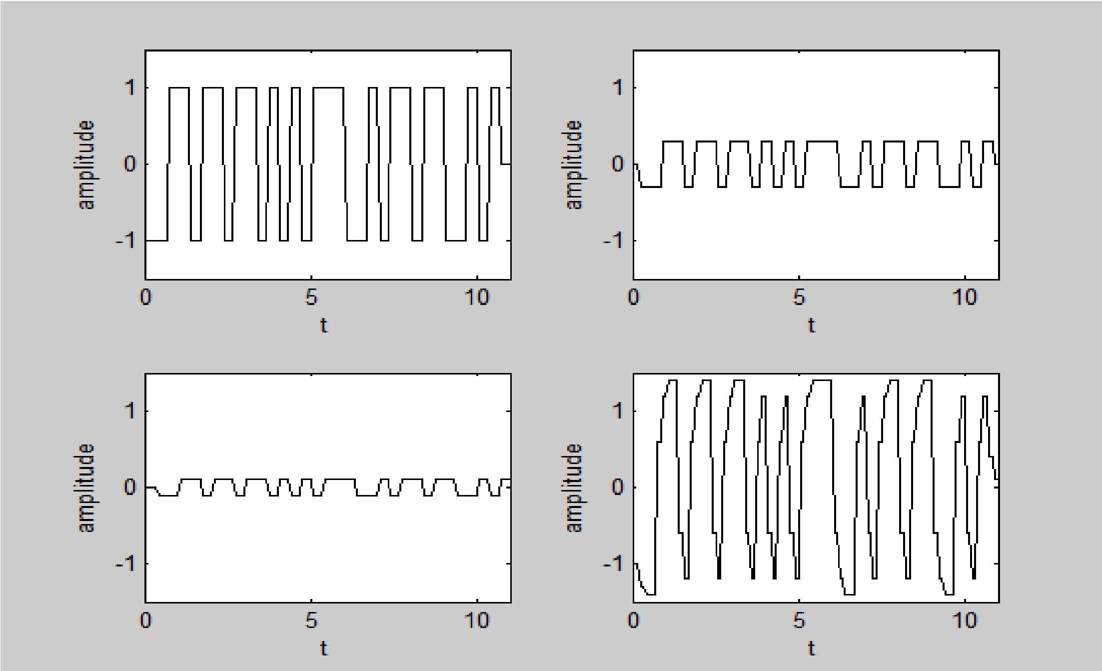

Fig 3 Arriving Paths of the Direct Path, Multipath 1, Multipath 2 and the Combined pat

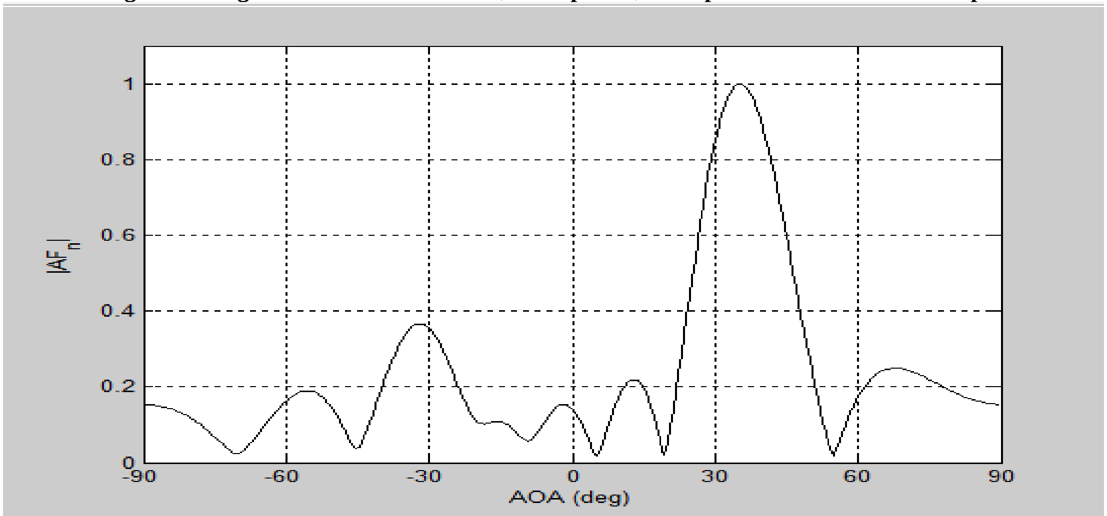

Fig 4 Array factor plot for CMA algorithm when the desired user with AOA 35°, the first multipath with AOA -30° and the second multipath with AOA -10°, the spacing between the elements is 0.5λ and step size is $\mu = 0.02$ and number of iteration is 25

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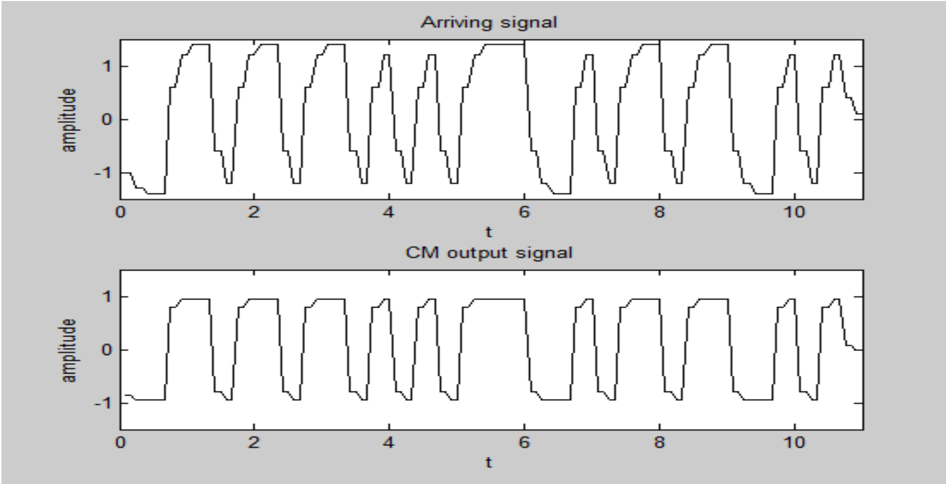

Fig 5 Arriving Signal and Output Signal

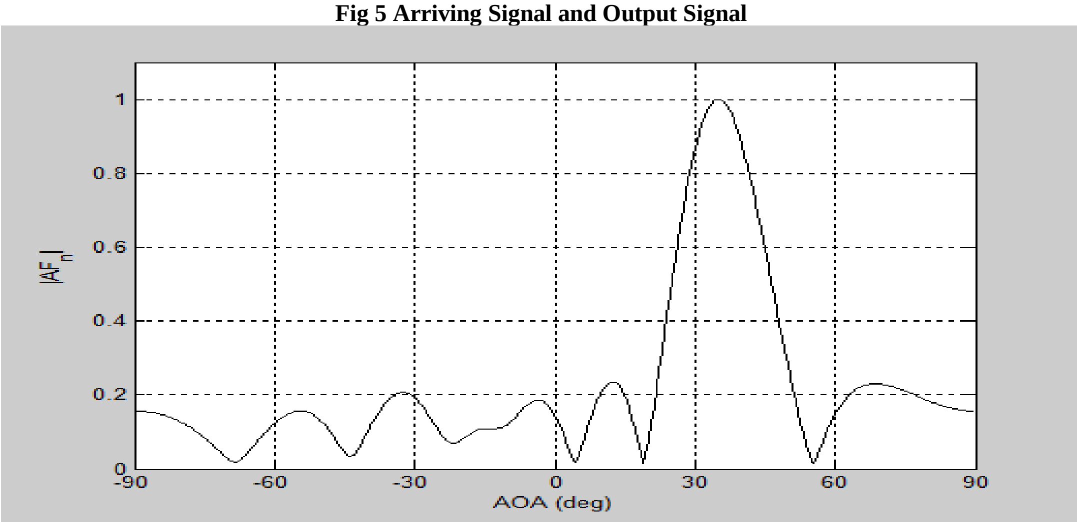

Fig 6 Array factor plot for CMA algorithm when the desired user with AOA 35°, the first multipath with AOA -30° and the second multipath with AOA -10°, the spacing between the elements is 0.5λ and step size is $\mu = 0.02$ and number of iteration is 50

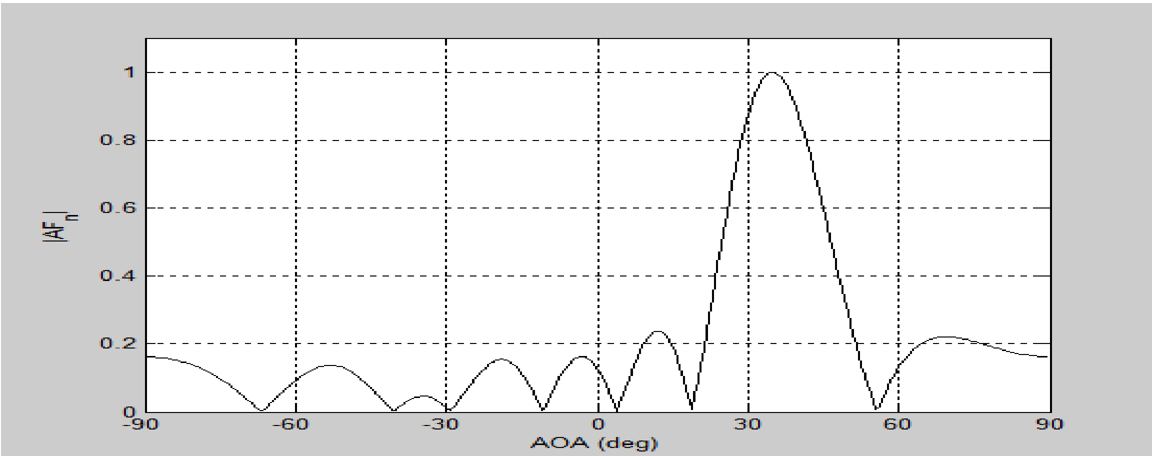

Fig 7 Array factor plot for a static LSCMA algorithm when the desired user with AOA 35°, the first multipath with AOA -30° and the second multipath with AOA -10°, the spacing between the elements is 0.5λ and step size is $\mu = 0.02$ and number of iteration is 5.

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V. CONCLUSION

This research worked analyzed the performance of the CMA blind adaptive algorithm for Smart Antennas in a W-CDMA network. Results obtained verify the improved performance of the Smart antenna system. The Smart antenna technology suggested in this research offers a significantly improved solution to reduce interference levels and improve the system capacity. With this approach, each user's signal is transmitted and received by the base station only in the direction of that particular user. This drastically reduces the overall interference in the system. Also the base station can form narrower beams towards the desired users and nulls towards interfering users, considerably improving the Signal-to-Interference-plus noise ratio. Such Smart antenna also can be used to achieve different benefits; such as higher network capacity by precise control of signal nulls quality and mitigation of interference. It also provides better range or coverage by focusing the energy sent out into the cell. The CMA algorithm is important because it does not require carrier synchronization and it can be applied successfully to non-constant modulus signal if the Kurtosis of the beam former output is less than two. One severe disadvantage of the Godard CMA algorithm is the slow convergence time. The slow convergence time limits the usefulness of the algorithm in dynamic environment where the signal must be captured quickly. A fast converging CMA is the Least Square CMA (LS-CMA) which is a block update iterative algorithm that is guaranteed to be stable and easily implemented. Typically LSCMA converges in 5 to 10 iteration regardless of the block size and as seen the static LSCMA can converge 100 times faster than the conventional CMA. However, the computational load makes the LSCMA impractical for a real-time application.

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