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Application of graph theory in determining the extent of cliques and dorminance in families in Rigati village Nyeri county-Kenya

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Abstract: Graph theory is moving into the mainstream of mathematics mainly because of its many applications. In this work, we applied two aspects of graph theory, namely, clique graphs and dominance graphs, to determine the patterns of cliques and dominance in a rural village set-up in Ragati village, Central Kenya. The research was carried out by administering questionnaires to members of ten different families to determine how the family members influence each other on different day to day activities. Analysis was then done and vertex matrices were then obtained at different influence levels. These were then used to determine the existence of cliques or dominance in the family. In this research, out of the ten families investigated, three had cliques and three had dominant members. In most of the cases, the cliques had a female majority and the females dominated the other family members which appear to confirm common belief that Nyeri females dominate their males.

Index Terms—**cliques, Dorminance, function, Graph.**

I. BACKGROUND INFORMATION

Anyone who has a basic knowledge in elementary mathematics is acquainted with graphs of various functions and their pictorial representation. In this project we shall be dealing with graphs of a different kind. The graphs considered in this project, like the ones encountered in elementary mathematics, may be represented in a diagram. Graph theory has a lot of applications in some areas of physics, chemistry, communication science, computer technology, electrical and civil engineering, architecture, operational research, genetics, psychology, sociology, economics, anthropology, and linguistics. It is intimately related to many branches of mathematics, including group theory, matrix theory, numerical analysis, probability, topology and combinatorics. It serves as a mathematical model for any system involving a binary operation.

Definition 1: Graph

A graph *G* is a finite non-empty set *V(G)* of objects called vertices, and a (possibly empty) set *(G)* of two elements subsets of *V(G)* called edges.

Definition 2: Directed graph (Digraph) D.

According to Chart and (1994) [2] , a diagraph is a finite non-empty set *V(D)* of vertices and a non-empty set *E(D)* of ordered pairs of distinct vertices. The elements of *E(D)* are called arcs. Digraphs can be represented by diagrams. The vertices of a digraph are represented by small circles and an arc (*u*,*v*) of *D* is represented by drawing a curve or line segment directed from vertex *u* to *v*. The diagram below is an example of a digraph.

Definition 3: In degree and out degree of a vertex

The in degree of a vertex *v* in a directed graph *D* is the number of edges leaving the vertex *v*. The out degree of a vertex *v* is the number of edges entering the vertex *v*.

Definition 4: Order and size of a digraph D

The **order** of a digraph *D* is the number of vertices in the digraph and the **size** of the digraph is the number of its edges.

Definition 5: Vertex matrix of a directed graph D

Let the vertex set of the digraph *D* be $\{v_1, v_2, v_3, \ldots\ldots, v_n\}$. Then the vertex matrix $M = [a_{ij}]$ is the $N \times N$ matrix defined by

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$$a_{ij} = \begin{cases} 1 & \text{if } (v_i, v_j) \in E(D) \\ 0 & \text{otherwise} \end{cases},$$

Vertex matrix of a directed graph is also referred to as its Adjacency matrix.

Definition 6: Degree of a vertex v

The degree of a vertex *v* is the number of edges incident with the vertex.

Definition 7: Source and Sink

A vertex *v* is called a sink if no arc leaves the vertex. It is called a source if no arc enters it.

Theorem 1:

Let *D* be a diagraph of order *p* and size *q*, with

$V(D) = \{v_1, v_2 ...., v_p\}$ Then

$$\sum_{i=1}^{p} od\ v_i = \sum_{i=1}^{p} id\ v_i = q$$

Where $od\ v_i$ = out degree of the vertex v_i

$id\ v_i$ = In degree of the vertex v_i

Proof

When the out degrees of vertices of *D* are summed up, each arc of *D* is counted exactly once. The same holds for the in degrees. (Chartrand, 1993) [2]

Corollary 1

The sum of entries in each row v_i of *M* gives the out degrees of v_i and the sum of entries in each column v_j of *M* gives the in degrees of v_j .

Proof

The sum of entries in each row v_i gives the total number of edges incident from v_i while the sum of entries in each column v_j of *M* gives the total number of edges incident to v_j (Harary, 1972) [3].

Corollary 2

The degree of a vertex *v*, $\deg v = od\ v + id\ v$

Proof

By definition, the degree of a vertex *v* is the number of vertices adjacent to it. In a diagraph, vertex *u* will be adjacent to vertex *v* if there is an arc to or from *v*. The total number of vertices with arc to *v* (in degrees of *v*) and the total number of vertices with arc from *v* (out degrees of *v*) will be equivalent to the degree of *v*. (Harary, 1972) [3].

Corollary 3:

A row of *M* with all zero entries corresponds to a sink while a column with all zero corresponds to a source.

Proof:

By corollary 1, the sum of entries in each row v_i gives the out degree of v_i . When $\sum od\ v_i = 0$, then no arc leaves vertex v_i.

When $\sum id\ v_i = 0$ then no arc enters v_i .

Definition 8

An **r-step connection** in a directed graph *D* from v_i to v_j $(v_i \to v_j)$ is the number of occurrences of arcs from the vertex v_i to v_j. If there are r-arcs from v_i to v_j, then this is an r-step connection.

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Theorem 2

Let *M* be the Vertex (Adjacency) matrix of a directed graph *D* and let $a_{ij}^{(r)}$ be the (*i*, *j*) element of M^r. Then $a_{ij}^{(r)}$ is equal to the number of r-step connections from v_i to v_j. (Parthasarathy 1994) [5].

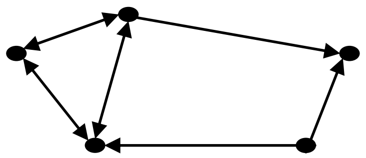
Definition 9: (Clique)

A subset of directed graph is called a clique if it satisfies the following conditions:

i) The subset contains at least three vertices

ii) For each pair of vertices v_i and v_j in the subset, both $v_i \to v_j$ and $v_j \to v_i$ are true

iii) The subset is as large as possible, that is, it is a maximal subset implying it is not possible to add another vertex to the subset and still satisfy condition (ii). (Howard, 2005) [4]



From the graph above, { v_1, v_2, v_5 } is a clique.

For simple directed graphs, cliques may be found by inspection. For large directed graphs, there is a systematic procedure for detecting cliques.

Definition 10: Clique Matrix v_3

A matrix $S = [s_{ij}]$ related to a given directed graph defined as

$$s_{ij} = \begin{cases} 1 & \text{if } v_i \leftrightarrow v_j \\ 0 & \text{otherwise} \end{cases}$$

helps identify the vertices that belong to a clique.

This matrix *S* above determines a directed graph, which is the same as the given directed graph with the exception that the directed edges with only one arrow are deleted. This is a modified graph of the original directed graph.

The matrix $S = [s_{ij}]$ may be obtained from the vertex matrix $M = [a_{ij}]$ of the original directed graph by setting

$$S = s_{ij} = \begin{cases} 1 & \text{if } a_{ij} = a_{ji} = 1 \\ 0 & \text{otherwise} \end{cases}$$

Theorem 3

Let $s_{ij}^{(3)}$ be the (*i*, *j*)-th element of S^3. Then a vertex v_i belongs to some clique if and only if $s_{ii}^{(3)} \neq 0$.(Howard, 2005) [4]

Definition 11: Dominance Directed graph

A dominance directed graph is a directed graph such that for any distinct pair of vertices v_i and v_j, either $v_i \to v_j$ or $v_j \to v_j$ but not both. Dominance directed graphs are sometimes called **tournaments.**

Theorem 4

In any dominance directed graph, there is at least one vertex from which there is a one-step or two-step connection to any other vertex. (Howard, 2005) [4]

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Definition 12: (Power of a vertex.)

The power of a vertex of a dominance directed graph is the total number of 1-step and 2-step connections from it to the other vertices. It is also the sum of entries of the i-th row of the matrix $A=M+M^2$, where M is the vertex matrix of the directed graph. The sum of entries in the i-th row of M is the total number of 1- step connection from vertex vi to other vertices and the sum of entries of the i-th row of M^2 is the total number of 2-step connections from v_i to other vertices. A row of the matrix $A=M+M^2$ with the largest row sum identifies the most powerful vertex. Some applications of Graph Theory.

According to Farary (1972)[3], Swiss mathematician Leonhard Euler in 1736 solved the Konigsberg bridge problem which sought to find out if it was possible to cross the city's network of seven bridges only once during a walk across town. He mathematized his question and represented each land area by a point and each bridge by a line joining corresponding points, thereby producing a graph.

According to Chartrand et al (1993)[2], Euler in 1766 solved the re-entrant's Knight's tour puzzle which sought to answer the question of whether it is possible for a Knight to tour the chess board, that is, visit each square exactly once and return to its initial square. He found out that given an n x n chessboard, a Knight's graph was defined with a vertex corresponding to each square of the chessboard and an edge connecting vertex i with vertex j if and only if there is a legal Knight's move from the square corresponding to vertex i to the square corresponding to vertex j. Thus, a re-entrant Knight's tour on the chessboard corresponds to a Hamiltonian circuit in the Knight's graph. The Hamiltonian circuit algorithm has been used to find re-entrant Knight's tours on chessboards of various dimensions.

According to Farary (1972)[3], Kirchhoff in 1847 developed the theory of trees in order to solve the system of simultaneous linear equations which give the current in each branch and around each circuit of an electric network. His work was carried on by Cayley (1857) who discovered this important class of graphs called trees by considering changes of variables in differential calculus. He used these graphs in enumerating isomers of saturated hydrocarbons $C_n H_{2n+2}$, with a given number n of hydrocarbons.

According to Farary (1972)[3], Sir William Hamilton in 1859 invented a game which uses a regular solid dodecahedron whose 20 vertices are labeled with names of famous cities. The player is challenged to travel "around the world" by finding a closed circuit along the edges which passes through each vertex exactly once. Graphically, this is to find a spanning cycle in the graph of the dodecahedron.

According to Chartrand et al (1993)[2], the Chinese Postman problem (where a letter carrier must deliver mail to every house in a small town and would like to cover the route in the most efficient way and then return to the Post-office) was solved by Guan (1960) by modeling the situation by using a graph. Graphically, he determined the shortest closed walk covering the edges of the graph.

Chai Wa Wu Chua (1995)[8] used algebraic graph theory to the synchronization in an array of coupled non linear oscillators. Sufficient conditions were derived from the connectivity graph which described how the oscillators were connected.

Samuel S. Katambi et al (2002)[7] used the theory to a Gross Error Detection for GPS Geodetic Control Network. GPS network is considered as a connected and directed network with three components. The gross error detection is undertaken through loops of different spanning trees using Loop law.

According to Murty (2002)[1], Agnes M. Herzberg and M.R Murty translated the problem of solving a sudoku puzzle into the language of graph theory. In a 9-by-9 sudoku grid, the 81 squares in the grid correspond to vertices in a mathematical graph and a line connects vertices that appear in the same row, column, or sub grid. They established that for a puzzle to have precisely one solution, the initial entries need to include at least eight of the nine digits.

Narsingh Deo et al (2002)[6] have used computational graph theory to solve problems in computer science. They are currently working on web graphs where World Wide Web can be modeled as a directed graph and each node

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is a web page and each hyperlink is an edge. Studying web graphs gives insights into things like web algorithms for crawling, searching or ranking web resources. If a virus spreads, graph theory can be used to see how it would travel through the web.

Statement of the problem

In this paper, we seek to apply two aspects of graph theory namely cliques and vertex power to determine patterns of cliques and dominance in a rural village set up to demystify the claim that women from Nyeri district of Central Kenya dominate men.

Objectives

i) To determine vertex matrix for each family.
ii) To use the vertex matrix obtained above to generate clique and dominance matrices.
iii) To determine the existence of a clique or dominance in each family.

II. METHODOLOGY

The sampling method was by random selection. The instrument used was a questionnaire. The questionnaire was issued to ten middle-class families with both parents. The questionnaire was designed to collect information on various aspects to show how the members of a family influence each other. The questionnaires were administered to each member of the family. This was done through face to face interview by the researcher.

Sample profile

The profiles of the family structures were as follows:-
Family 1 comprised of the father, mother, one son and one daughter.
Family 2 comprised of the father, mother, one son and two daughters.
Family 3 comprised of father, mother, one son and two daughters.
Family 4 comprised of father, mother, one daughter and two sons.
Family 5 comprised of father, mother, three daughters and one son.
Family 6 comprised of father, mother, one son and two daughters.
Family 7 comprised of father, mother, two sons and one daughter.
Family 8 comprised of father, mother, and three daughters.
Family 9 comprised of father, mother, two sons and two daughters.
Family 10 comprised of father, mother, two sons and one daughter.
In nine (9) out of ten families considered, all the parents were educated up to secondary school level and the children were either in secondary school or colleges. In one family, the children were in lower classes. All the families considered were Christians, that being the most dominant faith in the area. The influence between members was measured using common day to day family activities namely:-
A1: Preference for TV/Radio Programmes
A2: Choice of Clothing
A3: Meals preference
A4: Choice of friends
A5: Favourite Hobbies
A6: Choice of Religion/Church
A7: Careers choices
A8: Choice of school
A9: Control of daily routine
A10: Choice of destinations for outings / Leisure
Influence graphs (matrices) for the ten families for various levels of influence were then constructed starting with 20% influence, 30%, 40% and 50% influence.

Example 1

Suppose a family is made up of the father, mother, one son and one daughter. They influence one another as shown in the following table.

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Table 1

	F	M	S	D
A_1	S	D	D	S
A_2	D	D	F	M
A_3	M	D	D	M
A_4	D	F	M	M
A_5	M	D	D	S
A_6	M	F	M	M
A_7	M	-	M	F
A_8	M	F	D	F
A_9	M	F	M	M
A_{10}	S	S	F	F

By 20% influence level, we mean that a family member has influence on the other members in two or more activities out of the ten. From the above table, the son influences the father and the sister, the daughter influences the father, mother and the brother, the father influences the mother, the son and the daughter and the mother influences the father son and daughter. The vertex matrix for this influence will then be given as

$$M = \begin{matrix} & F & M & S & D \\ F & 0 & 1 & 1 & 1 \\ M & 1 & 0 & 1 & 1 \\ S & 1 & 0 & 0 & 1 \\ D & 1 & 1 & 1 & 0 \end{matrix}$$

In the family above we can determine the existence or not of a clique by construction of the clique matrix which is

$$S = \begin{matrix} & F & M & S & D \\ F & 0 & 1 & 1 & 1 \\ M & 1 & 0 & 0 & 1 \\ S & 1 & 0 & 0 & 1 \\ D & 1 & 1 & 1 & 0 \end{matrix}$$

The cube of S is given by

$$S^3 = \begin{matrix} & F & M & S & D \\ F & 4 & 5 & 5 & 5 \\ M & 5 & 2 & 2 & 5 \\ S & 5 & 2 & 2 & 5 \\ D & 5 & 5 & 5 & 4 \end{matrix}$$

Thus all the four family members belong to a clique or are members of a clique. We can tell if the members belong to the same clique or not by going back to the clique matrix S. We find that the mother and the son do not influence each other both ways. This implies that the mother and the son do not belong to the same clique. In this family there are two cliques, one consisting of the mother, father and daughter and the other one consisting of the father, son and the daughter. Suppose we increase the level of influence. Suppose we increase the influence level to 30%. The vertex matrix becomes

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$$M = \begin{matrix} & F & M & S & D \\ F & 0 & 1 & 0 & 0 \\ M & 1 & 0 & 1 & 1 \\ S & 0 & 0 & 0 & 0 \\ D & 0 & 1 & 1 & 0 \end{matrix}$$

Since some family members influence each other, a clique matrix can be worked out. This is found to be

$$S = \begin{matrix} & F & M & S & D \\ F & 0 & 1 & 0 & 0 \\ M & 1 & 0 & 0 & 1 \\ S & 0 & 0 & 0 & 0 \\ D & 0 & 1 & 0 & 0 \end{matrix}$$

Cubing this matrix yields

$$S^3 = \begin{matrix} & F & M & S & D \\ F & 0 & 2 & 0 & 0 \\ M & 2 & 0 & 0 & 2 \\ S & 0 & 0 & 0 & 0 \\ D & 0 & 2 & 0 & 0 \end{matrix}$$

There is no clique at this level of influence. All the elements in the leading diagonal of S^3 above are zeros. We cannot work out dominance in this family since some family members influence each other both ways. If dominance existed, then family member, say, A, would influence family member, say B, or member B would influence member A, but not both ways.

III. SURVEY RESULTS

After analyzing the ten Ragati village families, the following tables were obtained.

Family1

Family 1 which consisted of the father, mother, son and daughter generated the following influence table.

Table 2

	F	**M**	**S**	**D**
A1	S	S	D	S
A2	S	D	D	M
A3	D	D	M	M
A4	S	S	F	M
A5	D	D	F	S
A6	M		M	M
A7	M	-	M	F
A8	M	-	F	F
A9	M	F	M	M
A10	F	S	F	F

Where A_i, i = 1, 2….10, is the aspect in consideration.

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F: Father
M: Mother
S: Son
D: Daughter

20% Level of Influence

$$M = \begin{matrix} & F & M & S & D \\ F & 0 & 0 & 1 & 1 \\ M & 1 & 0 & 1 & 1 \\ S & 1 & 1 & 0 & 0 \\ D & 1 & 1 & 1 & 0 \end{matrix}$$

This is not a dominance matrix (graph) as some family members influence each other both ways, for example, $F \to S \text{ and } S \to F$ We need to find out if a clique exists in this family by working out the $S = [s_{ij}]$

where $S = s_{ij} = \begin{cases} 1 \text{ if } m_{ij} = m_{ji} \\ 0 \quad \text{otherwise} \end{cases}$

$$S = \begin{matrix} & F & M & S & D \\ F & 0 & 0 & 1 & 1 \\ M & 0 & 0 & 1 & 1 \\ S & 1 & 1 & 0 & 0 \\ D & 1 & 1 & 0 & 0 \end{matrix}$$

$$S^3 = \begin{matrix} & F & M & S & D \\ F & 0 & 0 & 4 & 4 \\ M & 0 & 0 & 4 & 4 \\ S & 4 & 4 & 0 & 0 \\ D & 4 & 4 & 0 & 0 \end{matrix}$$

Since the elements of the leading diagonal are all 0's, then there is no clique in this family at this level of influence. Existence of a clique at higher levels of influence is not expected as it missed at this lower level. Dominance cannot be worked out at this level as some family influence each other both ways. By definition of dominance, one member should influence the other but not both influencing each other.

Family 2

Family 2 had the father, mother, son and two daughters generated the following influence table.

	F	M	D_1	S	D_2
A1	D_1	D_1	S_1	D_1	S
A2	M	F	M	-	M
A3	M	D_1	M	D_2	D_1
A4	M	F	S_1	D_1	D_1
A5	M	-	F	-	D_1
A6	M	M	M	M	M
A7	-	F	F	M	F
A8	-	F	F	F	F
A9	-	F	M	M	F
A10	D_1	D_1	S_1	-	M

20% Level of Influence

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$$M = \begin{array}{c} \\ F \\ M \\ D_1 \\ S \\ D_2 \end{array} \begin{array}{c} \begin{array}{ccccc} F & M & D_1 & S & D_2 \end{array} \\ \begin{pmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

As some family members influence each other both ways, we can work out clique matrix to find out if a clique exists.

$$S = \begin{array}{c} \\ F \\ M \\ D_1 \\ S \\ D_2 \end{array} \begin{array}{c} \begin{array}{ccccc} F & M & D_1 & S & D_2 \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

$$S^3 = \begin{array}{c} \\ F \\ M \\ D_1 \\ S \\ D_2 \end{array} \begin{array}{c} \begin{array}{ccccc} F & M & D_1 & S & D_2 \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

There exists no clique in this family as all the elements in the leading diagonal of S^3 are all zeros. Since a clique does not exist at this level of influence, it cannot exist at higher levels of influence. Dominance does not exist as the condition that for any distinct pair of family members, either influences the other but not both, does not hold.

Family 3

Family 3 which consisted of the father, mother son and two daughters generated the following influence table.

Table 4

	F	**M**	**S**	**D_1**	**D_2**
A1	S	S	D_1	D_2	D_1
A2	S	D_1	D_1	M	M
A3	D_1	D_1	M	D_2	M
A4	M	F	D_1	S	D_1
A5	M	F	F	F	F
A6	M	F	M	M	M
A7	-	-	F	F	F
A8	-	-	F	F	F
A9	-	F	M	M	M
A10	S	S	F	S	D_1

20% Level of Influence

$$M = \begin{array}{c} \\ F \\ M \\ S \\ D_1 \\ D_2 \end{array} \begin{array}{c} \begin{array}{ccccc} F & M & S & D_1 & D_2 \end{array} \\ \begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

As some family members influence each other both ways, for example F ↔ M,

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F ↔ S, *M*↔*S, M*↔ *D*, we determine if any clique exists.

$$S = \begin{array}{c} \\ F \\ M \\ S \\ D_1 \\ D_2 \end{array} \begin{array}{c} \begin{array}{ccccc} F & M & S & D_1 & D_2 \end{array} \\ \begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

$$S^3 = \begin{array}{c} \\ F \\ M \\ S \\ D_1 \\ D_2 \end{array} \begin{array}{c} \begin{array}{ccccc} F & M & S & D_1 & D_2 \end{array} \\ \begin{pmatrix} 2 & 5 & 5 & 2 & 2 \\ 5 & 4 & 5 & 6 & 1 \\ 5 & 5 & 4 & 6 & 1 \\ 2 & 6 & 6 & 2 & 3 \\ 2 & 1 & 1 & 3 & 0 \end{pmatrix} \end{array}$$

S^3 has non-zero diagonal entries. This implies that in the corresponding directed graph, the father, mother, son and the eldest daughter belong to a clique or are members of a clique. From the clique matrix *S* above, the father and the eldest daughter do not influence each other both ways and therefore do not belong to the same clique. There are two cliques in this family, one consisting of the father, mother and son, and the other consisting of the mother, son and the eldest daughter.

30% Level of Influence

$$M = \begin{array}{c} \\ F \\ M \\ S \\ D_1 \\ D_2 \end{array} \begin{array}{c} \begin{array}{ccccc} F & M & S & D_1 & D_2 \end{array} \\ \begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

As some family members influence each other both ways, clique matrix can be worked out to find out if a clique still exists at this level of influence.

$$S = \begin{array}{c} \\ F \\ M \\ S \\ D_1 \\ D_2 \end{array} \begin{array}{c} \begin{array}{ccccc} F & M & S & D_1 & D_2 \end{array} \\ \begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

$$S^3 = \begin{array}{c} \\ F \\ M \\ S \\ D_1 \\ D_2 \end{array} \begin{array}{c} \begin{array}{ccccc} F & M & S & D_1 & D_2 \end{array} \\ \begin{pmatrix} 0 & 2 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

There is no clique at 30% Level of influence as all the elements in the leading diagonal of S^3 are all zeros. We therefore cannot get a clique at a higher level of influence. Dominance cannot be worked out as for any distinct pair of family members, the condition that only one member influences the other but not both does not hold.

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Family 4

Family 4 consisted of the father, mother, two sons and a daughter generated the following influence table.

Table 5

	F	**M**	**D**	**S_1**	**S_2**
A1	S_2	D	S_2	S_2	S_1
A2	S_2	D	F	F	S_1
A3	S_2	D	D	M	M
A4	M	F	F	M	S_1
A5	-	-	-	S_1	S_1
A6	F	-	F	F	F
A7	-	-	F	M	M
A8	-	-	S_2	F	S_1
A9	-	-	-	M	M
A10	M	S_2	S_2	-	D

20% Level of Influence

$$M = \begin{array}{c} \\ F \\ M \\ D \\ S_1 \\ S_2 \end{array} \begin{array}{c} F \quad M \quad D \quad S_1 \quad S_2 \\ \begin{pmatrix} 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

The graph represented by the vertex matrix above is a dominance directed graph as. For any distinct pair of family members, either member influences the other but not both. We therefore need to work out $M^2 + M$

$$M^2 + M = \begin{array}{c} \\ F \\ M \\ D \\ S_1 \\ S_2 \end{array} \begin{array}{c} F \quad M \quad D \quad S_1 \quad S_2 \\ \begin{pmatrix} 1 & 0 & 1 & 2 & 2 \\ 2 & 1 & 1 & 3 & 2 \\ 2 & 1 & 1 & 2 & 1 \\ 0 & 0 & 1 & 1 & 2 \\ 1 & 1 & 0 & 1 & 1 \end{pmatrix} \end{array} \qquad \begin{array}{c} Row \;\; Sum \\ 6 \\ 9 \\ 7 \\ 4 \\ 4 \end{array}$$

Since the second row has the largest row sum, the mother must then be the most influential member in this family.

30% Level of Influence

$$M = \begin{array}{c} \\ F \\ M \\ D \\ S_1 \\ S_2 \end{array} \begin{array}{c} F \quad M \quad D \quad S_1 \quad S_2 \\ \begin{pmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

Dominance fails at this level of influence. For any distinct pair of family members, not every member influences or is influenced by the other member, for example, the father and the mother do not influence each other either way. Since no two family members influence each other both ways, a clique cannot be worked out.

Family 5

Family 5 consisted of the father, mother, son and three daughters. It generated the following influence table.

Table 6

	F	**M**	**D_1**	**D_2**	**S**	**D_3**
A1	D_2	D_1	D_3	D_3	D_2	D_1
A2	M	D_2	D_2	D1	F	D_1
A3	D_2	D_1	M	D_3	D_2	D_1

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A4	M	F	D_3	D_1	F	D_2
A5	S	D_2	D_2	S	F	D_2
A6	M	-	M	M	M	M
A7	-	F	M	F	F	M
A8	-	-	M	S	F	D_1
A9	-	F	F	F	M	M
A10	M	D_2	D_2	D_1	D_2	D_2

20% Level of Influence

$$M = \begin{array}{c} \\ F \\ M \\ D_1 \\ D_2 \\ S \\ D_3 \end{array} \begin{array}{c} \begin{array}{cccccc} F & M & D_1 & D_2 & S & D_3 \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

Since some family members influence each other both ways, we need to work out the clique matrix.

$$S = \begin{array}{c} \\ F \\ M \\ D_1 \\ D_2 \\ S \\ D_3 \end{array} \begin{array}{c} \begin{array}{cccccc} F & M & D_1 & D_2 & S & D_3 \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

$$S^3 = \begin{array}{c} \\ F \\ M \\ D_1 \\ D_2 \\ S \\ D_3 \end{array} \begin{array}{c} \begin{array}{cccccc} F & M & D_1 & D_2 & S & D_3 \end{array} \\ \begin{pmatrix} 0 & 4 & 1 & 6 & 0 & 2 \\ 4 & 0 & 5 & 1 & 2 & 2 \\ 1 & 5 & 2 & 7 & 1 & 4 \\ 6 & 1 & 7 & 2 & 4 & 5 \\ 0 & 2 & 1 & 4 & 0 & 1 \\ 2 & 2 & 4 & 5 & 1 & 2 \end{pmatrix} \end{array}$$

The non- zero diagonal entries in S^3 imply that a clique exists in this family consisting of the three girls.

30% Level of Influence

$$M = \begin{array}{c} \\ F \\ M \\ D_1 \\ D_2 \\ S \\ D_3 \end{array} \begin{array}{c} \begin{array}{cccccc} F & M & D_1 & D_2 & S & D_3 \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

Since some family members influence each other both ways, clique matrix can be worked out.

$$S = \begin{array}{c} \\ F \\ M \\ D_1 \\ D_2 \\ S \\ D_3 \end{array} \begin{array}{c} \begin{array}{cccccc} F & M & D_1 & D_2 & S & D_3 \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

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$$S^3 = \begin{array}{c} \\ F \\ M \\ D_1 \\ D_2 \\ S \\ D_3 \end{array} \begin{array}{c} \begin{array}{cccccc} F & M & D_1 & D_2 & S & D_3 \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

There is no clique at 30% level of influence as elements in the leading diagonal of S^3 are all zeros. We therefore cannot get cliques at higher levels of influence no family members would influence each other both ways. Dominance does not exist as the condition that for any distinct pair of family members either member influences the other but not both does not hold.

Family 6

This family consists of the father, mother, son and the two daughters. The influence table for this family is as follows.

Table 7

	F	**M**	**S**	**D_1**	**D_2**
A1	D_2	D_1	D_2	D_2	D_1
A2	M	D_1	D_2	S	M
A3	D_2	D_1	M	D_2	M
A4	M	F	F	F	D_1
A5	-	-	D_2	S	S
A6	M	-	M	M	M
A7	-	-	F	S	M
A8	-	-	F	F	M
A9	M	-	M	F	F
A10	D_2	D_2	F	D_2	M

20% Level of Influence

The vertex matrix at 20% level of influence is

$$M = \begin{array}{c} \\ F \\ M \\ S \\ D_1 \\ D_2 \end{array} \begin{array}{c} \begin{array}{ccccc} F & M & S & D_1 & D_2 \end{array} \\ \begin{pmatrix} 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \end{pmatrix} \end{array}$$

Dominance can be worked out since, for any distinct pair of family members, one member influences the other but not both. We therefore need to work out $M+M^2$.

$$M^2 + M = \begin{array}{c} \\ F \\ M \\ S \\ D_1 \\ D_2 \end{array} \begin{array}{c} \begin{array}{ccccc} F & M & S & D_1 & D_2 \end{array} \\ \begin{pmatrix} 0 & 1 & 1 & 2 & 0 \\ 2 & 0 & 3 & 3 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 2 & 3 & 0 \end{pmatrix} \end{array} \quad \begin{array}{c} \text{Row sums} \\ 4 \\ 9 \\ 2 \\ 4 \\ 7 \end{array}$$

The second row has the largest row sum. This implies that the vertex M in the corresponding directed graph has the largest total number of 1-step and 2-step connections to any other vertex. Thus, the mother is the most influential person in this family.

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30% Level of Influence

$$M = \begin{array}{c} \\ F \\ M \\ S \\ D_1 \\ D_2 \end{array} \begin{array}{c} F \quad M \quad S \quad D_1 \quad D_2 \\ \begin{pmatrix} 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \end{pmatrix} \end{array}$$

This is the same as at 20% level of influence.

40% Level of Influence

$$M = \begin{array}{c} \\ F \\ M \\ S \\ D_1 \\ D_2 \end{array} \begin{array}{c} F \quad M \quad S \quad D_1 \quad D_2 \\ \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

As no family members influence each other both ways, no clique can be found at this and higher levels of influence. Dominance cannot be worked out since the condition for dominance, that for any distinct pair of family members, either member influences the other but not both, is not satisfied

Family 7

This family which consisted of the father, mother two sons and a daughter yielded the following influence table.

Table 8

	F	**M**	**S_1**	**S_2**	**D**
A1	S_1	S_1	S_2	D	-
A2	M	F	F	F	M
A3	M	S_1	M	M	-
A4	-	-	-	S_1	S_1
A5	-	-	-	-	S_2
A6	M	F	M	-	F
A7	-	-	M	-	-
A8	-	-	M	-	-
A9	M	F	M	-	-
A10	D	S_1	D	-	-

20% Level of influence

$$M = \begin{array}{c} \\ F \\ M \\ S \\ D_1 \\ D_2 \end{array} \begin{array}{c} F \quad M \quad S_1 \quad S_2 \quad D \\ \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

Since some family members influence each other both ways we can work out the clique matrix.

$$S = \begin{array}{c} \\ F \\ M \\ S_1 \\ S_2 \\ D \end{array} \begin{array}{c} F \quad M \quad S_1 \quad S_2 \quad D \\ \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

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$$S^3 = \begin{array}{c} \\ F \\ M \\ S_1 \\ S_2 \\ D \end{array} \begin{array}{c} \begin{array}{ccccc} F & M & S_1 & S_2 & D \end{array} \\ \begin{pmatrix} 0 & 2 & 0 & 0 & 0 \\ 2 & 0 & 2 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

There exists no clique in this family as the elements in the leading diagonal of S^3 are all zeros. Non-existence of a clique at this level implies that there would be clique even at higher levels of influence. Dominance cannot be worked out since some family members influence each other both ways and for dominance to exist, for any distinct pair of family members, either member influences the other but not both influence each other.

Family 8

This family is made up of the father, mother, and three daughters. It yielded the following influence table.

Table 9

	F	**M**	**D_1**	**D_2**	**D_3**
A1	D_3	D_2	D_2	D_3	D_2
A2	D_3	D_1	D_1	D1	D_1
A3	D_3	D_2	D_3	D_3	M
A4	-	F	M	D_1	D_1
A5	-	-	M	D_1	D_1
A6	M	F	M	M	M
A7	-	F	F	F	M
A8	-	-	F	F	D_1
A9	-	-	F	F	M
A10	D_3	D_2	M	D_3	F

At 20% Level of Influence, the vertex matrix M is

$$M = \begin{array}{c} \\ F \\ M \\ D_1 \\ D_2 \\ D_3 \end{array} \begin{array}{c} \begin{array}{ccccc} F & M & D_1 & D_2 & D_3 \end{array} \\ \begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

For any distinct pair of family members, either member influences the other but the two do not influence each other both ways. Dominance can therefore be worked out. We need to work out $M^2 + M$ in order to find out the most influential (powerful) person in the family.

$$M^2 + M = \begin{array}{c} \\ F \\ M \\ D_1 \\ D_2 \\ D_3 \end{array} \begin{array}{c} \begin{array}{ccccc} F & M & D_1 & D_2 & D_3 \end{array} \\ \begin{pmatrix} 0 & 2 & 2 & 2 & 2 \\ 1 & 0 & 1 & 2 & 2 \\ 1 & 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 2 & 1 & 2 & 0 \end{pmatrix} \end{array} \quad \begin{array}{c} \textit{Row Sum} \\ 8 \\ 6 \\ 5 \\ 3 \\ 6 \end{array}$$

The father is the most influential (powerful) person in the family as the first row has the largest row sum. A clique does not exist as the family members do not influence each other both ways.

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30% Level of Influence

$$M = \begin{array}{c} \\ F \\ M \\ D_1 \\ D_2 \\ D_3 \end{array} \begin{array}{c} F \; M \; D_1 \; D_2 \; D_3 \\ \begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

This vertex matrix is the same as at 20% Level of Influence Dominance therefore exists and the father is the most influential member of the family.

40% Level of Influence

$$M = \begin{array}{c} \\ F \\ M \\ D_1 \\ D_2 \\ D_3 \end{array} \begin{array}{c} F \; M \; D_1 \; D_2 \; D_3 \\ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

Dominance now fails as some family members are neither influenced nor influence any other family member, for example, $D_{2.}$ Since a clique failed at lower levels of influence, we cannot get one at this level.

Family 9

Family 9 consisted of the father, mother, two sons and two daughters and generated the following influence table.

Table 10

	F	M	S_1	D_1	D_2	S_2
A1	S_1	S_1	D_1	S_2	S_2	D_2
A2	M	F	F	S_1	D_1	D_2
A3	D_1	D_1	D_1	D_2	M	D_2
A4	M	F	D_1	S_1	S_1	S_1
A5	-	S_1	-	S_1	S_1	S_1
A6	M	-	M	M	M	M
A7	-	-	F	F	M	F
A8	-	-	F	F	M	S_1
A9	-	F	M	-	F	F
A10	S_1	S_1	F	F	F	F

20% Level of Influence

$$M = \begin{array}{c} \\ F \\ M \\ S_1 \\ D_1 \\ D_2 \\ S_2 \end{array} \begin{array}{c} F \; M \; S_1 \; D_1 \; D_2 \; S_2 \\ \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

Since some family members influence each other both ways, clique matrix can be worked out.

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$$S = \begin{array}{c} \\ F \\ M \\ S_1 \\ D_1 \\ D_2 \\ S_2 \end{array} \begin{array}{c} F \; M \; S_1 \; D_1 \; D_2 \; S_2 \\ \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

$$S^3 = \begin{array}{c} \\ F \\ M \\ S_1 \\ D_1 \\ D_2 \\ S_2 \end{array} \begin{array}{c} F \; M \; S_1 \; D_1 \; D_2 \; S_2 \\ \begin{pmatrix} 2 & 3 & 4 & 1 & 0 & 0 \\ 3 & 2 & 4 & 1 & 0 & 0 \\ 4 & 4 & 2 & 3 & 0 & 0 \\ 1 & 1 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

The non-zero entries in the leading diagonal imply that there is a clique in this family made up of father, mother and the eldest son .We cannot work out dominance as some family members influence each other both ways.

30% Level of Influence

$$M = \begin{array}{c} \\ F \\ M \\ S_1 \\ D_1 \\ D_2 \\ S_2 \end{array} \begin{array}{c} F \; M \; S_1 \; D_1 \; D_2 \; S_2 \\ \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

Since some family members influence each other both ways, then we determine if a clique exists. We work out the clique matrix, S

$$S = \begin{array}{c} \\ F \\ M \\ S_1 \\ D_1 \\ D_2 \\ S_2 \end{array} \begin{array}{c} F \; M \; S_1 \; D_1 \; D_2 \; S_2 \\ \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

$$S^3 = \begin{array}{c} \\ F \\ M \\ S_1 \\ D_1 \\ D_2 \\ S_2 \end{array} \begin{array}{c} F \; M \; S_1 \; D_1 \; D_2 \; S_2 \\ \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

There is no clique in this family at 30% level of influence and therefore no clique at higher influence levels. Dominance does not exist as some members influence each other both ways. As the existence of a clique and dominance fail at this level of influence, it is not possible to obtain them at higher levels of influence.

Family 10

Family 9 consisted of the father, mother, two sons and two daughters and generated the following influence table.

Table 11

	F	**M**	**S_1**	**S_2**	**D**
A1	S_2	S_1	S_2	D	S_2
A2	S_1	F	D	S_1	D

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A3	M	F	S_2	F	-
A4	-	F	F	S_1	S_1
A5	-	-	F	S_1	M
A6	-	F	M	-	M
A7	-	-	F	S_1	M
A8	-	-	D	S_1	M
A9	-	F	M	M	M
A10	S_1	D	D	S_1	F

20% Level of Influence

$$M = \begin{array}{c} \\ F \\ M \\ S_1 \\ S_2 \\ D \end{array} \begin{array}{c} F \; M \; S_1 \; S_2 \; D \\ \begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

We need to work out if a clique exists since some family members influence each other both ways.

$$S = \begin{array}{c} \\ F \\ M \\ S_1 \\ S_2 \\ D \end{array} \begin{array}{c} F \; M \; S_1 \; S_2 \; D \\ \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

$$S^3 = \begin{array}{c} \\ F \\ M \\ S_1 \\ S_2 \\ D \end{array} \begin{array}{c} F \; M \; S_1 \; S_2 \; D \\ \begin{pmatrix} 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 2 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

No clique exists in this family at this level of influence and therefore no clique exists even at higher influence levels. Dominance cannot be worked out as some family members influence each other both ways. There is no need to work out the existence of clique or dominance at higher levels of influence since they failed at this lower level.

IV. GENERAL OBSERVATION

From the ten families investigated, at 20% level of influence, it was found that three had cliques; three had a dominant (powerful) family member while the other four had neither cliques nor dominance.

Cliques were found in:

i) Family 3 which had two cliques, one comprised of the father, mother and the son, and the other comprised of mother, son and the eldest daughter

ii) Family 5 with the clique comprised of the three girls in the family

iii) Family 9 with the clique comprised of the father, mother and the eldest son.

No cliques were found at over 20% level of influence.

Dominance was found in:

i) Family 4 which comprised of the father, mother, one daughter and two sons.

The mother was the most dominant figure in this family.

ii) Family 6 which comprised of father, mother, one son and two daughters.

In this family, the mother was the most dominant figure.

iii) Family 8 which comprised of the father, mother, and three daughters.

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In this family, the father was the most dominant figure.

In family 6 and family 8, dominance was present at both 20% and 30% levels of influence but in family 4, it was found at only 20% influence level. There was no dominance above 30% influence level. The mother was the most dominant figure at both 20% and 30% levels of influence in family 8 while the father was the most dominant figure at the same influence levels in family 6.

Females were generally found to be the majority in the families where cliques and dominance were found. The mother was found to be present in almost every clique. She was also found to be the most influential family member in two of the three cases where dominance was found. This shows that in most families in Ragati village, the mother has a lot of influence on the members of the family. This appears to confirm the belief that Nyeri women are domineering.

V. RECOMMENDATIONS FOR FURTHER RESEARCH

i) Since this research was done in a small village in Nyeri, it is recommended that the project is expanded to the larger Nyeri.

ii) Influence levels should be restricted to 20% and 30% since there were no cliques and dominance above 30% influence level. This is because there were very few instances of influence of 40% and above. This was only found in family 6 and family 8.

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