Manual determination of the number of theoretical plates by MCCAB-Theil and by using MATLAB

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Abstract: Determination of the number of theoretical stages in stage wise distillation is rather important for the design of a fractionator’s column. There are many graphical and empirical correlations for the determination of the theoretical stages, among these methods, is the graphical method of MCcabe-theile which is reliable. In a recent work by Gasmelsed G. A. etal it is approved that McCabe-Theile can be used for multi component fractionation, taking the light and heavy keys as pseudo binary. McCabe-Thiele method requires the condition of the feed , the equilibrium data or relationship , the composition of the feed , top and bottom products mol fractions as well as the q – line. In this paper the number of theoretical stage were plotted manually by McCabe-Theile method and by software using MATLAB. The results of the number of stage were compared and found to by in a agreement. It is recommended that MATLAB software program has to be extended for a complete design of the fractionation column.

Keywords: Binary, Pseudo system, McCabe-Theile, Matlab.

I. INTRODUCTION
Distillation columns used to separate a mixture of 2 liquids or more depending on their relative volatility it is important to estimate the number of the stages that needed for pure distillate.

II. ESTIMATION OF MINIMUM NUMBER OF STAGES

Fig (1) A Distillation column

Fenske Equation
Fenske (1932) was the first to derive an Equation to calculate the minimum number of trays for multi-component distillation at total reflux. The derivation was based on the assumptions that the stages are equilibrium stages and can be written as
Where $N_{\text{min}}$ is the minimum number of stage at total reflux, including the reboiler $\alpha_{LK}$ is the average relative volatility of the component with respect to reference component. $x_{Lk}$, $x_{HK}$ the mole fraction of the light key and heavy key respectively in top and bottom of the column. Ponchon-Savarit Method Background Principle is concerned with the graphical analysis of calculating the theoretical stages by enthalpy balance required for the desired separation by distillation process (Hines and Maddox, 1984). In this method, the enthalpy balances are incorporated as an integral part of the calculation, this procedure combines the material balance calculations with enthalpy balance calculations. This method also provides the information on the condenser and reboiler duties. 

McCabe and Thiele graphical Method

The simplifying assumptions of constant molar heat of vaporization, no heat losses, and no heat of mixing, lead to a constant molar reflux flow in any section of the column. Using these simplifications, thus three ways of determining the equilibrium relationship between liquid and vapor phases which are first if a constant relative volatility $\alpha$. $\alpha_{AB} = \frac{k_A}{k_B} = \frac{y_A}{x_A} / \frac{y_B}{x_b}$ where $k_A$ and $k_b$ are the volatility of A and B respectively is assumed, the vapor composition $y_A$ can be determined at a given liquid composition $x_A$ by the relationship as follows:

$$y_A = \frac{\alpha_{AB}}{1 + x_A(\alpha_{AB} - 1)}$$

(2)

The two enrichment equations are obtained:

For rectifying section

$$V_n = V_{n+1} = V$$

(3)

$$L_n = L_{n+1} = L$$

(4)

$$y_n = \frac{L_n}{V_n} x_{n+1} + \frac{D}{V_n} x_d$$

(5)

For stripping section

$$V_m = V_{m+1} = V$$

(6)

$$L_m = L_{m-1} = L$$

(7)

$$y_m = \frac{L_m}{V_m} x_{m+1} + \frac{W}{V_m} x_w$$

(8)

McCabe-Thiele pointed out that, since these equations represent straight lines connecting on, they can be drawn on the same diagram $y_n$ with $x_{n+1}$ and $y_m$ with $x_{m+1}$ as equilibrium curve to give a simple graphical solution for the number of stages required.

Thus, if $x_{n+1} = x_d$ in equation (5) then

$$y_n = \frac{L_n}{V_n} x_d + \frac{D}{V_n} x_d = x_d$$

(9)

This equation represents straight line passing through the point $y_n = x_{n+1} = x_d$. If $x_{n+1}$ is put equal to zero, then $y_n = \frac{D}{V_n}$ giving a second easily determined point. The top operating line is therefore drawn...
through two points coordinates \( A(x_d, x_{d'}) \) and \( C(0, \frac{D_{ad}}{V_n}) \). For the bottom operating line, \( x_{m+1} = x_w \), then

\[
y_m = \frac{L_m}{V_m} x_w - \frac{W}{V_m} x_w.
\]

\[(10)\]

\[Fig (2) \text{ represent the trays of a distillation column.}\]

Since \( V_m = L_m - W \), it follows that \( y_m = x_w \). Thus the bottom operating line passes through the point C, that is \((x_u, x_u)\), and has a slope of \( \frac{L_m}{V_m} \). Then the two operating lines have been drawn in they intersected at point B together with q-line which is pass through the point \((x_i, x_i)\) and have slope of \(-q/(1-q)\). See fig(3) Secondly thermodynamic models can used to determine the equilibrium relationship. If the components A and B form an ideal mixture, and the vapor pressure for the pure compounds. \( P_{i_{\text{sat}}} \) can be obtained from Antoine equation \[(2)\]

\[
\log P_{i_{\text{sat}}} = A_i - \frac{B_i}{t + C_i}
\]

\[(11)\]

Finally, the liquid and vapor composition, \( x_A \) and \( y_A \) can be directly read from actual data with the liquid and vapor composition in equilibrium \( x_A \) and \( y_A \) equilibrium curve is plotted. Where \( A_i \), \( B_i \), and \( C_i \) are Antoine constant for component \( i \). These equations are used to calculate the relation between the
composition of the liquid on a plate and the composition of the vapor rising to that plate. By any method of those three, drawing equilibrium curve and operating lines, the number of stages required maybe found by

\[ a \quad \text{draw a horizontal line from } (x_D, x_D) \text{ to the equilibrium curve} \]

\[ b \quad \text{Drop a vertical line to the operating line} \]

\[ c \quad \text{Repeat } a \text{ and } b \text{ until } x \text{ reach } x_b \]

The operating lines in rectifying and stripping section intersected in the same point together with the q-line and the q-line joints the point \((x_f, x_b)\) in line 45 with the intersection of operating lines with slope of \(q/(1 - q)\) as shown in fig (4) which represent the q-line with different values of q. the above is the theoretical side of the main program idea. next is a problem for testing the program.

**III. PROBLEM**

A continuous fractionating column is to be designed to separate 30000kg/h of a mixture of 40% benzene and 60% toluene into an overhead product containing 97% benzene and a bottom product containing 98% toluene. These percentages are by weight. a reflux ratio of 3.5 is used. Benzene an toluene form an ideal system with a relative volatility of about 2.5; the feed is saturated liquid at its boiling point at pressure of one atm. Determine the number of ideal plates and the position of feed plate.

**IV. SOLUTION**

The molecular weight of benzene is 78 and that for toluene is 92. the concentration of feed \((x_f)\), overhead \((x_D)\) , and bottom \((x_B)\) respectively in mole fraction of benzene are:-

\[ x_f = \frac{40}{78 + 60} = 0.44 \]

\[ x_D = \frac{97}{78 + 3} = 0.974 \]

\[ x_B = \frac{2}{78 + 98} = 0.0235 \]
The feed condition saturated liquid at its boiling point so \( q = 1 \) relative volatility is given and equal to 2.5. The reflux ratio is 3.5 so we feed these data on the program the number of deal plates is found to be 12 and feed plate is number 6 from top see fig(5) below.

V. CONCLUSION

It is concluded that the number of theoretical plates for both binary and multi-component systems can confidently be determined using McCabe-Theile graphical method. As the graphical method can be inserted into a computer program to obtain the compositions profile, a program was developed using MATLAB. The program requires the equilibrium data as well as the composition of the feed, top and bottom mole fractions with the \( q \)-line values and the reflux ratio. The method compared quite well and the results using the program and manual technique are in very good agreement. The program can further be extended to determine the complete design parameters of a fractional column. The code is available upon request from the authors.

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REFERENCES


AUTHOR BIOGRAPHY

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APPENDIX

Fig (4) operating lines and feed line intersected at a, b, c, d and e with different values of \( q \).
Fig (5) $q=1, N_{\text{min}}=12$ program output for problem1 above

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