Abstract — Generally, the attitude and torque estimation and the measurement of these parameters are a requirement for the attitude control. As a result, the computational cost and the complexity of the control loop are relatively high. This paper presents a Linear Quadratic Regulator (LQR) employed to improve performance of an Electrical Power Steering (EPS) system applied in an autonomous mobile robot. In the case of the attitude estimation, one seeks to estimate the attitude and accelerations of a rigid body. Generally, EPS is a full electric system having an electrical motor which provides the assist torque on the steering mechanism in order to reduce the workload and to enhance the steering feel of the driver during steering process. Since the torque sensors are considerably expensive, the authors present an estimation strategy that eliminates the driver torque sensor by introducing a torque estimator. Three main technical areas are described in this paper. First, the principle and structure of EPS are presented including the dynamic model. Second, LQR and Kalman filter techniques are employed to derive an optimal controller for the EPS system. Finally, the simulations results are depicted.

Index Terms — Robot, Electric Power Steering, Linear Quadratic Regulator, Optimal control, Robust control.

I. INTRODUCTION

A steering system is a significant subsystem for an autonomous robotic system operation [1]. Since considerable steering effort is required with the increase vehicle weight and parking convenience for maneuvers, a power steering system was introduced to assist the drivers in turning the steering wheel in such driving conditions. Most power steering systems are hydraulic, which use a pump to supply hydraulic pressure.

Compared to the traditional hydraulic power steering on device, the EPS system eliminates the need for a hydraulic pump, hoses, hydraulic fluid, drive belt and pulley on the engine, therefore the total system is lighter. EPS has the advantages of safety, energy saving, and environmental protection. It has become the mainstream of power steering technology for passenger cars because the important role of the enhancement of handling stability and safety. Additional EPS features like active damping, active return, friction compensation, and angle finding logic are considered advantages for the driver.

Many papers on EPS topics have been published so far. Chen and Chen [2] used Newton’s law to build the dynamic model while Parmar and Hung [3] utilized the Lagrange’s equations to construct the dynamic equation of the EPS system. The authors present a strategy that eliminates the steering column torque sensor, a critical component in existing EPS controller designs. Liao and Du [4] combined Matlab/Simulink and Adams to simulate the behavior of the EPS system on the vehicle motion. In [1] associated with Matlab/Simulink to describe the effect of power electronics on the EPS system. The stability of the EPS system is analyzed by Li and Wenjiang on the basis of the mathematics model for pinion and rack steering system. A fuzzy control method [5] for actively reducing pre-sure ripples for EPS system is proposed by Li et al.

The major purpose of this research is to develop an EPS controller for an electrical vehicle (autonomous robot). The mechanical model of EPS is based on single pinion architecture suitable for light vehicles [5]-[6] and consists of following elements: a steering rack, a steering column coupled to the steering rack through a pinion gear, and the assist motor. Tie-rods connect the steering rack to the tires.
The content of this paper is organized as follows: Section II and III describe a state-space formulation that incorporates the dynamic model of the steering mechanism. Section IV illustrates the LQR estimator techniques to arrive at an optimal controller for the EPS system, comparison and analysis of the simulation data are discussed in Section V. Finally, the summary and conclusions of this paper are presented in Section VI.

II. MATHEMATICAL BACKGROUND

Consider two orthogonal right-handed coordinate frames: the body coordinate frame, $E^b = [e_1^b, e_2^b, e_3^b]$, located at the center of mass of the rigid body and the inertial coordinate frame, $E^f = [e_1^f, e_2^f, e_3^f]$, located at some point in the space. The rotation of the body frame $E^b$ with respect to the fixed frame $E^f$ is represented by a quaternion, consisting of a unit vector $\vec{e}$, known as the Euler axis, and a rotation angle $\beta$ about this axis. The quaternion $q$ is then defined as follows:

$$q = \left(\cos\frac{\beta}{2}, \frac{\beta}{2} \right) = \left(\begin{array}{c} q_0 \\ \vec{\theta} \end{array}\right) \in H$$

Where

$$H = \{ q | \|q\| = 1 \}$$

$$q = \left[q_0, -\vec{q}^T\right]^T, \quad q_0 \in \mathbb{R}, \vec{q} \in \mathbb{R}^3$$

$$\vec{q} = [q_3 q_2 q_1]^T$$

and $q_0$ are known as the vector and scalar parties of the quaternion respectively.

In attitude control applications, the unit quaternion represents the rotation from an inertial coordinate system $\mathcal{N}(x_N, y_N, z_N)$ located at some point in the space (for instance, the earth NED frame), to the body coordinate system $\mathcal{B}(x_B, y_B, z_B)$ located on the center of mass of a rigid body.

If $\vec{r}$ is a vector expressed in $\mathcal{N}$, then its coordinates in $\mathcal{B}$ are expressed by:

$$\vec{b} = \vec{q} \otimes \vec{r} \otimes \vec{q}$$

Where $\vec{b} = [0 \vec{b}^T]^T$ and $\vec{r} = [0 \vec{r}^T]^T$ are the quaternions associated to vectors $\vec{b}$ and $\vec{r}$ respectively. $\otimes$ denotes the quaternion multiplication and $\vec{q}$ is the conjugate quaternion multiplication of $q$, defined as:

$$\vec{q} = [q_0, -\vec{q}^T]^T$$

The rotation matrix $C(q)$ corresponding to the attitude quaternion $q$, is computed as:

$$C(q) = (q_0 \vec{e} - \vec{q}^T \vec{q})I_2 + 2(\vec{q} \vec{q}^T - q_0 [\vec{q}^2])$$

Where $I_2$ is the identity matrix and $[\vec{e} \vec{e}]$ is a skew symmetric tensor associated with the axis vector $\vec{e}$:

$$[\vec{e} \vec{e}] = \begin{pmatrix} 0 & -\xi_3 & \xi_2 \\ \xi_3 & 0 & -\xi_1 \\ -\xi_2 & \xi_1 & 0 \end{pmatrix}$$

Thus, the coordinate of vector $\vec{r}$ expressed in the $\mathcal{B}$ frame is given by:

$$\vec{b} = C(q)\vec{r}$$
The rotation matrix $C(q)$ corresponding to the attitude quaternion $q$, is computed as

$$C(q) = (q_0^2 - q^T q)I_3 + 2(q_0 [q] - q_0 [q])$$

where $I_3$ is the identity matrix and $[\xi]$ is the skew symmetric tensor associated with the axial vector $\xi$.

Denoting by $\overline{\omega} = [\omega_1 \omega_2 \omega_3]$ the angular velocity vector of the body frame B relative to the inertial frame N, expressed in B, the kinematics equation is given by:

$$\begin{bmatrix} \dot{q}_0 \\ \dot{q} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 \\ q \end{bmatrix} \overline{\omega}$$

The attitude error is used to quantify the mismatch between two attitudes. If $\hat{q}$ defines the current attitude quaternion and $q_d$ is the reference quaternion, i.e. the desired orientation, then the error quaternion that represents the attitude error between the current orientation and the desired one is given by

$$q_e = q \otimes q_d^{-1}$$

$\otimes$ denotes the quaternion multiplication and $q_d^{-1}$ is the complementary rotation of the quaternion $q_d$, which is the quaternion conjugate (see ([8]) for more details).

The attitude dynamics of a rigid body is described by

$$J \ddot{\phi} = -\overline{\omega} \times J \overline{\omega} + J$$

where $J \in \mathbb{R}^{3 \times 3}$ is the symmetric positive definite constant inertial matrix of the rigid body expressed in the B frame and $J \in \mathbb{R}^{3}$ is the vector of control torques. Note that this torques also depend on the environmental disturbance torques (aerodynamic, gravity gradient, etc.).

### III. PROBLEM STATEMENT

In the case of the attitude estimation, one seeks to estimate the attitude and accelerations of a rigid body [7] (figure 1). From now on it is assumed that the system is equipped with a tri axis accelerometer, three magnetometer and three rate gyros mounted orthogonally.
and for the assist motor the equation reads as

\[ J_M \cdot \ddot{\varphi}_M (t) = M_{MOR} (t) - c_M \cdot (\varphi_M (t) - \varphi_{LS} (t)) \cdot i_{motsc} \\
- d_M \cdot (\ddot{\varphi}_M (t) - \varphi_{LS} (t)) \cdot i_{motsc}. \tag{13} \]

The steering column part determines the equation

\[ J_{LS} \cdot \ddot{\varphi}_{LS} (t) = F_{RACK} (t) \cdot i_{rsc} - c_s \cdot (\varphi_{LS} (t) - \varphi_{SW} (t)) - d_s \cdot (\ddot{\varphi}_{LS} (t) - \varphi_{SW} (t)) - c_M \cdot (\varphi_{HLS} (t) \cdot i_{motsc} - \varphi_M (t)) \\\n- d_M \cdot (\ddot{\varphi}_{LS} (t) \cdot i_{motsc} - \varphi_M (t)). \tag{14} \]

The equation of tire loads is given by

\[ F_{RACK} (t) = -c_{SPS} \cdot x_R (t) - d_{SPS} \cdot \dot{x}_R (t) \tag{15} \]

and the rack movement can be expressed as

\[ x_R (t) = \varphi_{LS} (t) \cdot i_{rsc}. \tag{16} \]

Inserting the formulas (15) and (16) into (14), equation (14) can be rewritten as follows:

\[ J_{LS} \cdot \ddot{\varphi}_{LS} = -c_{SPS} \cdot \varphi_{LS} (t) \cdot i_{rsc} - d_{SPS} \cdot \varphi_{LS} (t) \cdot i_{rsc} - c_s \cdot (\varphi_{LS} (t) - \varphi_{SW} (t)) - d_s \cdot (\varphi_{LS} (t) - \varphi_{SW} (t)) \\
- c_M \cdot (\varphi_{HLS} (t) \cdot i_{motsc} - \varphi_M (t)) - d_M \cdot (\ddot{\varphi}_{LS} (t) \cdot i_{motsc} - \varphi_M (t)). \tag{17} \]

The linear EPS system is of the sixth-order and can be expresses in the state-space form

\[
\begin{align*}
\dot{x}(t) &= A \cdot x(t) + B \cdot u(t) + n_1(t) \\
y(t) &= C \cdot x(t) + D \cdot u(t) + n_2(t),
\end{align*}
\]

Where n1(t) and n2(t) are the random measurement noise terms for state and output, respectively. These signals are assumed to be stochastic Gaussian processes and are due to the noise in the sensor measurements and the disturbances that are transmitted from the road. The noise covariance has to be specified for Kalman filter design and will vary according to road conditions and vehicle type. The state assignment of the EPS derived model is

\[ x(t) = [\varphi_{SW} (t) \varphi_{LS} (t) \varphi_{SW} (t) \varphi_{LS} (t) \varphi_{M} (t) \varphi_{M} (t)]. \]

IV. TORQUE ESTIMATOR

Since the driver torque is not measured in line [09] [10], we introduce an estimator for \( \Gamma_{MOR} \). Essentially, the estimated value of the driver torque is
Where \( \tau_{Z} \) is the torque in the steering column part and \( \tau_{\text{Mot}} \) is the assist motor torque. In order that \( G^{-1} \) can be physically realizable (numerator degree of the transfer function is always less or equal than denominator degree), it is necessary to introduce a correction transfer function \( G_c(z) \) to maintain the properness. With this correction, the inverse transfer function becomes

\[
G_{\text{inv}} = G^{-1} \cdot G_c(z) \cdot \{ \tau_{Z}(z) - H(z), \tau_{\text{Mot}}(z) \}
\]

In order to improve the driver’s feel the desired motor assist motor current is stored in lookup tables for two different behaviors, named “Standard UAV curves” and “Sport UAV curves”. Totally, there are twenty six curves per standard or sport drive mode at different vehicle speeds selected by the signal. Finally, the determined motor current is multiplied by the motor constant \( k \) generating the motor torque. For this part of the estimation. The state assignment of the derived model is

\[
\mathbf{x}(t) = [\omega_{\text{input}}, \omega_{\text{val}}, \omega_{\text{motor}}, \dot{\omega}_{\text{input}}, \dot{\omega}_{\text{val}}, \dot{\omega}_{\text{motor}}]^T
\]

The motor torque and the applied driver torque are considered as the inputs to the system \[ [\tau(t), \tau_{\text{app}}(t)]^T \]. Output of the multi-input system is considered to be the column torque. The output and the two inputs of the system shown in Figure 2 are fed into an observer to generate \( \hat{\mathbf{x}}(k) \) an estimate of the system state. A Kalman filter is designed to obtain the required observer dynamics in the presence of stochastic disturbances. The state equation for the Kalman observer reads as.

\[
\hat{x}(k+1) = A_d \hat{x}(k) + B_d \hat{\mathbf{u}}(k) + L_d(y_k - \hat{y}_k)
\]

where \( L_d \) is the estimator gain matrix and \( \hat{y}_k \) is the estimated output. The equations of the closed loop UAV system including the observer are derived based on the assumption that, for simplicity, \( \Gamma_{\text{est}}(k) \) is approximately equal to \( \Gamma_{\text{app}}(k) \) and the random measurement noise terms are neglected. The formulas become

\[
\begin{align*}
\text{Plant:} & \quad \tau(k+1) = A_d x_k + B_{1d} \tau_{\text{Mot}}(k) + B_{2d} y_k \tau_{\text{est}}(k) \\
& \quad y(k) = C_d x_k + D_{1d} \tau_{\text{Mot}}(k) + D_{2d} \tau_{\text{est}}(k) \\
\text{Observer:} & \quad \hat{x}(k+1) = A_d \hat{x}(k) + B_{1d} \mathbf{u}(k) + B_{2d} \hat{\mathbf{y}}(k) \\
& \quad \hat{\mathbf{y}}(k) = C_d \hat{x}(k) + D_{1d} \tau_{\text{Mot}}(k) + D_{2d} \tau_{\text{est}}(k) \\
\text{Control:} & \quad \Gamma_{\text{Mot}}(k) = -K_d \hat{x}(k) \\
\end{align*}
\]

An equivalent closed-loop system can be derived and re-written as equation 27.
The selection of the discrete-time LQ weighting matrices $Q_d$ and $R_d$ is considered as follows.

One should select $Q_d$ to be positive semi-definite and $R_d$ to be positive definite. A reasonable choice for the $Q_d$ matrix is found to be:

$$Q_d = \begin{bmatrix} a_1 & 0 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 & 0 \\ 0 & 0 & 0 & a_4 \end{bmatrix}$$

using an appropriate selection of $a_i$. The $R_d$ matrix is chosen as:

$$R_d = b_i I$$

(28)

Within discrete-time LQR procedure, the optimal gain control matrix $k_d$ calculated such that the state-feedback law $u(k)$ minimizes the quadratic cost function.

$$J = \sum_{k=1}^{n} (x^T(k)Q_d x(k) + R_d I_{neet(k)})$$

(29)

where $Q_d$ and $R_d$ are the state and control weighting matrices. In addition to the state-feedback gain

$$k_d = (B_{1d}^T S_d B_{1d} + R_d)^{-1} (B_{1d}^T A_{d})$$

(30)

The solution $S_d$ of the associated discrete-time Riccati equation.

$$A_{d}^T S_d A_d - S_d - (A_{d}^T S_d B_{1d}) (B_{1d}^T S_d B_{1d} + R_d)^{-1} (B_{1d}^T S_d A_d) + Q_d = 0$$

(31)

and the closed-loop eigen values $Ed = \sigma g(A_d - B_d K_d)$ are returned.

V. EXPERIMENTAL RESULTS

Fig 3. Frequency characteristic curve of EPS system

The resulting control system is simulated in Matlab and the characteristics of the closed-loop system are compared with the open-loop system. Figure 3 displays the frequency response of the EPS system in terms of Bode diagrams proving the stability in closed-loop. The responses of the open-loop and the closed-loop system to an input step at
the driver torque $M_{SW}(t)$ are shown in Figure 4. It is observed, that the settling time of the closed-loop system is considerably lower than the open-loop system. Additionally, in order to prove the stability and robustness of the discrete time LQR controller, closed-loop step responses to driver torque have been plotted in Figure 5 when all model parameters varied with +20% and -20%. Combining Matlab/Simulink, this is used for designing, analyzing and simulating block diagrams models with specific block libraries provided by dSPACE for I/O hardware support, results in a real time functioning control environment.

![Figure 4. Step response of EPS system to driver torque](image)

This is then controlled and monitored interactively through dSPACE Control Desk. This is a comprehensive virtual instrument- oriented experiment environment that allows one to build the system implementation and interface for the DS1401 dSPACE MicroAutoBox.

The combination of the tools: Matlab, Simulink and Control Desk forms a very powerful environment for testing and developing real-time control systems experiment environment that allows one to build the system implementation and interface for the DS1401 dSPACE MicroAutoBox.

![Figure 5. Closed-loop step responses to driver torque at parameters variation](image)

V. CONCLUSIONS

In this paper, the authors presented simplified equations of motion for a single-pinion EPS system. An optimal discrete-time controller using LQR and Kalman filter techniques is designed for the system model along with the implementation of the boost-curves which provide an improved driver steering feel (provided by Volkswagen Mexico). Notable advances offered by the proposed controller are robust stability with respect to significant parameters variation. Furthermore, elimination of the driver torque sensor offers advantages in terms of both cost...
and mechanical performance. Matlab/Simulink simulations are compared and analyzed showing a very good correspondence.

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