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Causal Generalization of the Ohm's Law in Transient Phenomena Quasi-static

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Abstract*—The point form of Ohm's law has limitations when it is applied before the equilibrium is achieved in quasi-static conditions, because it represents a non-causal and instantaneous (static) relationship between the electric field and current density. We have proposed a generalized Ohm's law consistent with causality, under quasi-static conditions and in the absence of time-varying magnetic fields. As a result, emerges a dynamic model with electrostatic memory of the potential gradients applied, which incorporates to the usual Ohm law as a special case, when considering the instantaneous propagation. We have found the possibility of oscillations in the charge density and in the current density, that they may occur in the opposite direction to the one established by the electrical potential gradient, in transient phenomena quasi-static. Also, applications in electrical conductors, in PN semiconductor junctions and in lightning strikes are presented. We conclude that Ohm's law can be generalized for non-equilibrium phenomena, by incorporating causality according to the proposed generalized model.

Index Terms— **Causality, electrical conduction, Ohm's law, transient phenomena.**

I. INTRODUCTION

It is well known that the point form of Ohm's law establishes the proportionality between the conduction current density $\vec{J}$,descriptive of the flow of charge per unit time and per unit of cross-sectional area, and the electric field intensity $\vec{E}$ in the quasi-static case (with a constant magnetic induction $\vec{B}$). It was proposed by Kirchhoff in 1827 [1] and its modern form in quasi- static conditions is

$$\vec{J}(\vec{r},t) = \sigma\vec{E}(\vec{r},t) = -\sigma\vec{\nabla}\varphi \qquad (1)$$

Where σrepresents the electrical conductivity of the material medium and $\varphi$is the scalar electric potential. Historically, this expression was obtained from Ohm's pioneering work, and first appeared in a letter dated October 13, 1848, sent by Kirchhoff to Von Neumann [2]. Note that (1), when applied to any point $\vec{r}$ in homogeneous and isotropic materials under magneto static conditions, expresses an instant connection through the parameter σ, connecting the cause $\vec{E}$ with the consequence or phenomenological effect $\vec{J}$,exactly at the same time instant t for both magnitudes.

The static relationship (1) violates the principle of causality, according to which there must be a delay between cause (potential gradient) and its subsequent effect (current density). This delay or characteristic time is a consequence of the finite propagation speed of the information associated to the gradient of electric potential on its path through the volumetric material with finite dimensions.

In the usual conducting materials this characteristic time is near zero and negligible, but there are scenarios described in this research where its values are significant, justifying the proposal of a new model that generalizes the Ohm's law in (1) to be consistent with causality, in the sense that now includes the corresponding finite delay time of the perturbation through the material.

To date, it has not been observed in the specialized literature any reference over the evident mentioned problem about the non-causality of Ohm's law. However, a similar problem about non-causality has been detected, addressed and resolved in detail for at least two classical equations of elementary physics, analogous mathematically to Ohm's law (1) such as the Fourier's law for heat propagation and the Fick's law for transport theory (particle diffusion), when both are applied to situations before reached the steady state of thermal and diffusive equilibrium, respectively.

The widely known Fourier's law of heat propagation is

$$\vec{F}(\vec{r},t) = -k\,\vec{\nabla}T(\vec{r},t) \qquad (2)$$

where $\vec{F}$ represents the local heat flow density, k the thermal conductivity of the medium and $\vec{\nabla}T$ the temperature gradient. In Fourier's law (3) also emerges the same problem identified that compromises the causality, noticing the same time *t* on both sides of (2), as Cattaneo pointed [3], who modified the equation (2) by incorporating a thermal relaxation time representative of the causal delay due to the finite velocity of propagation of heat [4], [5]. A causal characteristic time, named τ_{cc}, and analogous to thermal relaxation time of Cattaneo, will be used in this investigation for the frame of a new causal generalization of Ohm's law (1).

And with regard to the Fick's law for the transport of particles by diffusion, taken for granted their expression

$$\vec{j}(\vec{r},t) = -D\,\vec{\nabla}c(\vec{r},t) \qquad (3)$$

Where $\vec{j}$ represents the massic current density of the particle flow, D the diffusion coefficient associated with the medium and $\vec{\nabla}c$ the volumetric particle concentration gradient. Notice again the same time instant *t* on both sides of (3). Falcón and Medina observed this problem of non-causality in Fick's law [6], and proposed a causal generalization model applicable before the thermodynamic equilibrium using a characteristic delay time between the cause (the gradient of concentration) and the effect (the current density), in an analogous way as Cattaneo.

Note that Ohm's law (1), Fourier's law (2) and Fick's law (3) are constitutive equations mathematically equivalent (analogous) in modeling different transport processes under the diffusion approximation for the electric charge, the heat and mass, respectively. These three models assume the instantaneous propagation (with infinite velocity) of the disturbance which causes the corresponding current density, which would not be valid before to equilibrium, in the transient state and therefore the aforementioned constitutive equations do not satisfy the principle of causality, which limits its application to certain physical situations.

The purpose of this research is to present a mathematical model to generalize the point form of Ohm's law consistent with causality, applicable to electrical conduction under quasi-static conditions and in the absence of time-varying magnetic field, by incorporating the finite delay between cause (the application of gradient potential), and the effect (establishing a current density). To solve the problem of the non-causality in Ohm's law (1), in Section II a general mathematical model will be constructed, using a similar methodology to the implemented by Cattaneo [3] and Falcón-Medina [6]. As a result emerges an electrostatic memory associated to the material (Section III), now interpreted as a linear dynamic system that cannot respond instantaneously, built around the definition of the causal characteristic time τ_{cc}. The new model replaces the static system defined in (1), by a new second order differential equation whose possible solutions are analyzed, yielding dynamic results for the transient such as the presence of oscillations in which the current density has the direction opposite to the usual direction set by the electrical potential gradient, a situation that is not predicted by (1). Over damped, critically damped and under damped solutions are obtained.

The paradigm under which the study was conducted is based on a quantitative research and the application of the deductive method, carried out in the framework of the classical physics corresponding to the electromagnetic theory in quasi-static conditions, providing a new falsifiable mathematical model as part of the results, and various physical scenarios for its experimental verification. In Section IV there will be presented three application examples where the proposed model is relevant, such as the electrical conduction in good conductors, the atmospheric electric discharges and the transport in semiconductors like the Schottky diode. Finally, conclusions are presented in Section V.

II. CAUSAL GENERAL EXPRESSION OF OHM'S LAW

Ohm's law, expressed as in (1), involves the appearance of the current density $\vec{J}(\vec{r},t)$ in the material at the same instant t at which the potential gradient $\vec{\nabla}\varphi$ is applied, thus violating this causality due to the instantaneous propagation of the disturbance, regardless of the dimensions of the conducting medium.

In this section, a new constitutive relationship between $\vec{E}$ and $\vec{J}$ is proposed, as a modification of (1), but considering now the finite delay time between these two magnitudes, defined as τ_{cc} or "causal characteristic time", so that the new relationship is now consistent with causality avoiding the instantaneous modeling implicit in (1). Under this approach, in quasi-static conditions, and neglecting the effects of polarization of the material medium, (1) becomes now in the proposed equation:

$$\vec{J}(\vec{r},t+\tau_{cc}) = -\sigma\vec{\nabla}\varphi(\vec{r},t) \qquad (4)$$

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notice that the time dependence in φ and in $\vec{J}$ is not the same, because the establishing of $\vec{E}$ (or equivalently of the potential gradient $-\vec{\nabla}\varphi$) at a point $\vec{r}$ of the material, at a given time t, causes the movement of the electric charges, which implies the subsequent effect of a current density $\vec{J}$ in a time $t' = t + \tau_{cc}$, where τ_{cc} is a positive quantity that represents the propagation delay associated with the finite velocity of the disturbance, forcing that causality is satisfied.

The causal characteristic time τ_{cc} is usually small compared to the time it takes an electrical current to travel around the entire macroscopic medium. The order of magnitude of τ_{cc} is similar to the effective transit time of the messenger particles that mediate the electromagnetic interaction (virtual photons), therefore:

$$t' = t + \tau_{cc} \approx t + \left(\frac{n}{c}\right) l \tag{5}$$

where n represents the refractive index, c is the speed of light in vacuum and l is the effective length of the material, it is clear that $\tau_{cc} \lesssim 10^{-10}$ s in material objects with dimensions of the order of some centimeters.

Doing the Taylor series expansion in the left side of (4) and considering that generally $\tau_{cc} \ll 1$ s, is obtained an expression that generalizes Ohm's law and prevents conflict with the causal principle:

$$\tau_{cc}\frac{\partial}{\partial t}\vec{J}(\vec{r},t) + \vec{J}(\vec{r},t) = -\sigma\vec{\nabla}\varphi(\vec{r},t) \tag{6}$$

In most of the applications, the first-order correction used in (6) can be ignored when the equilibrium is reached.

However, certain situations can be illustrated where (6) is important in transients phenomena with τ_{cc} on the order of picoseconds. In this sense, an example occurs in the field of high-speed switching in diodes and other semiconductor devices, as well as on the various response times of the systems to excitations with alternating signals, and in atmospheric electric discharges which occur in intervals on the order of milliseconds (Section III).

Taking the divergence (6), considering the continuity equation for the electric current where $\rho = \rho(\vec{r},t)$ is the electric charge density, and employing the Laplacian gives:

$$\tau_{cc}\frac{\partial^2\rho}{\partial t^2} + \frac{\partial\rho}{\partial t} = \sigma\nabla^2\varphi \tag{7}$$

If the Poisson's equation is used in (7), valid in the quasi-static case, where the permittivity of the medium is ε, is obtained for the density of electric charge:

$$\tau_{cc}\frac{\partial^2\rho}{\partial t^2} + \frac{\partial\rho}{\partial t} = -\frac{\sigma\rho}{\varepsilon} \tag{8}$$

This implies that before occurs the full electrical relaxation, the charge density ρ exhibits a dynamic behavior that is governed by (8). Clearly, the validity of (8) is applicable when it comes to times t less than or of the order of characteristic time τ_{cc}, since for the case of steady state or near it (when $t \gg \tau_{cc}$) the first term in the left side of (8) is negligible and the usual situation is obtained:

$$\frac{\partial\rho}{\partial t} \approx -\frac{\sigma\rho}{\varepsilon} \tag{9}$$

and the classical relation for the relaxation of the charge on the conductive medium is obtained:

$$\rho = \rho_0 e^{-\frac{\sigma}{\varepsilon}t} \approx \rho_0 e^{-\frac{t}{\tau_e}} \tag{10}$$

being τ_e the transitional (relaxation) period, defined as:

$$\tau_e \equiv \frac{\varepsilon}{\sigma} \tag{11}$$

proportional to the time in which the current density gradient is flattened. In a good conductor like silver $\tau_e \approx 10^{-19}$ s, and in a material with an electrical susceptibility χ_e and conductivity σ:

$$\tau_e = \frac{\varepsilon}{\sigma} = \frac{\varepsilon_0(1+\chi_e)}{\sigma} \approx \frac{\varepsilon_0\chi_e}{\sigma} \tag{12}$$

Returning to (6) by dividing it by τ_{cc} and then by taking simultaneously the limits $\tau_{cc} \to \infty$ and $\sigma \to \infty$ such that $\sigma/\tau_{cc} \approx \text{constant}$, is obtained:

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$$\frac{\partial \vec{J}}{\partial t} = -\frac{\sigma}{\tau_{cc}} \vec{\nabla} \varphi \quad (13)$$

If derived with respect to time both members of the continuity equation and if substituted (13) into the result, we obtain:

$$\frac{\partial^2 \rho}{\partial t^2} = -\nabla \cdot \left(-\frac{\sigma}{\tau_{cc}} \vec{\nabla} \varphi\right) \quad (14)$$

using the Poisson's equation in (14) and rearranging terms yields:

$$\frac{\partial^2 \rho}{\partial t^2} + \frac{\sigma}{\varepsilon \tau_{cc}} \rho = 0 \quad (15)$$

representing an oscillation of the current density, which takes place before the equilibrium, when considered the ideal case of non-loss electrical conduction. So, in the transient regime the charge density fluctuates harmonically, with natural frequency:

$$\omega_0 = \sqrt{\frac{\sigma}{\varepsilon \tau_{cc}}} \quad (16)$$

and accordingly, the current density displaces in opposite direction to the one of the potential gradient during the mid-cycle, and in the direction of the potential gradient in the other half of the cycle, and is null instantly in the transient regime. These prescriptions do not appear in the usual modeling, not causal, of the Ohm's Law in (1).

III. ELECTROSTATIC CONDUCTION MEMORY

Next, a general model of electrical conduction in quasi-static transient phenomena is presented, incorporating the history of the potential gradients by solving (6). Multiplying both sides of (6) by the integrating factor $e^{t/\tau_{cc}}$, the solution for $\vec{J}$ in the differential equation is obtained, in the form

$$\vec{J}(\vec{r},t) = -\frac{\sigma}{\tau_{cc}} \int_{-\infty}^{t} \vec{\nabla} \varphi(\vec{r},t') e^{-(t-t')/\tau_{cc}}\, dt' \quad (17)$$

The equation (6) states that the instantaneous current density (measured at instant t) is the result of the historical accumulation of all potential gradients, applied in earlier times to t. Note that the exponential factor in the kernel of the integral at (17) imposes the fact that the most recent gradients mostly contribute to current density when compared to the later ones, in any case this imply that the system can be considered with memory, a typical situation in dynamic systems which are physically unable to respond instantly.

In general, we can be conceived different constitutive models characterizing the conduction story or memory of the ohmic material, through the generalization of a functional $K(t,t')$ in the integral kernel of (17):

$$\vec{J}(\vec{r},t) = -\frac{\sigma}{\tau_{cc}} \int_{-\infty}^{t} K(t,t') \vec{\nabla} \varphi(\vec{r},t')\, dt' \quad (18)$$

Thus, the original Ohm's law is obtained according to (1) when it is assumed:

$$K(t,t') = \delta(t-t') \quad (19)$$

If $K(t,t')$is assumed constant:

$$K(t,t') = K_0 \quad (19)$$

Equation (20) would correspond to the special case in which all potential gradients contribute with equal weighting to the electrical conduction; this corresponds to the case discussed in (15).

Obviously, if the kernel is of the form:

$$K(t,t') = e^{-(t-t')/\tau_{cc}} \quad (21)$$

is obtained (17) again.

The different constitutive models given in expressions (19), (20) and (21) can be used, as discussed in the next section, to describe the transient regime of the current density in certain particular physical situations.

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IV. RESULTS AND DISCUSSION

Considering the homogeneous differential equation (12), the general solution for the charge density is determined, subject to initial conditions $\rho(0) = \rho_0$ and $\dot{\rho}(0) = \dot{\rho}_0$. Using the Laplace transform is obtained the pair of roots s_1 and s_2 of the characteristic equation of the system, in the form:

$$s_{1,2} = -\frac{1}{2\tau_{cc}}\left[1 \pm \left(1 - \frac{4\tau_{cc}}{\tau_\varepsilon}\right)^{\frac{1}{2}}\right] \tag{22}$$

Where $s_{1,2} \equiv s_1$ and $s_{1,2} \equiv s_2$ are the roots corresponding to the plus and minus sign in (22), respectively.

Depending on the values acquired by the roots in (22) (real roots or complex-conjugated roots, according to the value taken by the term $\tau_{cc}/\tau_\varepsilon$), three possible cases for the dynamics of the transient regime of the charge density ρ are presented: Over damped (different real roots), critically damped (identical real roots) and under damped (complex conjugate roots), discussed below:

A. Over damped case: $\tau_{cc}/\tau_\varepsilon < 1/4$***, the solution is obtained:***

$$\rho(t) = \rho_0\left[\frac{1}{2}\left(1 - \frac{1}{\Lambda}\right) - \frac{\dot{\rho}_0}{\rho_0}\frac{\tau_{cc}}{\Lambda}\right]e^{-\frac{1}{2}(1+\Lambda)t/\tau_{cc}} + \rho_0\left[\frac{1}{2}\left(1 + \frac{1}{\Lambda}\right) + \frac{\dot{\rho}_0}{\rho_0}\frac{\tau_{cc}}{\Lambda}\right]e^{-\frac{1}{2}(1-\Lambda)t/\tau_{cc}} \tag{23}$$

Where the parameter Λ is defined as

$$\Lambda \equiv \left(1 - \frac{4\tau_{cc}}{\tau_\varepsilon}\right)^{1/2} \tag{24}$$

Since in the previous deduction we have assumed $t \leq \tau_{cc}$ the charge density decreases monotonically from its initial value ρ_0 during the transient regime. Thus, the time constant τ associated to the dominant pole is given by:

$$\tau = \frac{2\tau_{cc}}{1-\Lambda} \tag{25}$$

B. Critically damped case: $\tau_{cc}/\tau_\varepsilon = 1/4$ **,with solution:**

$$\rho(t) = \rho_0\left[1 + \left(\frac{1}{2} + \frac{\dot{\rho}_0\tau_{cc}}{\rho_0}\right)\frac{t}{\tau_{cc}}\right]e^{-t/2\tau_{cc}} \tag{26}$$

In this case the time constant associated with the transient period is $\tau = 2\tau_{cc}$.Note that (26) implies that for $t \ll \tau_{cc}$ the charge density is greater than its initial value ρ_0 in the transient regime, with a linear increasing rate.

C. Under damped case $\tau_{cc}/\tau_\varepsilon > 1/4$***, with solution:***

$$\rho(t) = \rho_0 e^{-t/2\tau_{cc}}\left[\cos\omega t + \frac{1}{\omega\tau_{cc}}\left(\frac{1}{2} + \frac{\dot{\rho}_0\tau_{cc}}{\rho_0}\right)\sin\omega t\right] \tag{27}$$

Where the angular frequency ω is defined as:

$$\omega = \frac{1}{2\tau_{cc}}\left(\frac{4\tau_{cc}}{\tau_\varepsilon} - 1\right)^{1/2} \tag{28}$$

And the time constant associated with the transient period is identical to the one of the critically damped case.

Note that the solutions offered in the three cases *A*, *B* and *C* are only valid for times that simultaneously verify the conditions $t \ll \tau_{cc}$ and $t \ll \tau_\varepsilon$, in accordance with the assumed hypothesis of diffusion approximation in the transient regime. The oscillation in the charge density, given by (27) implies that in the transient regime could result a movement of charges in the opposite direction to the gradient of potential, depending on the values of the roots of the characteristic equation (22).

In order to study the various dynamic cases in specific materials, the roots in (22) are evaluated by replacing the definition of the causal characteristic time τ_{cc} of (5) and the transient time τ_ε of (11), obtaining:

$$s_{1,2} \cong -\frac{c}{2nl}\left[1 \pm \left(1 - \frac{4nl\sigma}{c\varepsilon}\right)^{1/2}\right] \tag{29}$$

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where n, ε, l, and σ represent the refractive index, the electric permittivity, the characteristic length and the electrical conductivity of the medium, respectively.

In a pure silicon semiconductor, with electrical conductivity $\sigma \sim 10^{-2}$ S/m [7], and a length scale of the order of a millimeter, the dynamics of the charge density in the transient regime, associated to the roots of (29), would correspond to the over damped case given by (23), resulting the transient in times of the order from picoseconds to nanoseconds, according to the doping. Similarly for a crystal of InP or GaAs, for which $\sigma \sim 10^{-5}$ S/m and $\sigma \sim 10^{-6}$ S/m respectively [7], [8], there would be an over damped solution which in principle would be experimentally measureable, in crystals with lengths of the order of a millimeter, and microsecond lapses.

Clearly in good conductors such as copper, aluminum, gold and silver, where it is well known that their electrical conductivities are of the order of $\sigma \sim 10^{7}$ S/m, the roots of (29) correspond to the under damped case given by (27), with oscillations of a decreasing amplitude in the charge density according (28), but the equilibrium is reached almost immediately, in periods of the order of $\tau_\varepsilon \approx 10^{-19} s$ and the oscillations predicted by (27) are not observable. However, we can imagine examples where these oscillations occur with larger times. Such is the case of a silicon PN semiconductor junction, with characteristic length of about 1 cm and electrical conductivities as large as $\sigma \sim 10^{-1}$ S/m [7]. That material would present under damped oscillations in the charge density, observables in times of the order of nanoseconds.

Another plausible application of the proposed model is when a semiconductor diode is driven from the reversed biasing condition to the forward biasing state, or in the opposite direction: The diode response is accompanied by a transient, and elapses an interval of time before the diode reaches to the steady state, this lapse is frequently named "reverse recovery time". Fig. 1 shows oscillograms measured by the authors, in the transient regime of a half-wave rectifier, implemented with diodes (1N5822 and 1N5401), excited with a square wave voltage, at several frequencies.

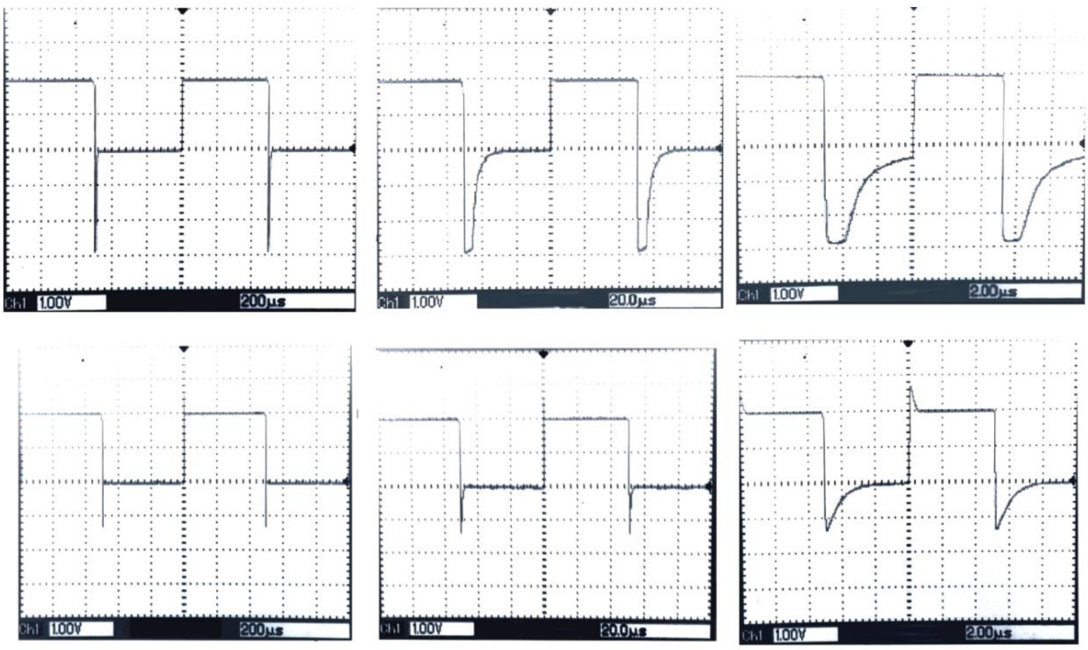

Fig. 1. Response of a half-wave rectifier driven with a square wave voltage waveform, with frequencies (from left to right) of 10 kHz, 100 kHz and 1000 kHz, respectively Above: 1N5401 diode. Below: 1N5822 Schottky diode. Note in both cases the quasi-statics transient regime (with a reverse recovery time) which is applicable in the generalization of Ohm's law in a manner consistent with causality.

Note that the oscillograms show that the reverse recovery time is on the order of 20 µs for the 1N5401 diode and only of 3 µs for the 1N5822 Schottky diode. It is noted in the two devices a reverse recovery time composed of two distinct zones phenomenological, one of quasi-constant voltage, commonly associated with storage times of the minority carriers, followed by another region with an exponential transient (transition time) corresponding to the diffusion process through the PN junction and to the capacitance of the reverse biased junction. Note that the

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transition time is about 1.6 µs for a 1N5401 diode and is about 0.4 µs for a 1N5822 Schottky diode. The characteristic times of these transients phenomena in semiconductors have the same order of magnitude as the associated periods to the time constant corresponding to the dominant pole τ in (25) or associated with the respective period of the frequency of oscillation (28), depending on the roots for the typical values in (29) for the physical length of the PN junction l and the specific conductivity of the doped silicon. So, the proposed causal generalization of Ohm's law in quasi-static transient's phenomena provides a complementary explanation to the reverse recovery time, raised in macroscopic terms, from the temporal behavior of the charge density.

Additionally, another application scenario for the proposed model emerges in the study of the ionized gases as they occur in atmospheric electrical discharges (lightning), where the electrical conductivity is a function of temperature for electronic densities in the order of 10^{12} particles/cm^3: $\sigma \sim 10^{-5} T^{3/2}$ S/m [9], and damped oscillations would be obtained on the charge density according to (27) and (28) with periods in the range of the picoseconds, which could suggest an analytical frame that explain how occur the stepped leaders observed in lightning, before the establishment of the return strokes [10].

V. CONCLUSION

The approximation of Ohm's law in its usual form (1) is valid only after the equilibrium is achieved, but this law has a non-causal behavior in quasi-static transient phenomena. To correct the non-causality implicit in Ohm's law (1), detected originally here, we conceived different constitutive models that characterize the history of electrical conduction (memory of the ohmic material). Depending on how the conductive history is modeled through the functional K, we obtained the prescriptions of behavior memory less (19), with infinite memory (20) or with a Maxwell-Boltzmann statistics memory (21); specifying the response of the conductive medium to the potential gradient applied in the times previous to the establishment of the equilibrium.

In order to describe the electric conduction in transients quasi-static phenomena, is necessary to introduce a new parameter denominated causal characteristic time τ_{cc}, as a function of the dimensions considered. Conversely, in transient phenomena with a duration of the order of τ_{cc} or greater, the charge density exhibits a dynamic behavior

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REFERENCES

[1] G. R. Kirchoff, (1849). "Über eine Ableitung der Ohm's schen Gesetze, welche sich an die Theorie der Elecktrostatik anschliesst," Annalen der Physik, 154, 12, pp. 506-513, 1849.

[2] O. Darrigol, "Electrodynamics from Ampere to Einstein," Oxford, England: Oxford University Press, 2002, pp. 70–71.

[3] C. Cattaneo, "Sulla conduzione del calore," Atti del Semin. Mat. e Fis., Univ. Modena, vol. 3, pp. 83–101, 1948.

[4] D. Jou, J. Casas-Vásquez and G. Lebon, "Extended irreversible thermodynamics revisited," Rep. Prog. Phys., vol. 62, pp. 1035-1142, 1998.

[5] L. Herrera and N. Falcón, "Heat Waves and Thermohaline Instability in a Fluid," Phys. Lett. A, vol. 201, pp. 33-37, 1995.

[6] N. Falcón and A. Medina, "Consideraciones de causalidad en el transporte de partículas por difusión," Bistua: Revista de la Facultad de Ciencias Básicas 10, 2, pp. 22-27, 2012.

[7] G. L. Pearson and J. Bardeen (1949). "Electrical Properties of Pure Silicon and Silicon Alloys Containing Boron and Phosphorus," Phys. Rev., vol. 75, pp. 865-871, 1949.

[8] D. Fisher and I. Bahl, "Gallium Arsenide IC Applications Handbook," vol. 1, Roanoke, Virginia: ITT GaAs Technology Center, 1995, pp. 22–385.

[9] J. L. Delcroix, "Introducción a la Teoría de los Gases Ionizados," Madrid, Spain, Alhambra, 1968, pp. 139.

[10] M. Uman, "The Lightning Discharge," New York, Dover Pub., 1987, pp. 8-16.

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